B-Trees

(4.7 in Weiss)

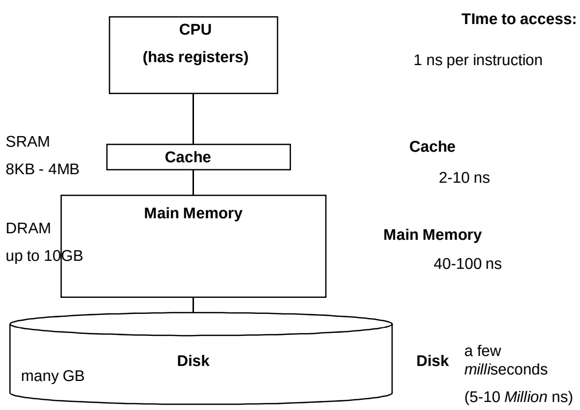
CSE 373
Data Structures & Algorithms
Ruth Anderson

Today's Outline

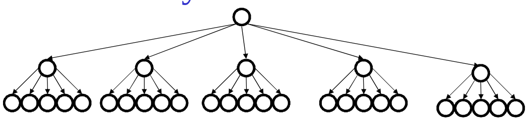
- **Admin**:
  - Final Exam – Tuesday December 11th, topic list posted soon
  - HW #6 – Sorting, due Thurs December 6 at 11pm
- **Sorting**
  - In-place and Stable Sorting
- **Dictionaries**
  - B-Trees

Trees so far

- BST
- AVL



M-ary Search Tree



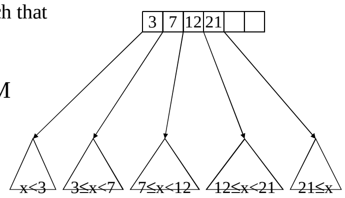
- Maximum branching factor of `M`
- <u>Complete</u> tree has height =

\# disk accesses for *find*:

Runtime of *find*:

Solution: B-Trees

- specialized *M*-ary search trees
- Each **node** has (up to) M-1 keys:
  - subtree between two keys *x* and *y* contains leaves with *values* *v* such that $x \le v < y$
- Pick branching factor M such that each node takes one full {*page, block*} of memory



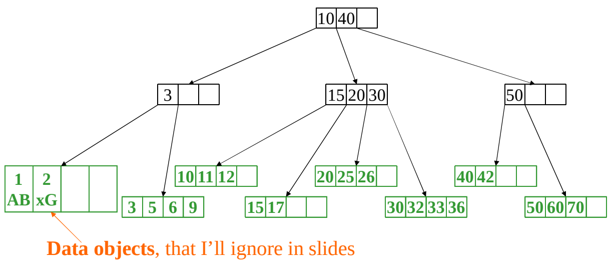
B-Trees

What makes them disk-friendly?

1. **Many keys stored in a node**
   - All brought to memory/cache in one access!
2. Internal nodes contain *only* keys;
   Only leaf nodes contain keys and actual *data*
   - The tree <u>structure</u> can be loaded into memory irrespective of data object size
   - Data actually resides in disk

B-Tree: Example

B-Tree with `M = 4` (# **<u>pointers</u>** in internal node)
and `L = 4` (# **<u>data items</u>** in <u>L</u>eaf)



Data objects, that I'll ignore in slides

Note: All leaves at the same depth!

B-Tree Properties ‡

– Data is stored at the leaves

– All leaves are at the same depth and contain between $\lceil L/2 \rceil$ and L data items

– Internal nodes store up to M-1 keys

– Internal nodes have between $\lceil M/2 \rceil$ and M children

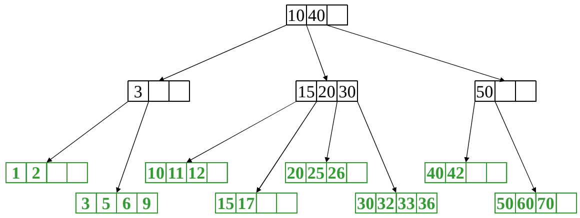
– Root (special case) has between 2 and M children (or root could be a leaf)

‡These are technically B+-Trees

Example, Again

B-Tree with **`M = 4`**

and **`L = 4`**



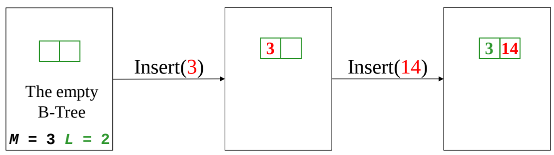
(Only showing keys, but leaves also have data!)

B-trees vs. AVL trees

Suppose we have 100 million items (100,000,000):

- Depth of AVL Tree
- Depth of B+ Tree with M = 128, L = 64

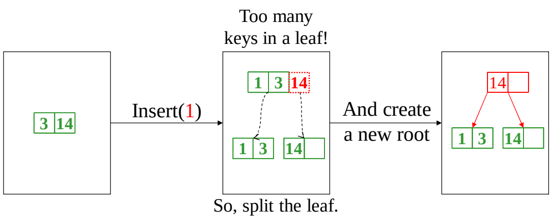
Building a B-Tree



Now, Insert(1)?

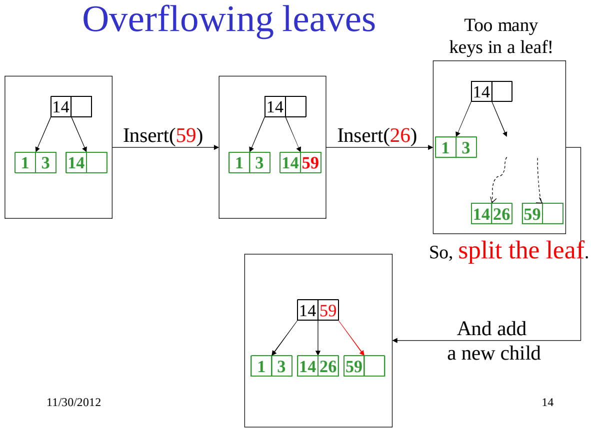
M = 3 *L* = 2

Splitting the Root



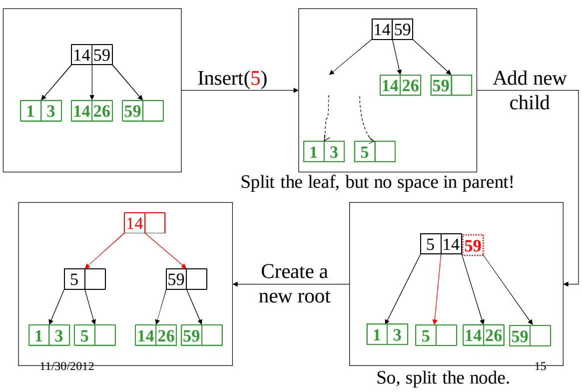
M = 3 *L* = 2

Overflowing leaves



M = 3 *L* = 2

Propagating Splits



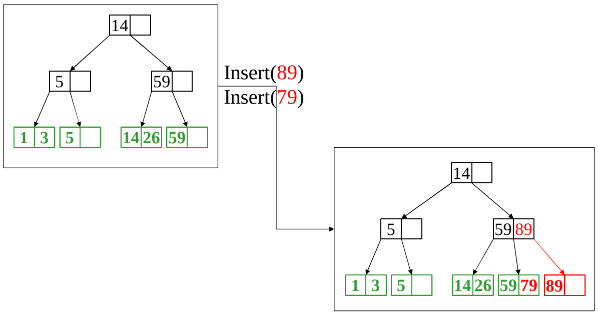
Insertion Algorithm

1. Insert the key in its leaf
2. If the **leaf** ends up with L+1 items, **overflow**!
   - **Split** the leaf into two nodes:
     - original with $\lceil (L+1)/2 \rceil$ items
     - new one with $\lfloor (L+1)/2 \rfloor$ items
   - Add the new child to the parent
   - If the parent ends up with ***M*+1** items, **overflow**!
3. If an **internal node** ends up with M+1 items, **overflow**!
   - **Split** the node into two nodes:
     - original with $\lceil (M+1)/2 \rceil$ items
     - new one with $\lfloor (M+1)/2 \rfloor$ items
   - Add the new child to the parent
   - If the parent ends up with ***M*+1** items, **overflow**!
4. **Split** an overflowed **root** in two and hang the new nodes under a new root

This makes the tree deeper!

M = 3 L = 2

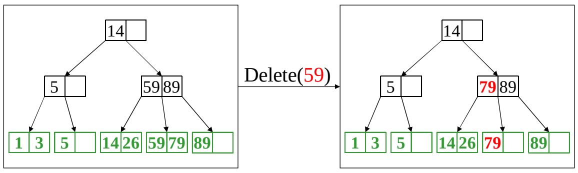
After More Routine Inserts



M = 3 L = 2

Deletion

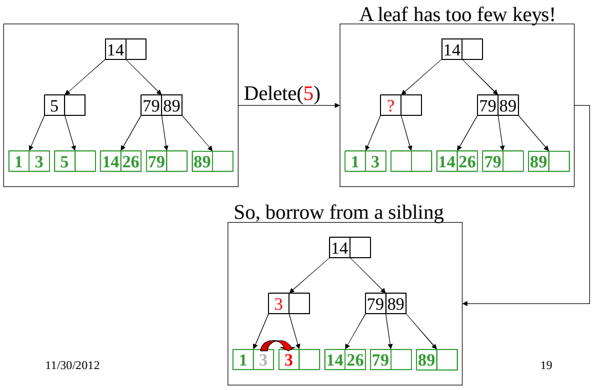
1. Delete item from leaf
2. Update keys of ancestors if necessary



What could go wrong?

M = 3 L = 2

Deletion and Adoption



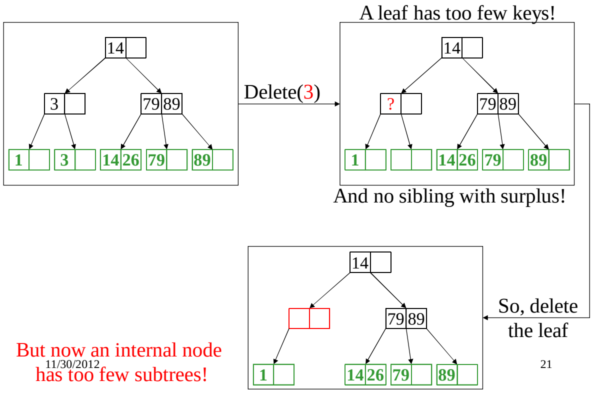
Does Adoption Always Work?

- What if the sibling doesn't have enough for you to borrow from?

  e.g. you have $\lceil L/2 \rceil$-1 and sibling has $\lceil L/2 \rceil$?

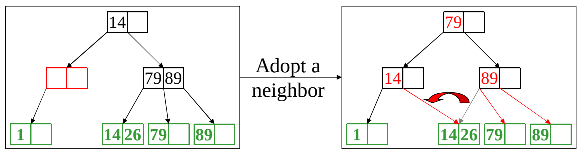
M = 3 L = 2

Deletion and Merging



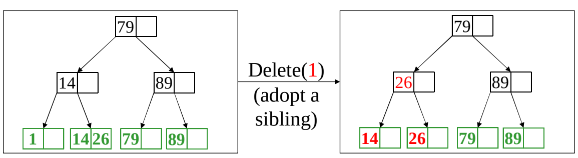
M = 3 L = 2

Deletion with Propagation (More Adoption)



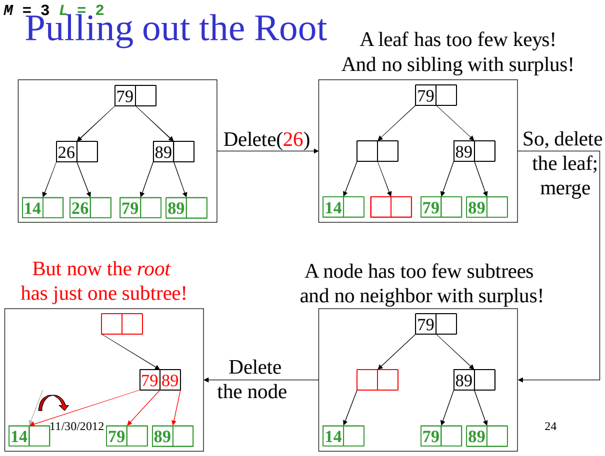
M = 3 L = 2

A Bit More Adoption



M = 3 L = 2

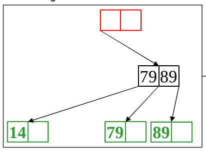
Pulling out the Root



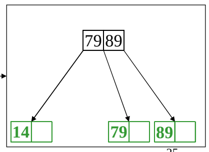
M = 3 **L = 2**

Pulling out the Root (continued)

The *root*
has just one subtree!



Simply make
the one child
the new root!



Deletion Algorithm

1. Remove the key from its leaf

2. If the **leaf** ends up with fewer than ⌈`L/2`⌉ items, **underflow**!
   - **Adopt** data from a sibling; update the parent
   - If adopting won't work, delete node and **merge** with neighbor
   - If the parent ends up with fewer than ⌈`M/2`⌉ items, **underflow**!

Deletion Slide Two

3. If an **internal** node ends up with fewer than ⌈`M/2`⌉ items, **underflow**!
   - **Adopt** from a neighbor; update the parent
   - If adoption won't work, **merge** with neighbor
   - If the parent ends up with fewer than ⌈`M/2`⌉ items, **underflow**!

4. If the **root** ends up with only one child, make the child the new root of the tree

This reduces the height of the tree!

Thinking about B-Trees

- B-Tree **insertion** can cause (expensive) splitting and propagation
- B-Tree **deletion** can cause (cheap) adoption or (expensive) deletion, merging and propagation
- Propagation is rare if **`M`** and **`L`** are large
  (Why?)
- If **`M = L = 128`**, then a B-Tree of height 4 will store at least 30,000,000 items

Tree Names You Might Encounter

FYI:

- B-Trees with ***M* = 3**, ***L* = x** are called **2-3 trees**
  - Nodes can have 2 or 3 pointers
- B-Trees with ***M* = 4**, ***L* = x** are called **2-3-4 trees**
  - Nodes can have 2, 3, or 4 pointers

Determining M and L for a B-Tree

1 Page on disk = 1 KByte

Key = 8 bytes, Pointer = 4 bytes

Data = 256 bytes per record (includes key)

M =

L =

Student Activity