Name: Key

Email address: ____________________

CSE 373 Sample Final Exam

(closed book, closed notes, calculators o.k.)

Instructions Read the directions for each question carefully before answering. We may give partial credit based on the work you **write down**, so if time permits, show your work! Use only the data structures and algorithms we have discussed in class or which were mentioned in the book so far.

Note: For questions where you are drawing pictures, please circle your final answer for any credit. There is one extra page at the end of the exam that you may use for extra space on any problem. If you detach this page it must still be turned in with your exam when you leave.

Advice You have 110 minutes, **do the easy questions first**, and work quickly!

1) Short Answer Problems: *Be sure to answer all parts of the question!!*

a) For large input sizes, mergesort will always run faster than insertion sort (on the same input). **True** [✓ **False**]

b) You could use an AVL tree to do a sort. Describe how you would do this. What is the worst-case running time for your sort and why? (***Be specific***)

1) Insert all values into AVL tree ← { N insertions
{ $O(\log N)$ per insertion

2) Do an inorder traversal ← $O(N)$

Worst Case = $O(N \log N + N) = O(N \log N)$

c) Suppose we choose the median of five items as the pivot in quicksort. If we have an N element array, then we find the median of the elements located at the following positions: left (= 0), right (= N – 1), center (the average of left and right, rounded down), leftOfCenter (the average of left and center, rounded down), and rightOfCenter (the average of right and center, rounded down). The median of these elements is the pivot.

What is the worst case running time of *this version* of quicksort?

$O(N^2)$

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2) Minimum spanning tree (MST)

Given a weighted, undirected graph with |V| nodes, answer the following as best as possible, with a brief explanation. Assume all weights are non-negative.

a) If each edge has weight ≤ *w*, what can you say about the cost of an MST?

Total Cost $\leq \left(|V| - 1 \right) \cdot w$

b) If the cost of an MST is *c*, what can you say about the shortest distances returned by Dijkstra's algorithm when run with an arbitrary vertex *s* as the source?

Max Distance $= c$

3) Sorting:

a) Show the steps required to do a radix sort on the following set of values when using base 10.

346, 22, 31 212 157 102 568 435 8 14 5

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
	31	22 212 102		14	435 5	346	157	568 8	

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
102 5 8	212 14	22	31 435	346	157	568			

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
5 8 14 22 31	102 157	212	346	435	568				

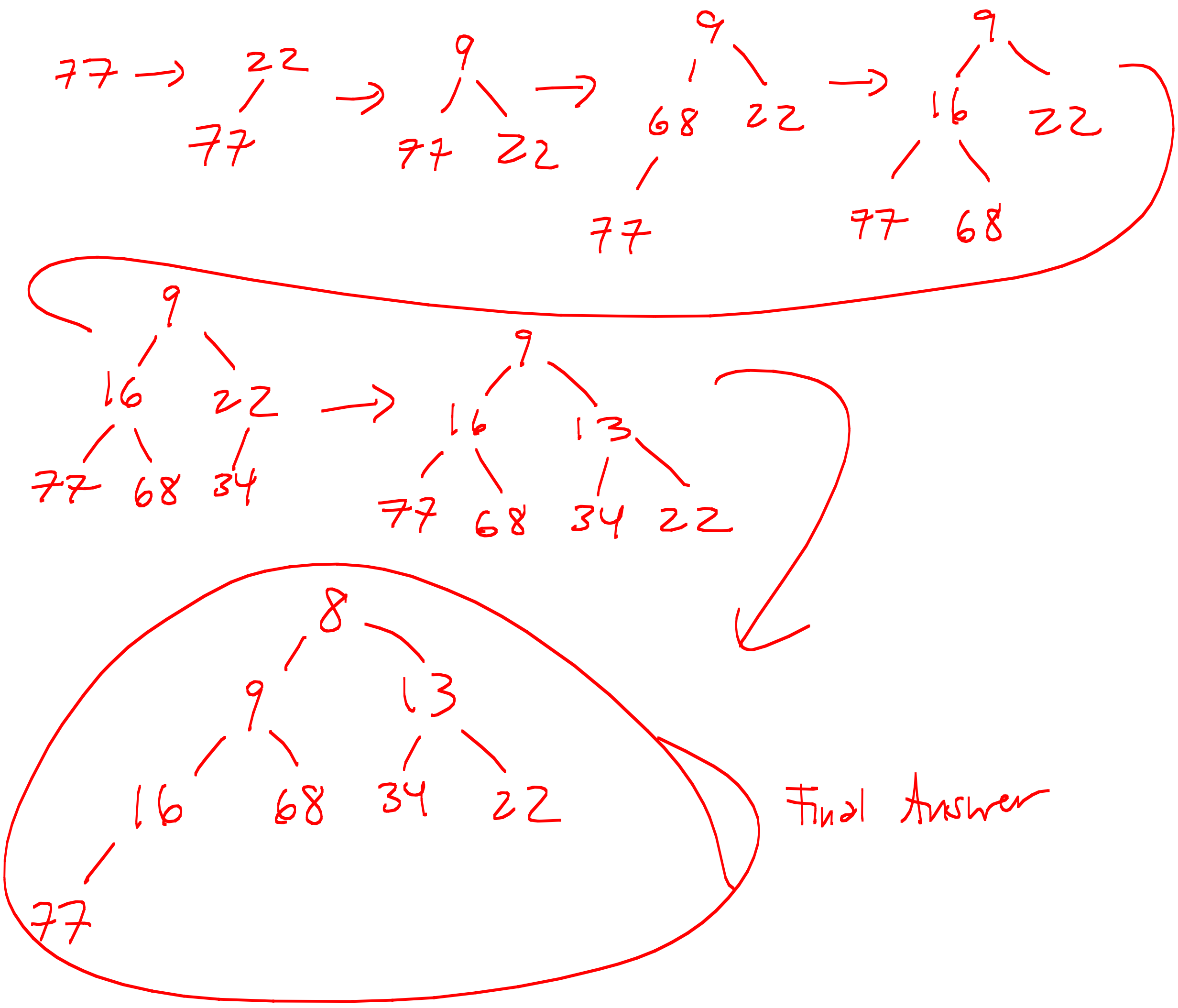
b) What is the running time of radix sort? Describe any variables you use other than N.

$O(P \cdot (N+K))$ Radix (in this case 10)

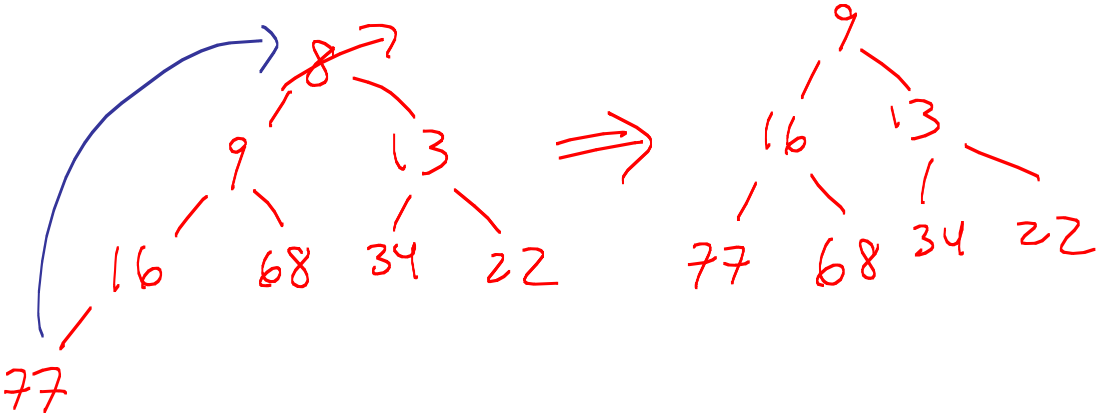
\# phases = $\log_K(\text{MaxNum})$

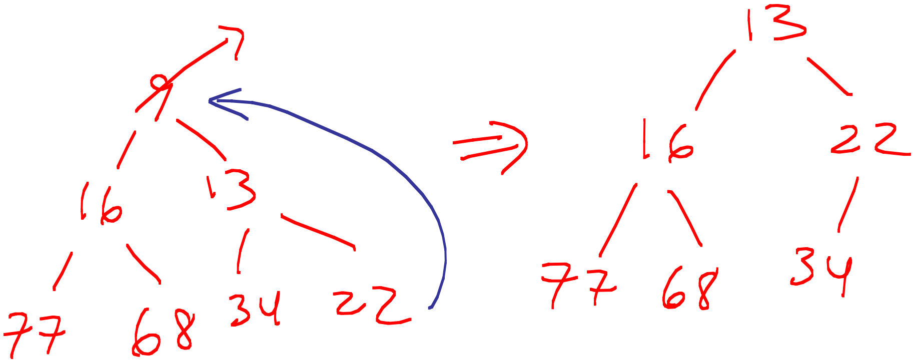
4) Heaps

a) Draw the binary min heap that results from inserting: 77, 22, 9, 68, 16, 34, 13, 8 in that order into an initially empty binary min heap. You do not need to show the array representation of the heap. You are only required to show the final heap, although if you draw intermediate heaps, ***please circle your final result for ANY credit.***



b) Draw the binary min heap that results from doing 2 deletemins on the heap you created in part a). You are only required to show the final heap, although if you draw intermediate heaps **please circle your final result for ANY credit**.



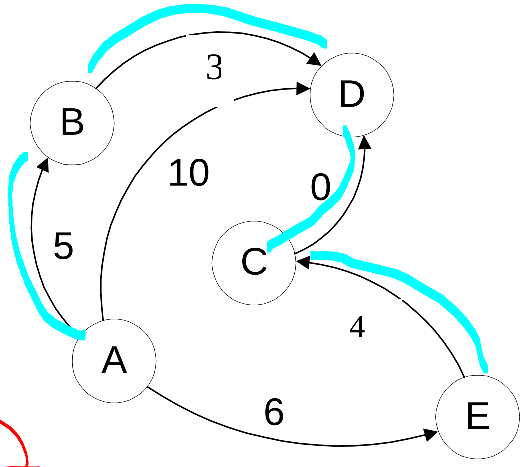


c) What is the null path length of the root node in the last heap you drew in part b) above?

one

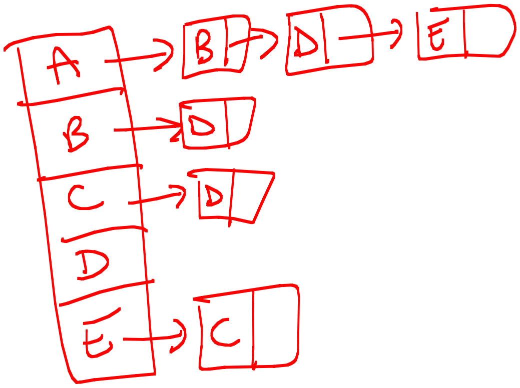
5) Graph Manipulations:

Use the following graph for this problem. **Where needed and not determined by the algorithm, assume that any algorithm begins at node A.**



a) Draw both the adjacency matrix and adjacency list representations of this graph. Be sure to specify which is which.

	A	B	C	D	E
A	F	T	F	T	T
B	F	F	F	T	F
C	F	F	F	T	F
D	F	F	F	F	F
E	F	F	T	F	F



b) Give two valid topological orderings of the nodes in the graph.

ABECD

AEBCD

AECBD

c) Step through Dijkstra's Algorithm to calculate the single source shortest path from A to every other vertex. You only need to show your final table, but you should show your steps in the table below for partial credit. Show your steps by crossing through values that are replaced by a new value. *Note that the next question asks you to recall what order vertices were declared known.*

Vertex	**Known**	**Distance**	**Path**
A	T	0	—
B	T	5	A
C	T	~~∞~~ 10	E
D	T	~~10~~ 8	~~A~~ B
E	T	6	A

d) In what order would Dijkstra's algorithm mark each node as *known***?**

A B E D C

e) What is the shortest (weighted) path from A to D?

ABD

f) What is the length (weighted cost) of the shortest path you listed in part (e)?

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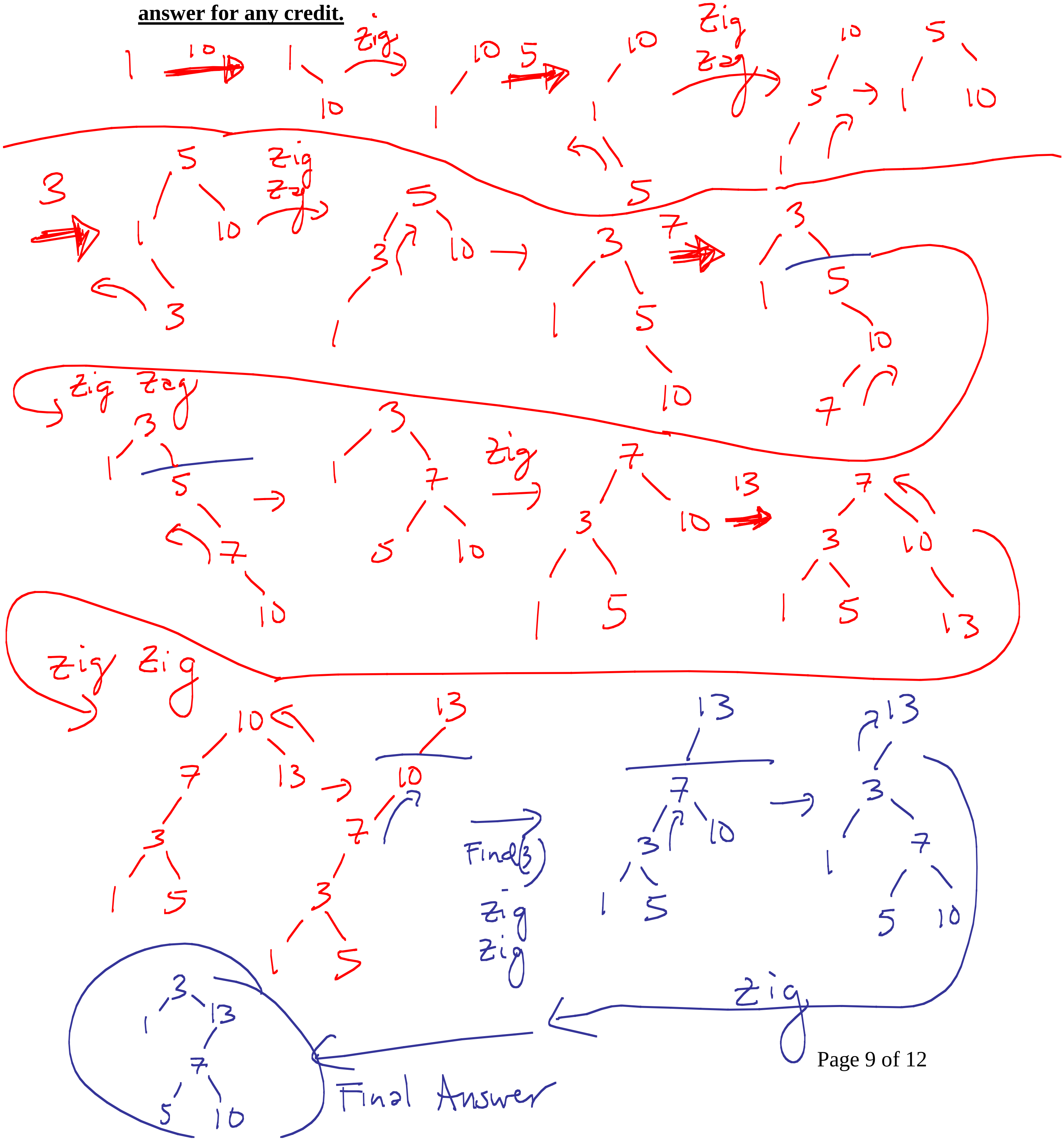
g) Imagine that the graph were undirected (i.e., ignore the directions of the edges). Highlight the MST(minimum Spanning Tree) on the graph above or redraw it here.

(see above)

6) Splay trees

Imagine that the following operations are performed on an initially empty splay tree:
Insert(1), Insert(10), Insert (5), Insert (3), Insert (7), Insert (13), Find (3).
Show the state of the splay tree after performing each of the above operations. Be sure to label each of your trees with what operations you have just completed. **Circle your final answer for any credit.**



7) Hash Tables: Draw the contents of the hash table in the boxes below given the following conditions:

The size of the hash table is 11.
Open addressing and double hashing is used to resolve collisions.
The hash function used is H(k) = k mod 11
The second hash function is H2(k) = 5 – (k mod 5)

What values will be in the hash table after the following sequence of insertions? Draw the values in the boxes below, and show your work for partial credit.

16, 23 9, 34, 12, 56

0		
1	23	← 34_0 ← 12_0
2	34_1	← 56_0 56_3
3		
4	12_1	
5	16	← 56_1
6	56_4	
7		
8		
9	9	← 56_2
10		

16 mod 11 = 5
23 mod 11 = 1
9 mod 11 = 9
34 mod 11 = 1
H2(34) = 5 − (34 mod 5)
= 5 − 4 = 1
1 + (1 * 1) = 2
12 mod 11 = 1
H2(12) = 5 − (12 mod 5)
= 5 − 2 = 3
1 + (1 * 3) = 4
56 mod 11 = 1
H2(56) = 5 − (56 mod 5)
= 5 − 1 = 4
1 + (1 * 4) = 5
1 + (2 * 4) = 9
1 + (3 * 4) = 13 mod 11
= 2
1 + (4 * 4) = 17 mod 11
= 6

8) Hash Tables:

What is the suggested load factor (according to our text) for a hash table that uses open addressing?

$$\lambda = \frac{1}{2}$$

9) Memory

Pick one of the data structures we have studied so far this quarter and describe when/whether it has **spatial** and **temporal** locality:

(Several possible answers)

$L = \frac{2048}{512} = \frac{2^{11}}{2^{9}} = 2^{2} = 4$

10) B-trees

a) Given the following parameters:

1 Page on disk = 2048 bytes

Disk access time = 1milli-sec per byte

Pointer = 4 bytes

Key = 8 bytes

Data = 512 bytes per record (includes key)

$4 \cdot M + 8(M-1) = 2048$

$12M = 2056$

$M \le 171$

What are the best values for:

M = 171

L = 4

b) Insert the following values in this order, into a B-tree where M = 3 and L =2.
8, 4, 6, 2, 3, 1, 7, 9