CSE373: Data Structures & Algorithms

Lecture 5: Dictionary ADTs; Binary Trees

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Winter 2015

Today's Outline

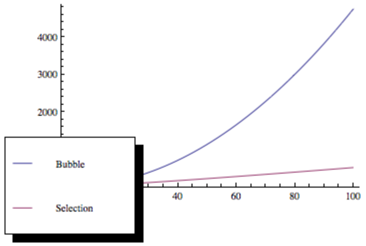
Announcements

- Homework 1 due TODAY at 11:59 pm ☺
- Homework 2 out (paper and pencil assignment)
  - Due in class Wednesday Jan. 21 at the START of class

Today's Topics

- Finish Asymptotic Analysis
- Dictionary ADT (a.k.a. Map): associate keys with values
  – Extremely common
- Binary Trees

Summary of Asymptotic Analysis



Analysis can be about:

- The problem or the algorithm (usually algorithm)
- Time or space (usually time)
  - Or power or dollars or …
- Best-, worst-, or average-case (usually worst)
- Upper-, lower-, or tight-bound (usually upper)

- The most common thing we will do is give an O upper bound to the worst-case running time of an algorithm.

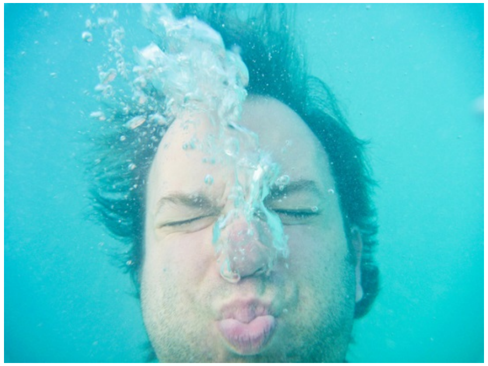
Big-Oh Caveats

- Asymptotic complexity focuses on behavior for large *n* and is independent of any computer / coding trick
- But you can "abuse" it to be misled about trade-offs
- Example: $n^{1/10}$ vs. **`log`** *n*
  - Asymptotically $n^{1/10}$ grows more quickly
  - But the "cross-over" point is around $5 * 10^{17}$
  - So if you have input size less than 2^{58}, prefer $n^{1/10}$
- For *small n*, an algorithm with worse asymptotic complexity might be faster
  - If you care about performance for small *n* then the constant factors can matter

Addendum: Timing vs. Big-Oh Summary

- Big-oh is an essential part of computer science's mathematical foundation
  - Examine the algorithm itself, not the implementation
  - Reason about (even prove) performance as a function of *n*
- Timing also has its place
  - Compare implementations
  - Focus on data sets you care about (versus worst case)
  - Determine what the constant factors "really are"

Let's take a breath



- So far we've covered
  - Some simple ADTs: stacks, queues, lists
  - Some math (proof by induction)
  - How to analyze algorithms
  - Asymptotic notation (Big-Oh)

- Coming up….
  - Many more ADTs
    - Starting with dictionaries

The Dictionary (a.k.a. Map) ADT

- Data:
  - set of (key, value) pairs
  - keys must be comparable

- Operations:
  - **`insert(key,value)`**
  - **`find(key)`**
  - **`delete(key)`**
  - **…**

insert(eden, ….) →

find(megan) ←

Megan Hopp, …

- **eden**
  Eden Ghirmai
  OH: Fri 4.30-5.20
  …
- **rama**
  Rama Gokhale
  OH: Fri 1.30-2.20
  …
- **megan**
  Megan Hopp
  OH: Mon 10:30-11:20
  …

Will tend to emphasize the keys;
don't forget about the stored values

A Modest Few Uses

Any time you want to store information according to some key and be able to retrieve it efficiently

- Lots of programs do that!

• Search:	inverted indexes, phone directories, …
• Networks:	router tables
• Operating systems:	page tables
• Compilers:	symbol tables
• Databases:	dictionaries with other nice properties
• Biology:	genome maps
• …	

Possibly the most widely used ADT

Simple implementations

For dictionary with *n* key/value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list	$O(1)$*	$O(n)$	$O(n)$
• Unsorted array	$O(1)$*	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O(\log n)$	$O(n)$

* Unless we need to check for duplicates

We'll see a Binary Search Tree (BST) probably does better

but not in the worst case (unless we keep it balanced)

Lazy Deletion

10	**12**	**24**	**30**	**41**	**42**	**44**	**45**	**50**
✓	✗	✓	✓	✓	✓	✗	✓	✓

A *general technique* for making **`delete`** as fast as **`find`**:

- Instead of actually removing the item just mark it deleted

Plusses:

- Simpler
- Can do removals later in batches
- If re-added soon thereafter, just unmark the deletion

Minuses:

- Extra *space* for the “is-it-deleted” flag
- Data structure full of deleted nodes wastes *space*
- May complicate other operations

Better dictionary data structures

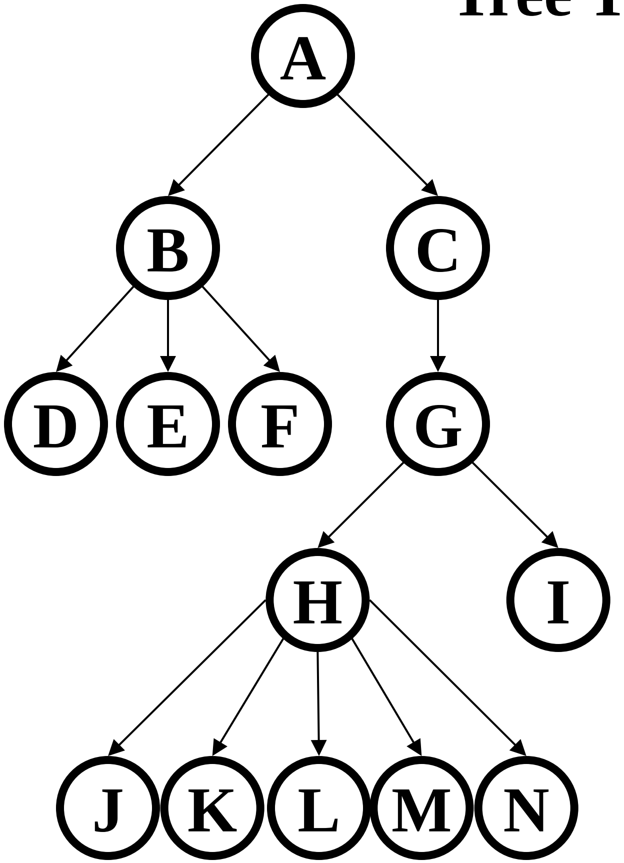
There are many good data structures for (large) dictionaries

1. Binary trees
2. AVL trees
   - Binary search trees with *guaranteed balancing*
3. B-Trees
   - Also always balanced, but different and shallower
   - B-Trees are not the same as Binary Trees
     - B-Trees generally have large branching factor
4. Hash Tables
   - Not tree-like at all

Skipping: Other balanced trees (e.g., red-black, splay)

Tree terms (review?)

Tree T



Root **(tree)**

Leaves **(tree)**

Children **(node)**

Parent **(node)**

Siblings **(node)**

Ancestors **(node)**

Descendents **(node)**

Subtree **(node)**

Depth **(node)**

Height **(tree)**

Degree **(node)**

Branching factor **(tree)**

More tree terms

- There are many kinds of trees
  - Every binary tree is a tree
  - Every list is kind of a tree (think of “next” as the one child)

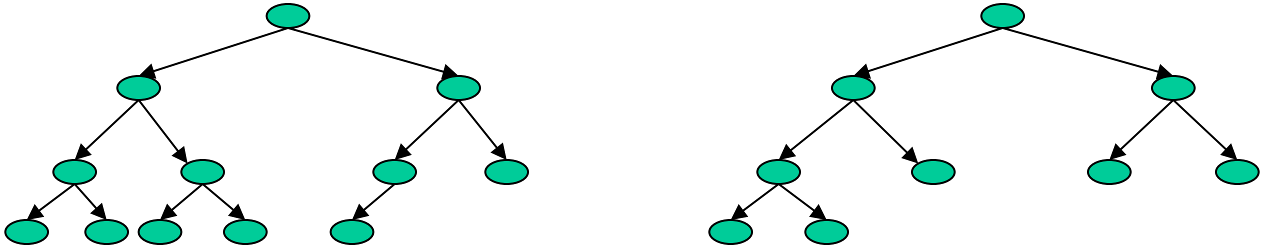
- There are many kinds of binary trees
  - Every binary search tree is a binary tree
  - Later: A binary heap is a different kind of binary tree

- A tree can be balanced or not
  - A balanced tree with *n* nodes has a height of $O(\mathbf{log}\ n)$
  - Different tree data structures have different “balance conditions” to achieve this

Kinds of trees

Certain terms define trees with specific structure

- Binary tree: Each node has at most 2 children (branching factor 2)
- *n*-ary tree: Each node has at most *n* children (branching factor *n*)
- Perfect tree: Each row completely full
- Complete tree: Each row completely full except maybe the bottom row, which is filled from left to right



What is the height of a perfect binary tree with n nodes?

A complete binary tree?

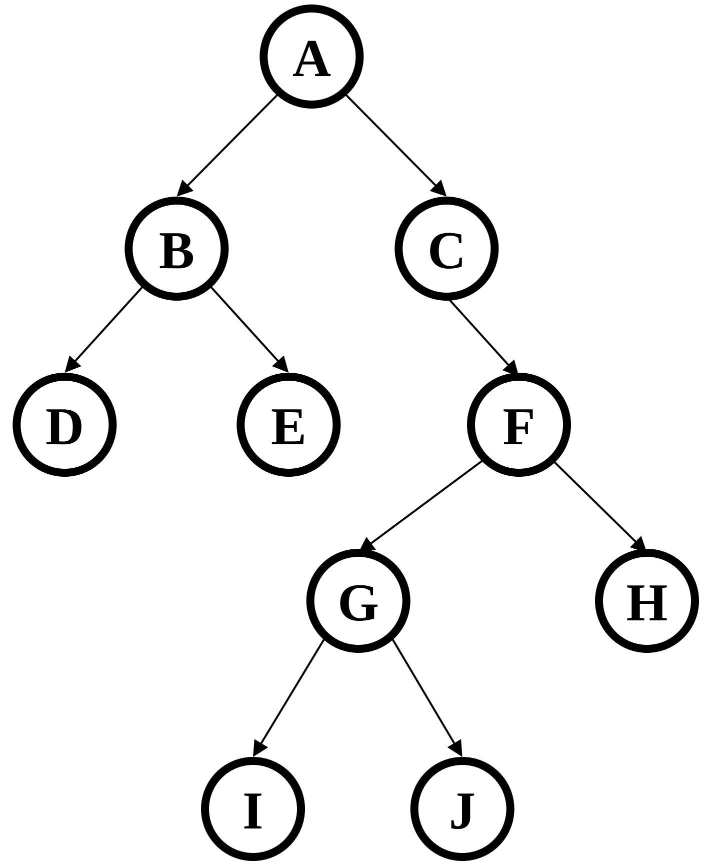
Binary Trees

- Binary tree: Each node has at most 2 children (branching factor 2)

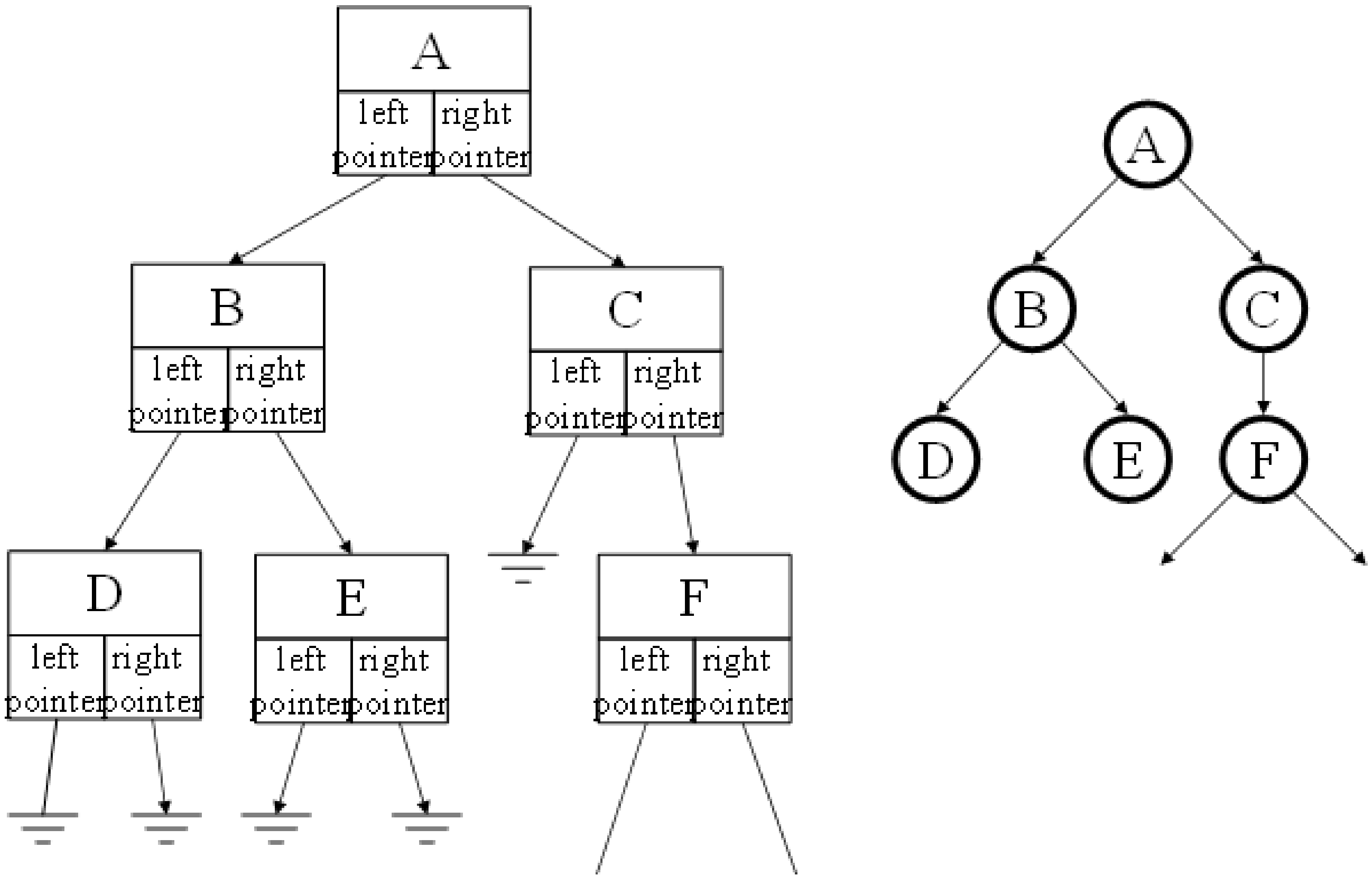
- Binary tree is
  - A root *(with data)*
  - A left subtree that's a binary tree
  - A right subtree that's a binary tree
- *These subtrees may be empty.*
- Representation:

Data	
left pointer	**right pointer**

- For a dictionary, data will include a key and a value



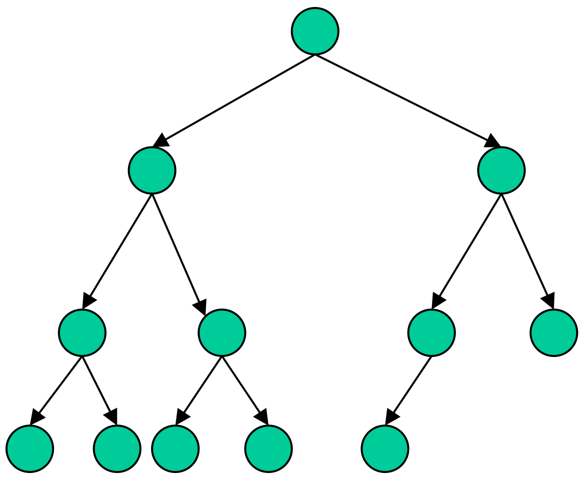
Binary Tree Representation



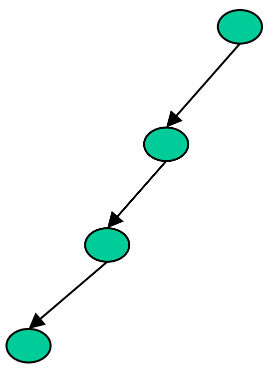
Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:



- max # of leaves: 2^h
- max # of nodes: $2^{(h + 1)} - 1$
- min # of leaves: 1
- min # of nodes: $h + 1$



For n nodes, we cannot do better than O(`log` n) height and we want to avoid O(n) height

Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {

              ???


}
```

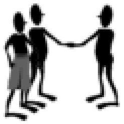
Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {
  if(root == null)
    return -1;
  return 1 + max(treeHeight(root.left),
                 treeHeight(root.right));
}
```

Running time for tree with n nodes: $O(n)$ – single pass over tree

Note: non-recursive is painful – need your own stack of pending nodes; much easier to use recursion's call stack



Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

- *Pre-order*: root, left subtree, right subtree

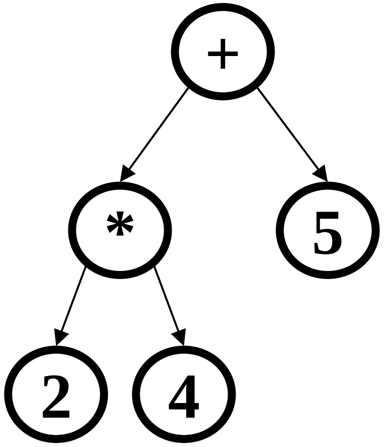
  + * 2 4 5

- *In-order*: left subtree, root, right subtree

  2 * 4 + 5

- *Post-order*: left subtree, right subtree, root

  2 4 * 5 +



(an expression tree)

More on traversals

```
void inOrderTraversal(Node t){
  if(t != null) {
    inOrderTraversal(t.left);
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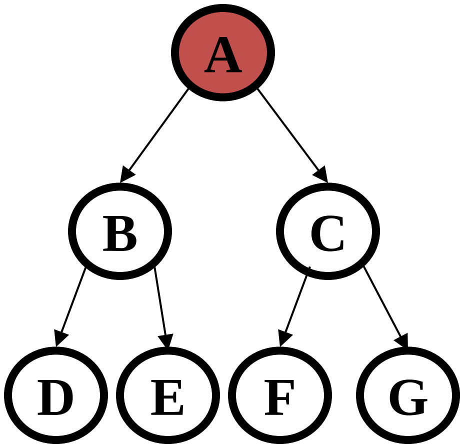

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A = completed node ✓= element has been processed

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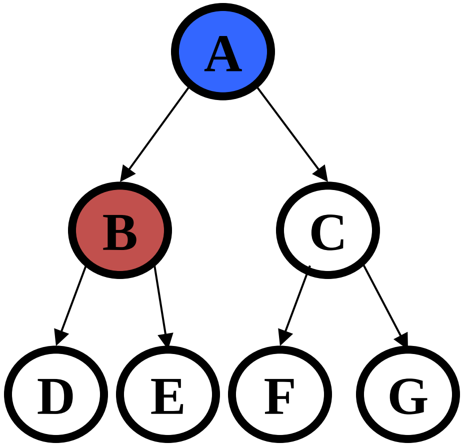

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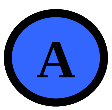

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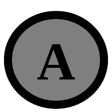

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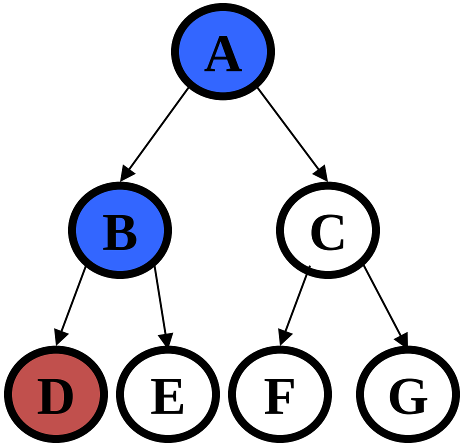

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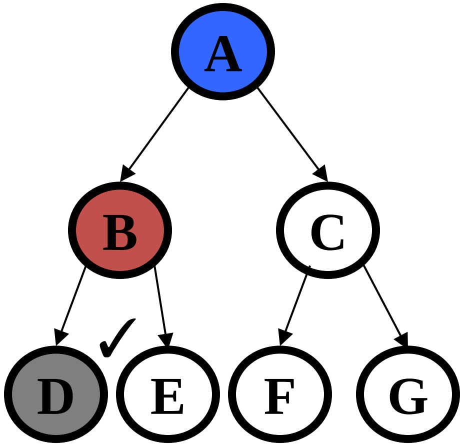

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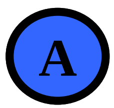

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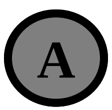

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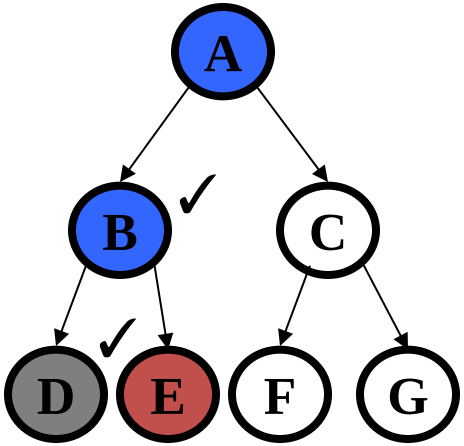

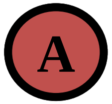

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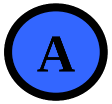

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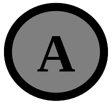

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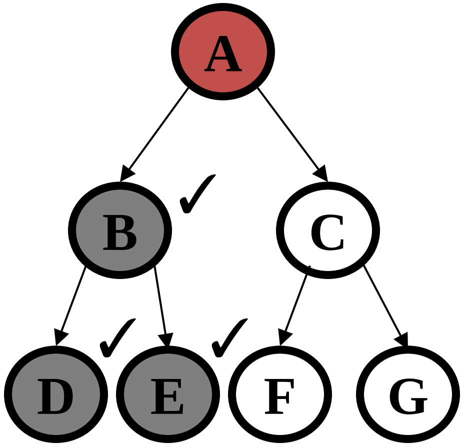

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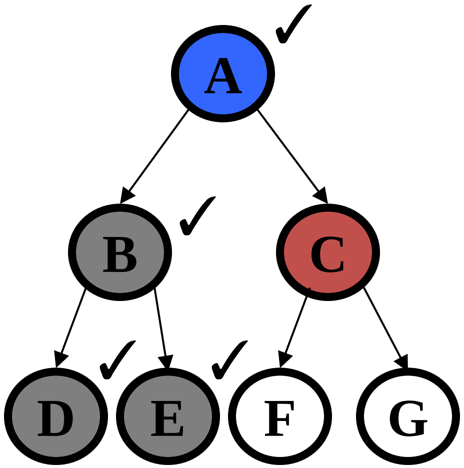

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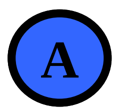

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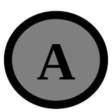

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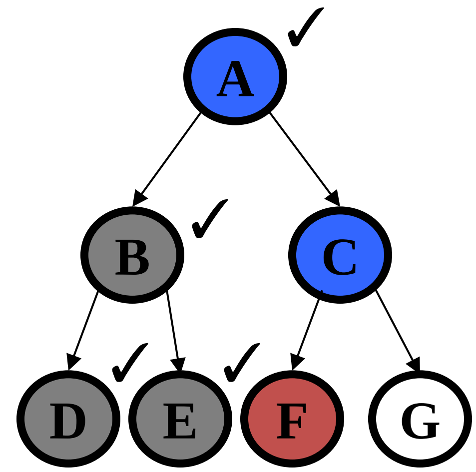

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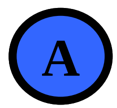

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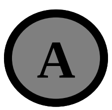

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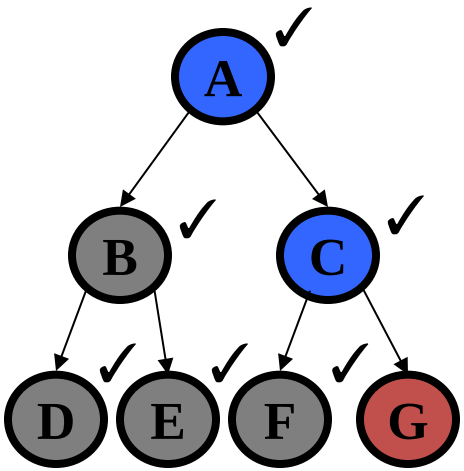

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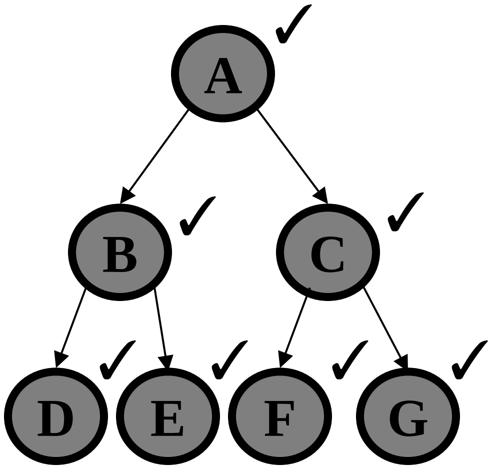

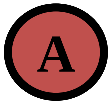

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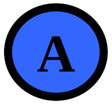

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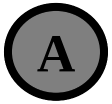

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