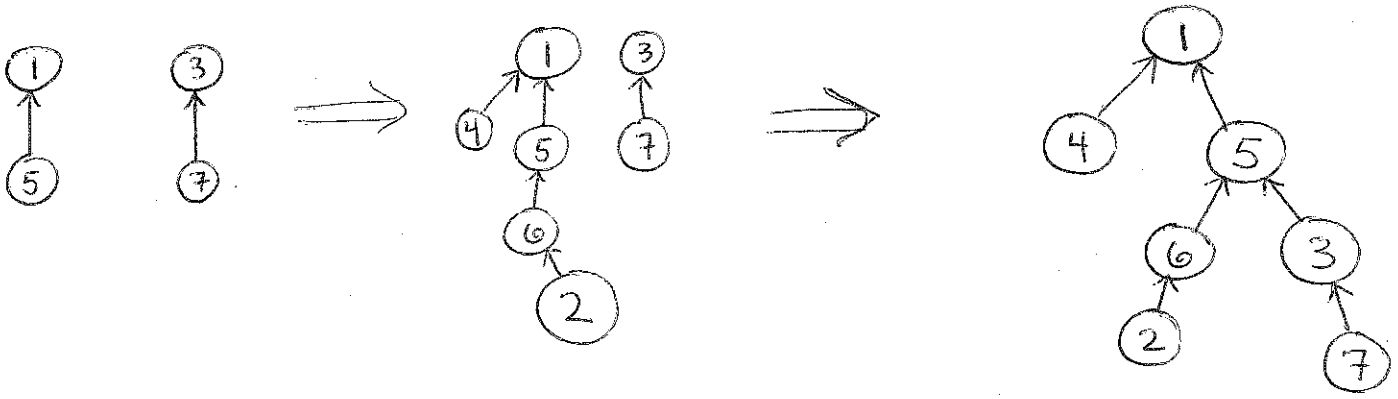
T.A. Help Session: Union Find
ANSWER KEY

1. a. Show the resulting up Up-Tree data structure after the following unions (no union-by-size):

union(1,5), union(3,7), union(5,6), union(1,4), union(6,2), union(5,3)



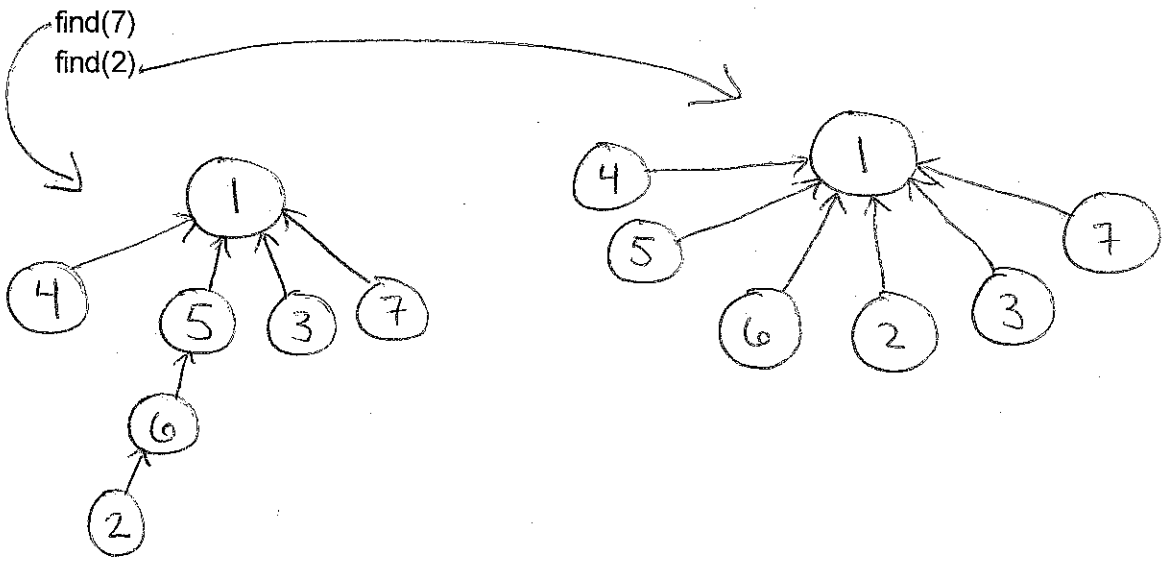
b. Fill in the resulting up-tree array implementation with

	1	2	3	4	5	6	7
up	**0**	**6**	**5**	**1**	**1**	**5**	**3**
weight	**7**	**0**	**0**	**0**	**0**	**0**	**0**

What is the worst-case runtime of a union operation? O(1)

find operation? O(N)

c. Now show the resulting up-tree data structure after the following finds, utilizing path compression:



2. Pseudocode

a. Write pseudocode for a basic union operation:

```
int[] up;
public void union(int x, int y) {
      up[y] = x;
}
```

b. Write pseudocode for a union-by-weight operation:

```
int[] up;
int[] weight;
// Assume x and y are roots
public void unionByWeight(int x, int y) {
      if (weight[x] > weight[y]) {
            up[y] = x;
            weight[x] += weight[y];
      } else {
            up[x] = y;
            weight[y] += weight[x];
      }
}

// OPTIMIZED, assumption that x and y are roots
int[] up
public void unionByWeight(int x, int y) {
      if (up[x] < up[y]) {  // x has heavier *negative* weight
            up[x] += up[y];
            up[y] = x;
      } else {
            up[y] += up[x];
            up[x] = y;
      }

}
```

c. Write pseudocode for a basic find operation:

```
int[] up;
public int find(int x) {
      while (up[x] > 0) {
            x = up[x];
      }
      return x;
}
```

d. Write pseudocode for a find operation, implementing path compression:

```
int[] up;
public int findWithPathCompression(int x) {
      int root = x;
      while (up[root] > 0)
            root = up[root];

      if (root == x)
            return root;

      int oldParent = up[x];
      while (oldParent != root) {
            up[x] = root;
            x = oldParent;
            oldParent = up[x];
      }
      return root;
}
```