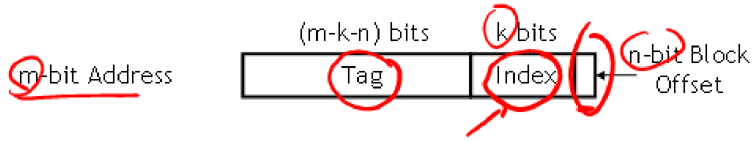
Lecture 16

- Today:
  - We can do a lot better than direct mapped!
  - Save 10 minutes for midterm questions?

Finding the location within the cache

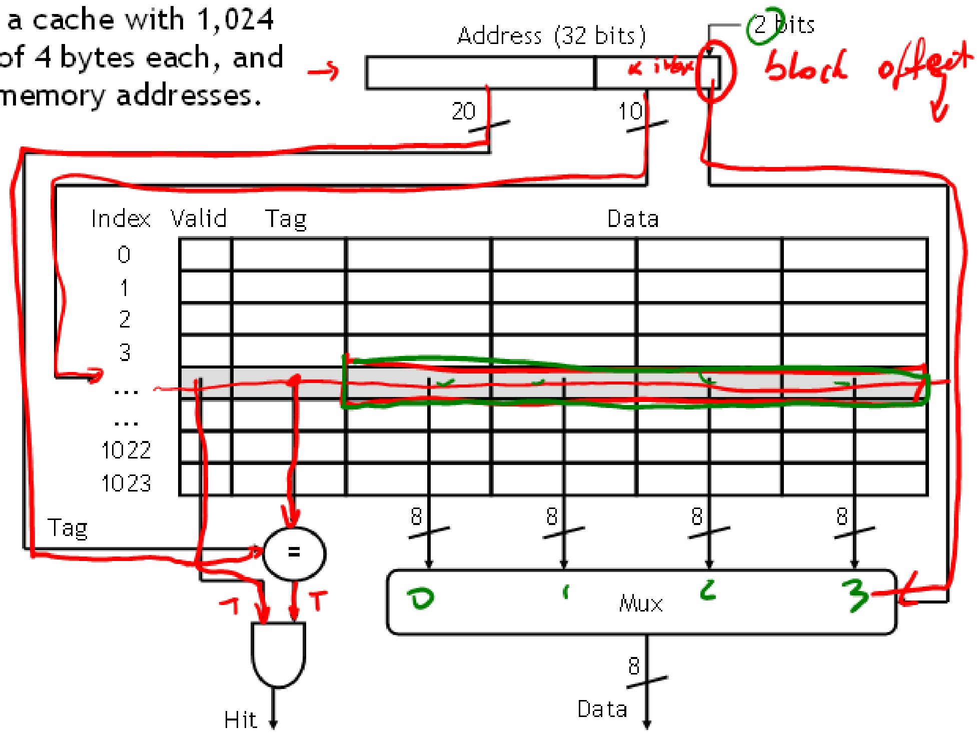
- An equivalent way to find the right location within the cache is to use arithmetic again.



- We can find the index in two steps, as outlined earlier.
  - Do integer division of the address by 2^n to find the block address.
  - Then mod the block address with 2^k to find the index.
- The block offset is just the memory address mod 2^n.
- For example, we can find address 13 in a 4-block, 2-byte per block cache.
  - The block address is 13 / 2 = 6, so the index is then 6 mod 4 = 2.
  - The block offset would be 13 mod 2 = 1.

A diagram of a larger example cache

- Here is a cache with 1,024 blocks of 4 bytes each, and 32-bit memory addresses.

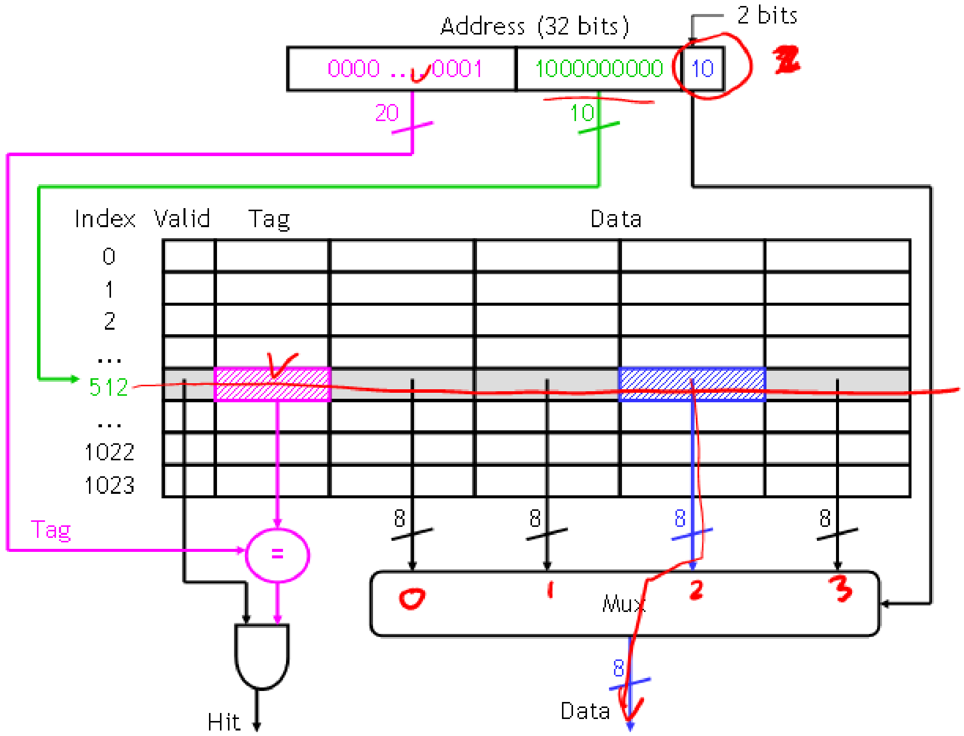


A larger example cache mapping

- Where would the byte from memory address 6146 be stored in this direct-mapped 2^{10}-block cache with 2^2-byte blocks?

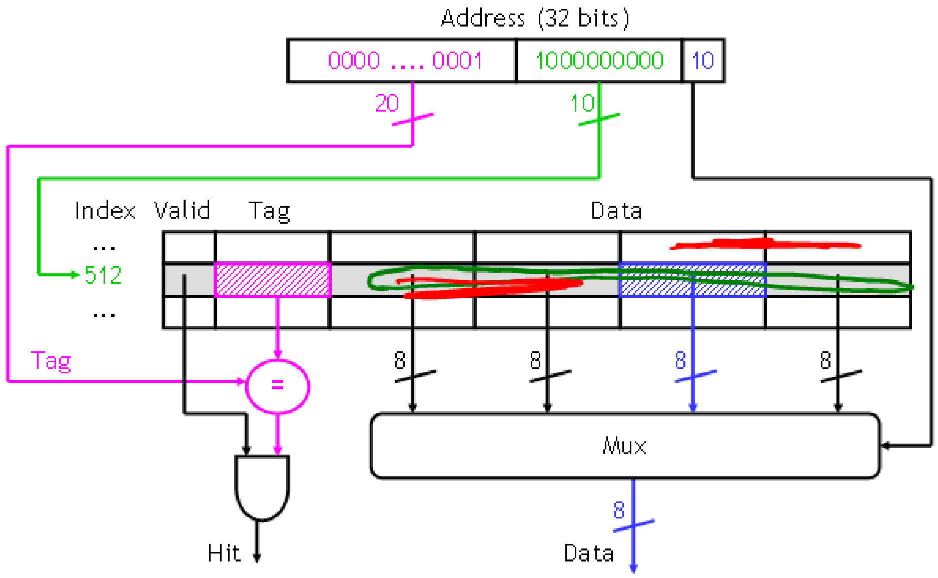
- 6146 in binary is 00...01 1000 0000 00 10.

A larger diagram of a larger example cache mapping



What goes in the rest of that cache block?

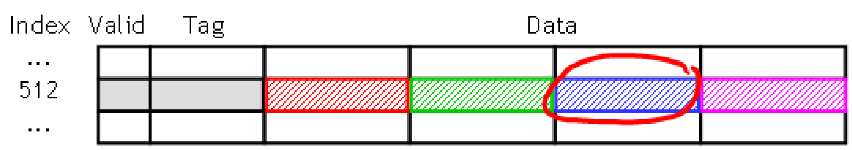
- The other three bytes of that cache block come from the same memory block, whose addresses must all have the same index (1000000000) and the same tag (00...01).



The rest of that cache block

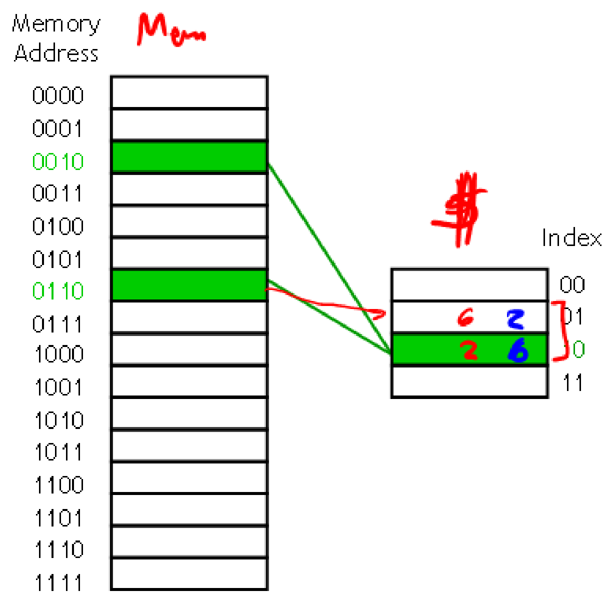
- Again, byte *i* of a memory block is stored into byte *i* of the corresponding cache block.
  - In our example, memory block 1536 consists of byte addresses 6144 to 6147. So bytes 0-3 of the cache block would contain data from address 6144, 6145, 6146 and 6147 respectively.
  - You can also look at the lowest 2 bits of the memory address to find the block offsets.

Block offset	Memory address	Decimal
00	00..01 1000000000 00	6144
01	00..01 1000000000 01	6145
10	00..01 1000000000 10	6146
11	00..01 1000000000 11	6147



Disadvantage of direct mapping

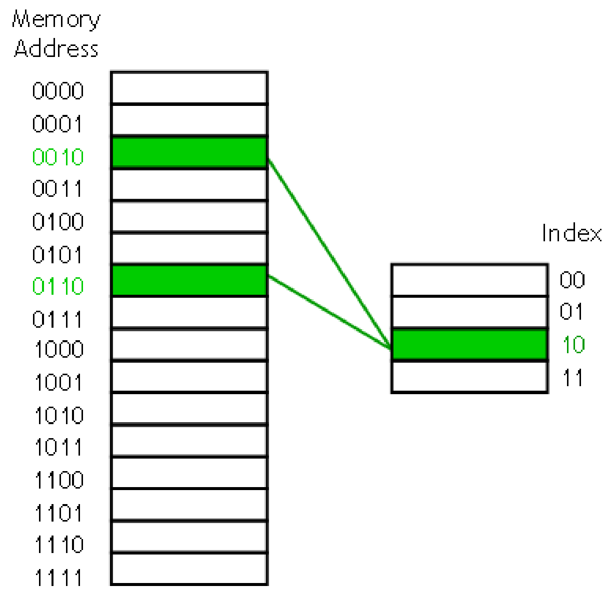
- The direct-mapped cache is easy: indices and offsets can be computed with bit operators or simple arithmetic, because each memory address belongs in exactly one block.
- But, what happens if a program uses addresses 2, 6, 2, 6, 2, …?



How do we solve this problem?

Disadvantage of direct mapping

- The direct-mapped cache is easy: indices and offsets can be computed with bit operators or simple arithmetic, because each memory address belongs in exactly one block.
- However, this isn't really flexible. If a program uses addresses 2, 6, 2, 6, 2, ..., then each access will result in a cache miss and a load into cache block 2.
- This cache has four blocks, but direct mapping might not let us use all of them.
- This can result in more misses than we might like.

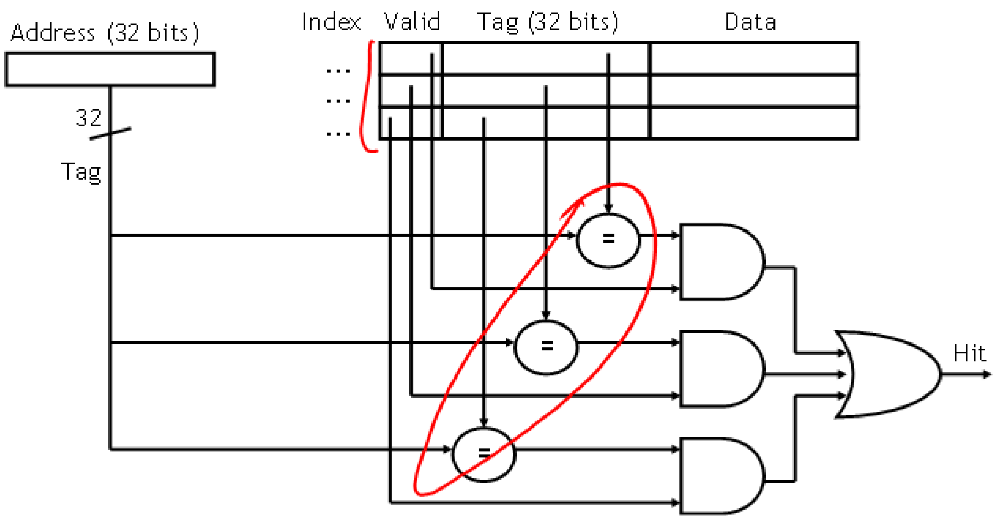


A fully associative cache

- A fully associative cache permits data to be stored in *any* cache block, instead of forcing each memory address into one particular block.
  - When data is fetched from memory, it can be placed in *any* unused block of the cache.
  - This way we'll never have a conflict between two or more memory addresses which map to a single cache block.
- In the previous example, we might put memory address 2 in cache block 2, and address 6 in block 3. Then subsequent repeated accesses to 2 and 6 would all be hits instead of misses.
- If all the blocks are already in use, it's usually best to replace the least recently used one, assuming that if it hasn't used it in a while, it won't be needed again anytime soon.

The price of full associativity

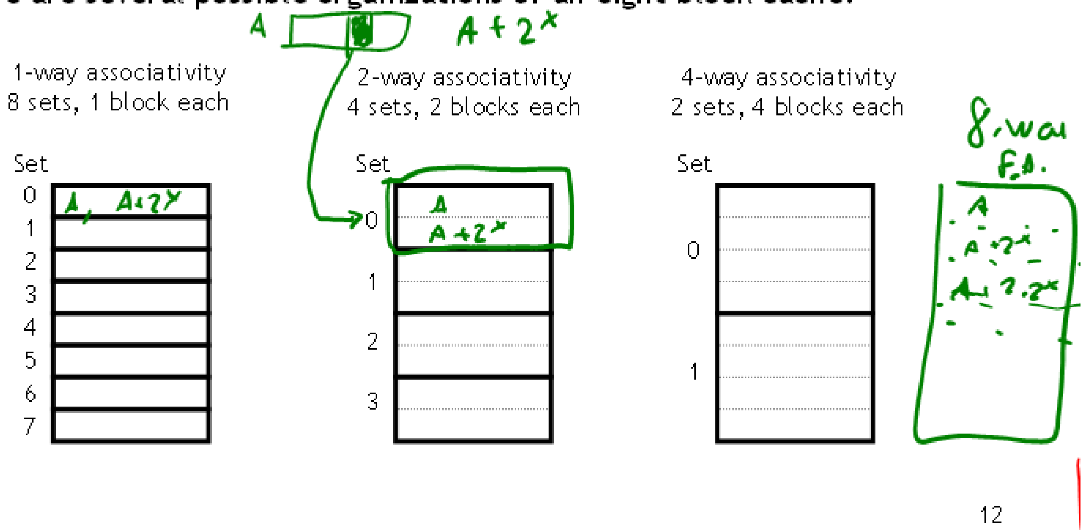
- However, a fully associative cache is expensive to implement.
  - Because there is no index field in the address anymore, the *entire* address must be used as the tag, increasing the total cache size.
  - Data could be anywhere in the cache, so we must check the tag of *every* cache block. That's a lot of comparators!



Hmm, how do we get the best of both worlds?

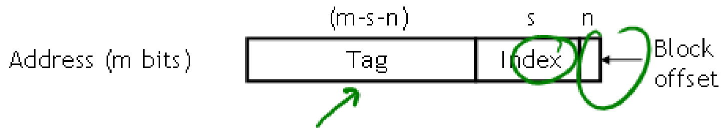
Set associativity

- An intermediate possibility is a set-associative cache.
  - The cache is divided into *groups* of blocks, called sets.
  - Each memory address maps to exactly one set in the cache, but data may be placed in any block within that set.
- If each set has 2^x blocks, the cache is an 2^x-way associative cache.
- Here are several possible organizations of an eight-block cache.



Locating a set associative block

- We can determine where a memory address belongs in an associative cache in a similar way as before.
- If a cache has 2^s sets and each block has 2^n bytes, the memory address can be partitioned as follows.



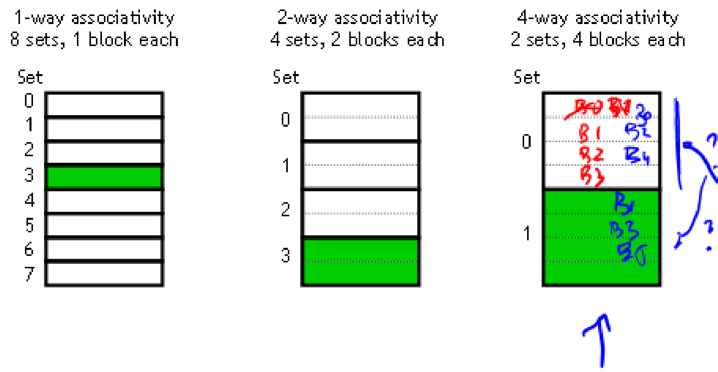
- Our arithmetic computations now compute a set index, to select a *set* within the cache instead of an individual block.

Block Offset = Memory Address mod 2^n

Block Address = Memory Address / 2^n
Set Index = Block Address mod 2^s

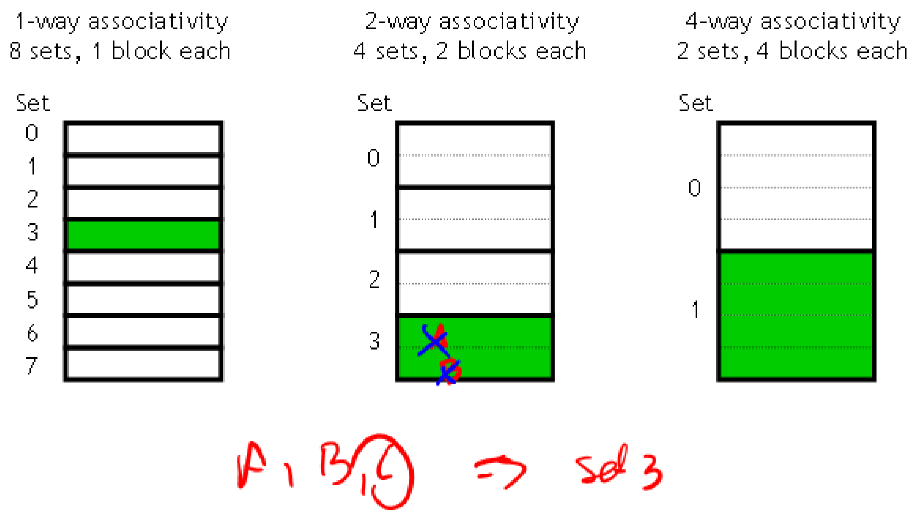
Example placement in set-associative caches

- Where would data from memory byte address 6195 be placed, assuming the eight-block cache designs below, with 16 bytes per block?
- 6195 in binary is 00...0110000 011 0011.
- Each block has 16 bytes, so the lowest 4 bits are the block offset.
- For the 1-way cache, the next three bits (011) are the set index.
  For the 2-way cache, the next two bits (11) are the set index.
  For the 4-way cache, the next one bit (1) is the set index.
- The data may go in *any* block, shown in green, within the correct set.



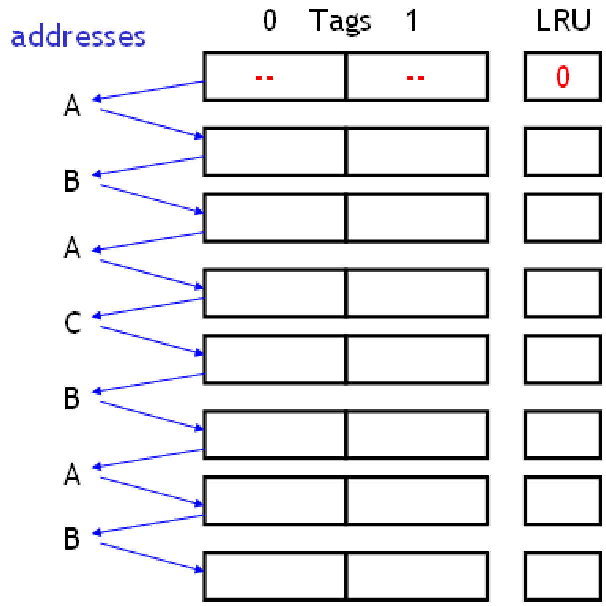
Block replacement

- Any empty block in the correct set may be used for storing data.
- If there are no empty blocks, the cache controller will attempt to replace the least recently used block, just like before.
- For highly associative caches, it's expensive to keep track of what's really the least recently used block, so some approximations are used. We won't get into the details.



LRU example

- Assume a fully-associative cache with two blocks, which of the following memory references miss in the cache.
  - assume distinct addresses go to distinct blocks

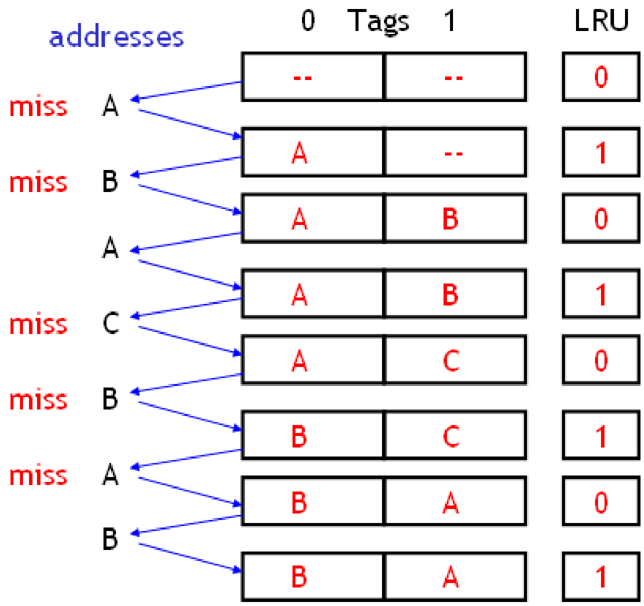


LRU example

- Assume a fully-associative cache with two blocks, which of the following memory references miss in the cache.
  - assume distinct addresses go to distinct blocks

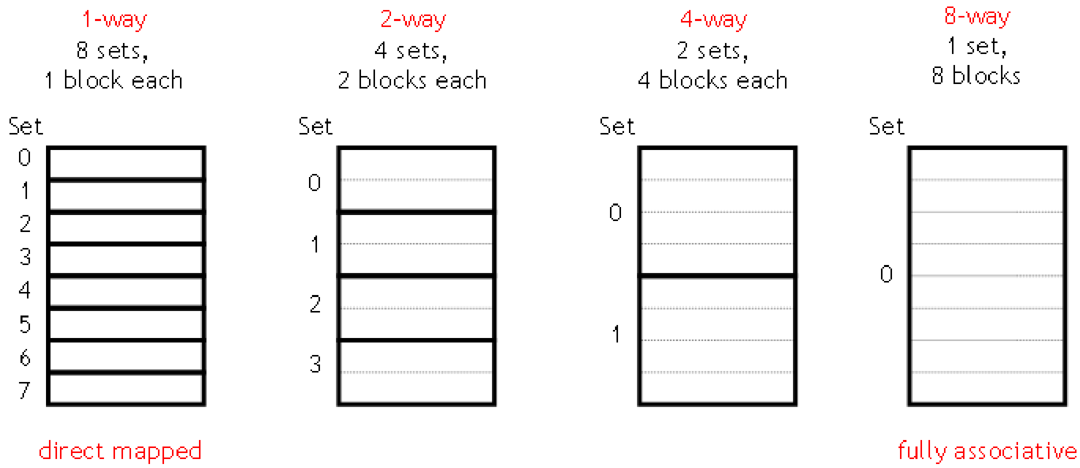
On a miss, we replace the LRU.

On a hit, we just update the LRU.



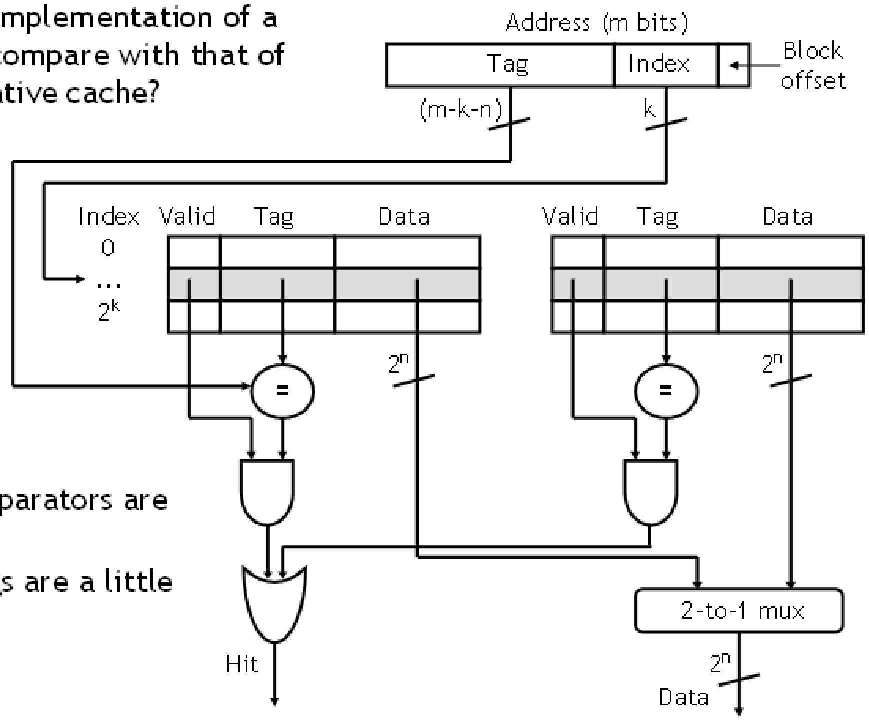
Set associative caches are a general idea

- By now you may have noticed the 1-way set associative cache is the same as a direct-mapped cache.
- Similarly, if a cache has 2^k blocks, a 2^k-way set associative cache would be the same as a fully-associative cache.



2-way set associative cache implementation

- How does an implementation of a 2-way cache compare with that of a fully-associative cache?



- Only two comparators are needed.
- The cache tags are a little shorter too.

Summary

- Larger block sizes can take advantage of spatial locality by loading data from not just one address, but also nearby addresses, into the cache.
- Associative caches assign each memory address to a particular set within the cache, but not to any specific block within that set.
  - Set sizes range from 1 (direct-mapped) to 2^k (fully associative).
  - Larger sets and higher associativity lead to fewer cache conflicts and lower miss rates, but they also increase the hardware cost.
  - In practice, 2-way through 16-way set-associative caches strike a good balance between lower miss rates and higher costs.
- Next, we'll talk more about measuring cache performance, and also discuss the issue of *writing* data to a cache.