# Binary Numbers

CSE 410, Spring 2009

Computer Systems

http://www.cs.washington.edu/410

# Binary, Hex, and Decimal

| $Binary_2$ | $Hex_{16}$ | $10^3=1000_{10}$ | $10^2=100_{10}$ | $10^1=10_{10}$ | $10^0=1_{10}$ |
|---|---|---|---|---|---|
| 11 | 0x3 | | | | 3 |
| 1001 | 0x9 | | | | 9 |
| 1010 | 0xA | | | 1 | 0 |
| 1111 | 0xF | | | 1 | 5 |
| 1 0000 | 0x10 | | | 1 | 6 |
| 1 1111 | 0x1F | | | 3 | 1 |
| 111 1111 | 0x7F | | 1 | 2 | 7 |
| 1111 1111 | 0xFF | | 2 | 5 | 5 |

# General Anything-to-Base 10 Conversion

Base = number of unique symbols per alphabet

Given a base and a string of digits…

- Digit[ i ] contributes $base^i$ * Digit[ i ]
- $\sum$ all contributions == base10 value
- RtoL: Digit[ 0 ] is LSB and Digit[ size-1 ] is MSB

$15 = 1 * 10^1 + 5 * 10^0$

$234 = 2 * 10^2 + 3 * 10^1 + 4*10^0$

# Digit[ i ] contributes base$^{i}$ * Digit[ i ]

- Binary = {0,1} = Base 2
- $01 = 2^1*0 + 2^0*1 = 1$ (Base 10)
- $11 = 2^1*1 + 2^0*1 = 3$ (Base 10)
- 101 = ?
- 1101 = ?

# Unsigned binary numbers

- Each bit represents a power of 2
- For unsigned numbers in a fixed width field
  - » $2^n$ distinct values
  - » the minimum value is 0
  - » the maximum value is $2^n$-1, where n is the number of bits in the field
  - » So, for binary, if we have one bit: $2^1 = 2$ values = {0,1}
- Fixed field widths determine many limits
  - » 5 bits = 32 possible values ($2^5 = 32$)
  - » 10 bits = 1024 possible values ($2^{10} = 1024$)

# Binary, Hex, and Decimal

## Digit[ i ] contributes base$^{i}$ * Digit[ i ]

| $2^8=256_{10}$ | $2^7=128_{10}$ | $2^6=64_{10}$ | $2^5=32_{10}$ | $2^4=16_{10}$ | $2^3=8_{10}$ | $2^2=4_{10}$ | $2^1=2_{10}$ | $2^0=1_{10}$ | $Hex_{16}$ | $Decimal_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1 | 1 | 0x3 | 3 |
| | | | | | 1 | 0 | 0 | 1 | 0x9 | 9 |
| | | | | | 1 | 0 | 1 | 0 | 0xA | 10 |
| | | | | | 1 | 1 | 1 | 1 | 0xF | 15 |
| | | | | 1 | 0 | 0 | 0 | 0 | 0x10 | 16 |
| | | | | 1 | 1 | 1 | 1 | 1 | 0x1F | 31 |
| | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0x7F | 127 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0xFF | 255 |

# Digit[ i ] contributes base$^{i}$ * Digit[ i ]

- Hex = {0,…,9,A,B,C,D,E,F} = Base 16
  - » Where A = 10, B = 11, …, F = 15
  - » 16 unique symbols, ranging in (base10) values 0-15
- 0x3 = $16^0 * 3$
- 0x33 = $16^1 * 3 + 16^0 * 3$ ($=51_{10}$)
- 0xF = $16^0 * 15$
- 0x40 = ?
- 0x0016 = ?
- 0xCAFE = ?

# Binary, Hex, and Decimal

| $Binary_2$ | $16^4=65536_{10}$ | $16^3=4096_{10}$ | $16^2=256_{10}$ | $16^1=16_{10}$ | $16^0=1_{10}$ | $Decimal_{10}$ |
|---|---|---|---|---|---|---|
| 11 | | | | | 3 | 3 |
| 1001 | | | | | 9 | 9 |
| 1010 | | | | | A | 10 |
| 1111 | | | | | F | 15 |
| 1 0000 | | | | 1 | 0 | 16 |
| 1 1111 | | | | 1 | F | 31 |
| 111 1111 | | | | 7 | F | 127 |
| 1111 1111 | | | | F | F | 255 |