CSE 417: Algorithms and Computational Complexity

I: Organization & Overview

Winter 2007

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What you'll have to do

Homework (~55% of grade)

- Programming
  - Several small projects
- Written homework assignments
  - English exposition and pseudo-code
  - Analysis and argument as well as design

Midterm / Final Exam (~15% / 30%)

Late Policy:

- Papers and/or electronic turnins are due at the *start* of class on the due date. 10% off for one day late (Monday, for Friday due dates); 20% per day thereafter.

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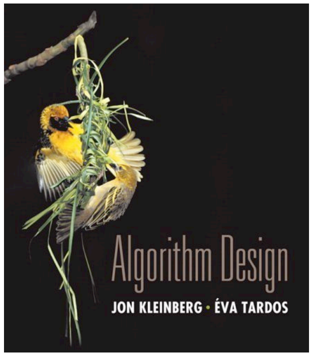
Textbook



Algorithm Design by Jon Kleinberg and Eva Tardos. Addison Wesley, 2006.

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What the course is about

Design of Algorithms

- design methods
- common or important types of problems
- analysis of algorithms - efficiency
- correctness proofs

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What the course is about

Complexity, NP-completeness and intractability

- solving problems in principle is not enough
  - algorithms must be *efficient*
- some problems have *no efficient solution*
- NP-complete problems
  - important & useful class of problems whose solutions (seemingly) cannot be found efficiently, but *can* be checked easily

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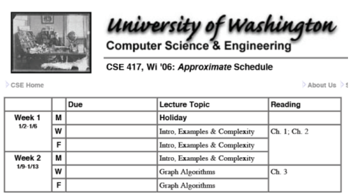
Very Rough Division of Time

Algorithms (7 weeks)

- Analysis of Algorithms
- Basic Algorithmic Design Techniques
- Graph Algorithms

Complexity & NP-completeness (3 weeks)

Check online schedule page for (evolving) details


University of Washington
Computer Science & Engineering
CSE 417, Wi '06: *Approximate* Schedule

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		Due	Lecture Topic	Reading
Week 1 1/2-1/6	M		Holiday	
	W		Intro, Examples & Complexity	Ch. 1; Ch. 2
	F		Intro, Examples & Complexity	
Week 2 1/9-1/13	M		Intro, Examples & Complexity	
	W		Graph Algorithms	Ch. 3
	F		Graph Algorithms	

Complexity Example

Cryptography (e.g. RSA, SSL in browsers)

- Secret: p,q prime, say 512 bits each
- Public: n which equals p x q, 1024 bits

In principle

- *there is an algorithm* that given n will find p and q: try all 2^{512} possible p's, an astronomical number

In practice

- *no efficient algorithm* is known for this problem
- security of RSA depends on this fact

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Algorithms versus Machines

We all know about Moore's Law and the exponential improvements in hardware...

Ex: sparse linear equations over 25 years

10 orders of magnitude improvement!

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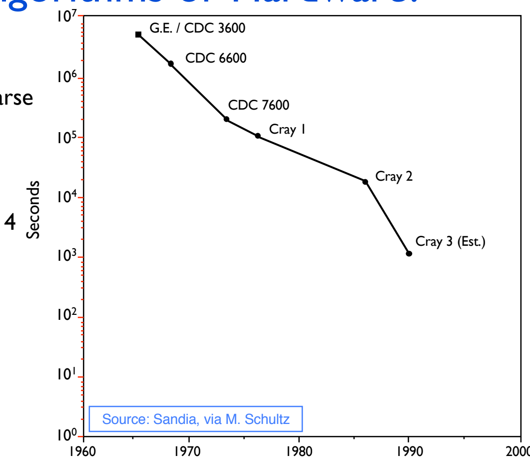
Algorithms or Hardware?

25 years progress solving sparse linear systems

hardware: 4 orders of magnitude

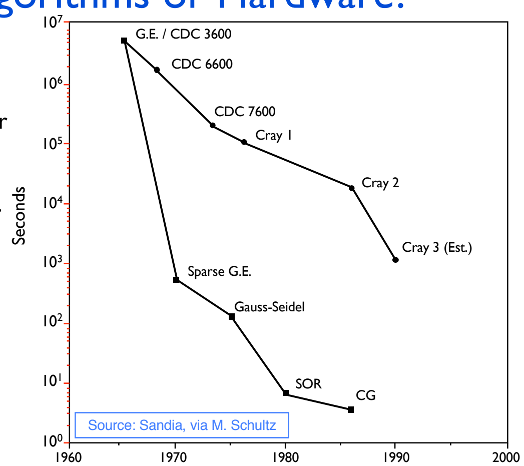


Algorithms or Hardware?

25 years progress solving sparse linear systems

hardware: 4 orders of magnitude

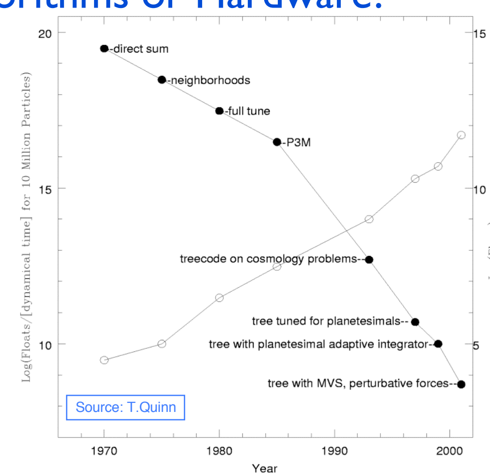
software: 6 orders of magnitude



Algorithms or Hardware?

The N-Body Problem:

in 30 years
10^7 hardware
10^{10} software



Algorithm: definition

Procedure to accomplish a task or solve a well-specified problem

- Well-specified: know what all possible inputs look like and what output looks like given them
- "accomplish" via simple, well-defined steps
- Ex: sorting names (via comparison)
- Ex: checking for primality (via +, -, *, /, ≤)

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Algorithms: a sample problem

Printed circuit-board company has a robot arm that solders components to the board

Time: proportional to total distance the arm must move from initial rest position around the board and back to the initial position

For each board design, find best order to do the soldering

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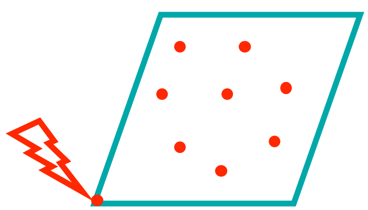
Printed Circuit Board



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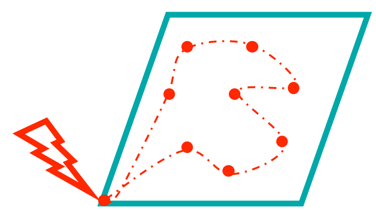
Printed Circuit Board



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A Well-defined Problem

Input: Given a set *S* of *n* points in the plane

Output: The shortest cycle tour that visits each point in the set *S*.

Better known as "TSP"

How might you solve it?

Nearest Neighbor Heuristic

heuristic: A rule of thumb, simplification, or educated guess that reduces or limits the search for solutions indomains that are difficult and poorly understood. May be good, but usually *not* guaranteed to give the best or fastest solution.

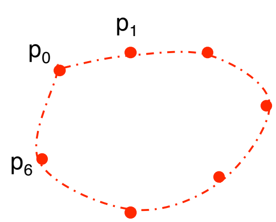
Start at some point p_0

Walk first to its nearest neighbor p_1

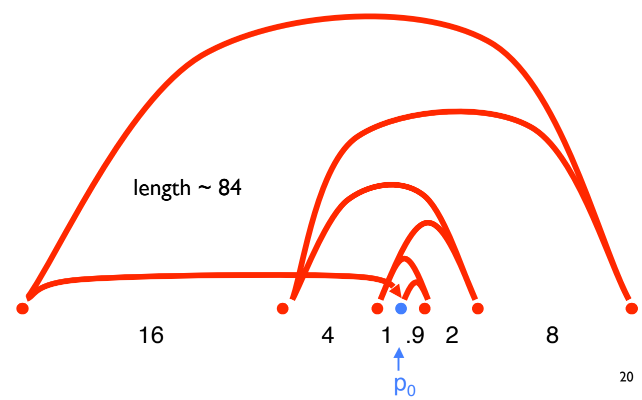
Repeatedly walk to the nearest unvisited neighbor p_2, then p_3,… until all points have been visited

Then walk back to p_0

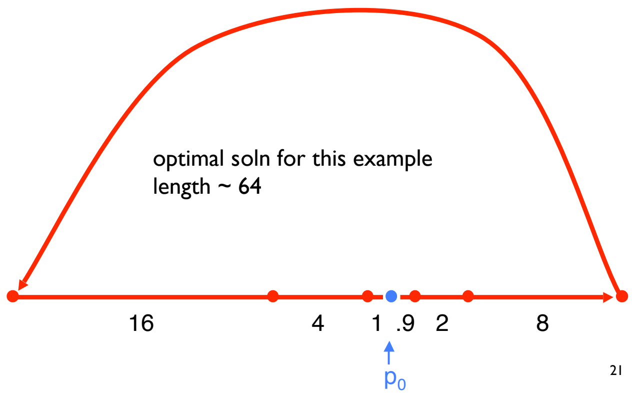
Nearest Neighbor Heuristic



An input where it works badly



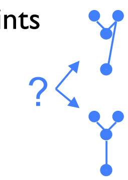
An input where it works badly



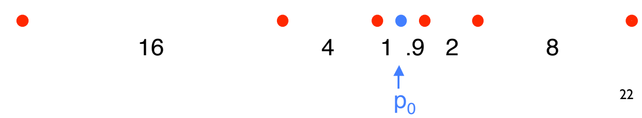
Revised idea - Closest pairs first

Repeatedly join the closest pair of points

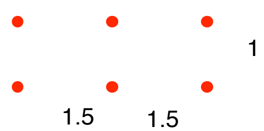
(s.t. result can still be part of a single loop in the end. I.e., join endpoints, but not points in middle, of path segments already created.)



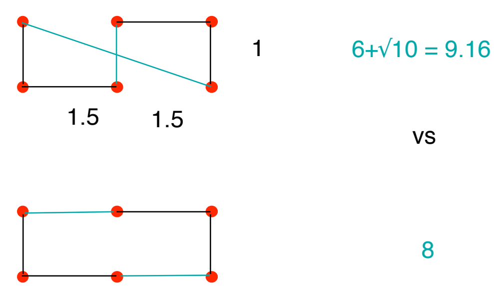
How does this work on our bad example?



Another bad example



Another bad example



Something that works

For each of the n! = n(n-1)(n-2)…1 orderings of the points, check the length of the cycle you get

Keep the best one

Two Notes

The two *in*correct algorithms were greedy

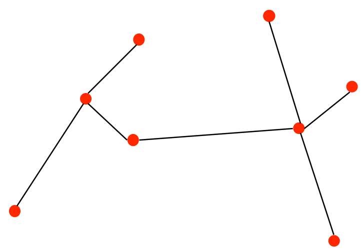
- Often very natural & tempting ideas
- They make choices that look great “locally” (and never reconsider them)
- When greed works, the algorithms are typically efficient
- BUT: often does not work - you get boxed in

Our correct alg avoids this, but is incredibly slow

- 20! is so large that checking one billion per second would take 2.4 billion seconds (around 70 years!)

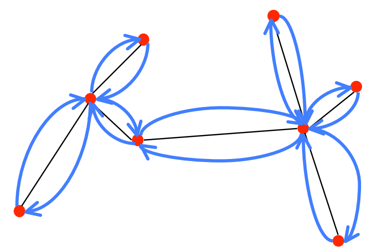
Something that “works” (differently)

1. Find Min Spanning Tree



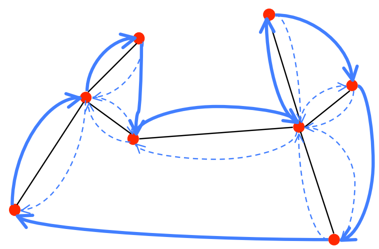
Something that “works” (differently)

2. Walk around it



Something that "works" (differently)

3. Take shortcuts (instead of revisiting)



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Something that "works" (differently): Guaranteed Approximation

Does it seem wacky?

Maybe, but it's *always* within a factor of 2 of the best tour!

deleting one edge from best tour gives *a* spanning tree, so *Min* spanning tree < best tour

best tour ≤ wacky tour ≤ 2 * MST < 2 * best

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The Morals of the Story

Simple problems can be hard
- Factoring, TSP

Simple ideas don't always work
- Nearest neighbor, closest pair heuristics

Simple algorithms can be very slow
- Brute-force factoring, TSP

Changing your objective can be good
- Guaranteed approximation for TSP

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