CSE 421
Algorithms

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Lecture 4

Announcements

- Homework 2, Due October 11, 1:30 pm.
- Reading
  - Chapter 2.1, 2.2
  - Chapter 3 (Mostly review)
  - Start on Chapter 4

Today

- Finish discussion of asymptotics
  - O, Ω, Θ
- Graph theory terminology
- Basic graph algorithms

Formalizing growth rates

- T(n) is O(f(n)) $[T : Z^+ \rightarrow R^+]$
  - If sufficiently large n, T(n) is bounded by a constant multiple of f(n)
  - Exist c, n_0, such that for $n > n_0$, $T(n) < c\, f(n)$
- T(n) is O(f(n)) will be written as:
  T(n) = O(f(n))
  - Be careful with this notation

Order the following functions in increasing order by their growth rate

a) $n \log^4 n$
b) $2n^2 + 10n$
c) $2^{n/100}$
d) $1000n + \log^8 n$
e) n^{100}
f) 3^n
g) $1000 \log^{10} n$
h) $n^{1/2}$

Ordering growth rates

- For b > 0 and x > 0
  - $\log^b n$ is $O(n^x)$
- For r > 1 and d > 0
  - n^d is $O(r^n)$

Lower bounds

- T(n) is Ω(f(n))
  - T(n) is at least a constant multiple of f(n)
  - There exists an n_0, and $\varepsilon > 0$ such that $T(n) > \varepsilon f(n)$ for all $n > n_0$
- Warning: definitions of Ω vary
- T(n) is Θ(f(n)) if T(n) is O(f(n)) and T(n) is Ω(f(n))

True or False

- n log n is O(n^2)
- n^3 is O($4n^3 + 2n + n$)
- n^{-1} is O(n^{-2})
- n^{-1} is Ω(n^{-2})
- f(n) = n^2 if n is even, 0 if n is odd
  f(n) is Ω(n^2)

Useful Theorems

- If lim (f(n) / g(n)) = c for c > 0 then f(n) = Θ(g(n))
- If f(n) is O(g(n)) and g(n) is O(h(n)) then f(n) is O(h(n))
- If f(n) is O(h(n)) and g(n) is O(h(n)) then f(n) + g(n) is O(h(n))

Graph Theory

- G = (V, E)
  - V – vertices
  - E – edges
- Undirected graphs
  - Edges sets of two vertices {u, v}
- Directed graphs
  - Edges ordered pairs (u, v)
- Many other flavors
  - Edge / vertices weights
  - Parallel edges
  - Self loops

Definitions

- Path: $v_1, v_2, \ldots, v_k$, with (v_i, v_{i+1}) in E
  - Simple Path
  - Cycle
  - Simple Cycle
- Distance
- Connectivity
  - Undirected
  - Directed (strong connectivity)
- Trees
  - Rooted
  - Unrooted

Graph search

- Find a path from s to t

```
S = {s}
While there exists (u, v) in E with u in S and v not in S
        Pred[v] = u
        Add v to S
        if (v = t) then path found
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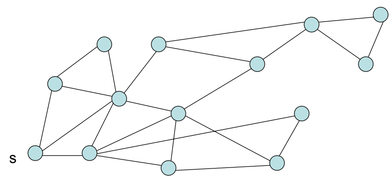
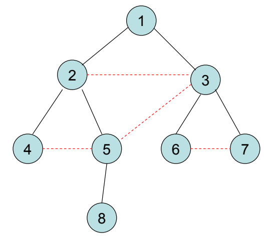
Breadth first search

- Explore vertices in layers
  - s in layer 1
  - Neighbors of s in layer 2
  - Neighbors of layer 2 in layer 3 . . .



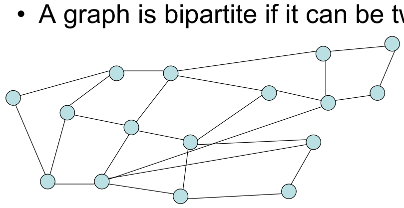
Key observation

- All edges go between vertices on the same layer or adjacent layers



Bipartite

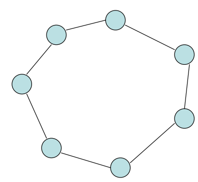
- A graph V is bipartite if V can be partitioned into V_1, V_2 such that all edges go between V_1 and V_2
- A graph is bipartite if it can be two colored



Two color this graph

Testing Bipartiteness

- If a graph contains an odd cycle, it is not bipartite



Algorithm

- Run BFS
- Color odd layers red, even layers blue
- If no edges between the same layer, the graph is bipartite
- If edge between two vertices of the same layer, then there is an odd cycle, and the graph is not bipartite