CSE 421
Algorithms

Richard Anderson
Lecture 19
Longest Common Subsequence

Longest Common Subsequence

- $C=c_1 \ldots c_g$ is a subsequence of $A=a_1 \ldots a_m$ if C can be obtained by removing elements from A (but retaining order)
- LCS(A, B): A maximum length sequence that is a subsequence of both A and B

ocurranec
occurrence

attacggct
tacgacca

Determine the LCS of the following strings

BARTHOLEMEWSIMPSON

KRUSTYTHECLOWN

String Alignment Problem

- Align sequences with gaps

```
CAT TGA  AT
CAGAT AGGA
```

- Charge δ_x if character x is unmatched
- Charge γ_{xy} if character x is matched to character y

Note: the problem is often expressed as a minimization problem, with $\gamma_{xx} = 0$ and $\delta_x > 0$

LCS Optimization

- $A = a_1 a_2 \ldots a_m$
- $B = b_1 b_2 \ldots b_n$

- Opt[j, k] is the length of $LCS(a_1 a_2 \ldots a_j, b_1 b_2 \ldots b_k)$

Optimization recurrence

If $a_j = b_k$, Opt[j,k] = 1 + Opt[j-1, k-1]

If $a_j \neq b_k$, Opt[j,k] = max(Opt[j-1,k], Opt[j,k-1])

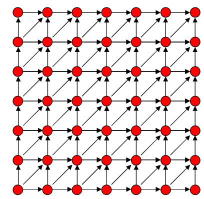
Give the Optimization Recurrence for the String Alignment Problem

- Charge δ_x if character x is unmatched
- Charge γ_{xy} if character x is matched to character y

Opt[j, k] =

Let a_j = x and b_k = y
Express as minimization

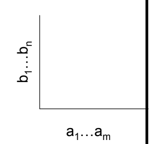
Dynamic Programming Computation



Code to compute Opt[j,k]

Storing the path information

```
A[1..m], B[1..n]
for i := 1 to m    Opt[i, 0] := 0;
for j := 1 to n    Opt[0,j] := 0;
Opt[0,0] := 0;
for i := 1 to m
        for j := 1 to n
                if A[i] = B[j] { Opt[i,j] := 1 + Opt[i-1,j-1]; Best[i,j] := Diag; }
                else if Opt[i-1, j] >= Opt[i, j-1]
                        { Opt[i, j] := Opt[i-1, j], Best[i,j] := Left; }
                else    { Opt[i, j] := Opt[i, j-1], Best[i,j] := Down; }
```



How good is this algorithm?

- Is it feasible to compute the LCS of two strings of length 100,000 on a standard desktop PC? Why or why not.

Observations about the Algorithm

- The computation can be done in O(m+n) space if we only need one column of the Opt values or Best Values
- The algorithm can be run from either end of the strings