CSE 421
Algorithms

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Lecture 26

Network Flow Applications

Today's topics

- More network flow reductions
  - Airplane scheduling
  - Image segmentation
  - Baseball elimination

Airplane Scheduling

- Given an airline schedule, and starting locations for the planes, is it possible to use a fixed set of planes to satisfy the schedule.
- Schedule
  - [segments] Departure, arrival pairs (cities and times)
- Approach
  - Construct a circulation problem where paths of flow give segments flown by each plane

Example

- Seattle->San Francisco, 9:00 – 11:00
- Seattle->Denver, 8:00 – 11:00
- San Francisco -> Los Angeles, 13:00 – 14:00
- Salt Lake City -> Los Angeles, 15:00-17:00
- San Diego -> Seattle, 17:30-> 20:00
- Los Angeles -> Seattle, 18:00->20:00

- Flight times:
  - Denver->Salt Lake City, 2 hours
  - Los Angeles->San Diego, 1 hour

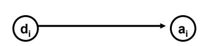
Can this schedule be full filled with two planes, starting from Seattle?

Compatible segments

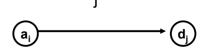
- Segments S_1 and S_2 are compatible if the same plane can be used on S_1 and S_2
  - End of S_1 equals start of S_2, and enough time for turn around between arrival and departure times
  - End of S_1 is different from S_2, but there is enough time to fly between cities

Graph representation

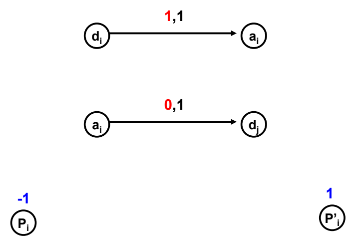
- Each segment, S_i, is represented as a pair of vertices (d_i, a_i, for departure and arrival), with an edge between them.



- Add an edge between a_i and d_j if S_i is compatible with S_j.



Setting up a flow problem



Result

- The planes can satisfy the schedule iff there is a feasible circulation

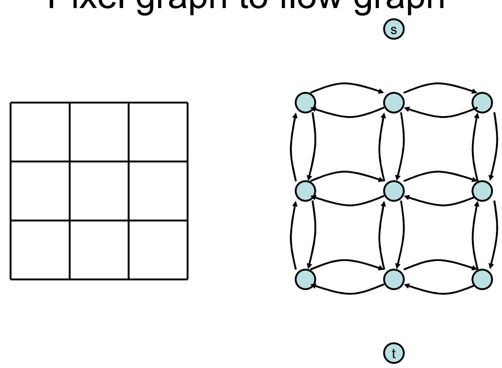
Image Segmentation

- Separate foreground from background

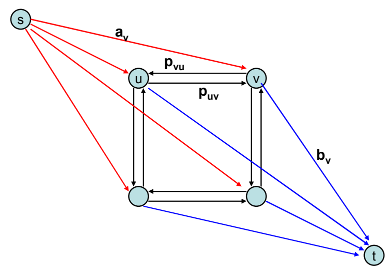
Image analysis

- a_i: value of assigning pixel i to the foreground
- b_i: value of assigning pixel i to the background
- p_{ij}: penalty for assigning i to the foreground, j to the background or vice versa
- A: foreground, B: background
- $Q(A,B) = \Sigma_{\{i \text{ in } A\}} a_i + \Sigma_{\{j \text{ in } B\}} b_j - \Sigma_{\{(i,j) \text{ in } E,\, i \text{ in } A,\, j \text{ in } B\}} p_{ij}$

Pixel graph to flow graph



Mincut Construction



Baseball elimination

- Can the Dinosaurs win the league?
- Remaining games:
  - AB, AC, AD, AD, AD, BC, BC, BC, BD, CD

	W	L
Ants	4	2
Bees	4	2
Cockroaches	3	3
Dinosaurs	1	5

A team wins the league if it has strictly more wins than any other team at the end of the season
A team ties for first place if no team has more wins, and there is some other team with the same number of wins

Baseball elimination

- Can the Fruit Flies win the league?
- Remaining games:
  - AC, AD, AD, AD, AF, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, BF, CE, CE, CE, CF, CF, DE, DF, EF, EF

	W	L
Ants	17	12
Bees	16	7
Cockroaches	16	7
Dinosaurs	14	13
Earthworms	14	10
Fruit Flies	12	15

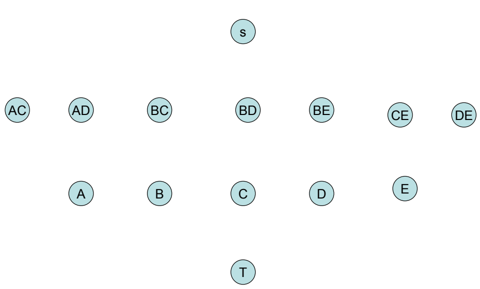
Assume Fruit Flies win remaining games

- Fruit Flies are tied for first place if no team wins more than 19 games
- Allowable wins
  - Ants (2)
  - Bees (3)
  - Cockroaches (3)
  - Dinosaurs (5)
  - Earthworms (5)
- 18 games to play
  - AC, AD, AD, AD, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, CE, CE, CE, DE

	W	L
Ants	17	13
Bees	16	8
Cockroaches	16	9
Dinosaurs	14	14
Earthworms	14	12
Fruit Flies	19	15

Remaining games

AC, AD, AD, AD, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, CE, CE, CE, DE



Network flow applications summary

- Bipartite Matching
- Disjoint Paths
- Airline Scheduling
- Survey Design
- Baseball Elimination
- Project Selection
- Image Segmentation