CSE 421 Algorithms

Richard Anderson
Lecture 29
NP-Completeness and course wrap-up

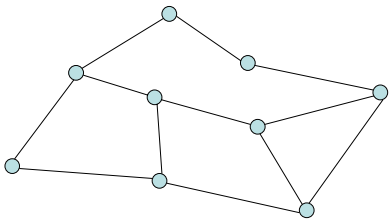
Today

- NP-completeness
  - Reductions
  - Problem Thresholds
  - Number problems
  - What is NP?
  - What we don't know about NP-completeness
- Course summary
- Evaluations

NP-Completeness Reductions

- If X is NP-Complete, Y is in NP, and $X <_P Y$, then Y is NP-Complete

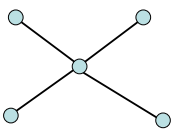
Hamiltonian Circuit, Hamiltonian Path

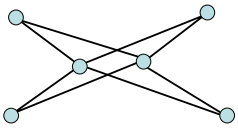


How do you show that Hamiltonian Path is NP-Complete?

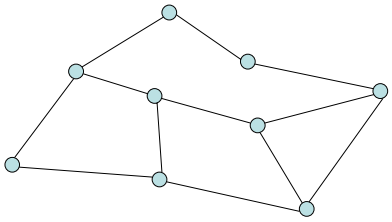
Local Modification

- Convert G to G'
- Pick a vertex v
  - Replace v by v' and v''
  - If (u,v) is an edge, include edges (u, v'), (u, v'')
- G' has a Hamiltonian Path from v' to v'' iff G has a Hamiltonian Circuit





HamPath $<_P$ DirHamPath



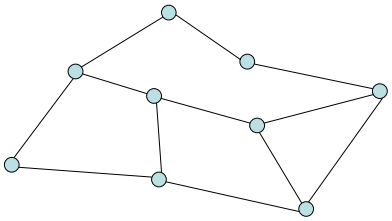
How do you show that Directed Hamiltonian Path is NP-Complete?

Problem definition

- Given a graph G, does G have an independent set?
- Given a graph G, does G have an independent set of size 7?
- Given a graph G, and an integer K, does G have an independent set of size K?

Graph Coloring

- NP-Complete
  - Graph K-coloring
  - Graph 3-coloring
- Polynomial
  - Graph 2-Coloring



Number Problems

- Subset sum problem
  - Given natural numbers $w_1, \ldots, w_n$ and a target number W, is there a subset that adds up to exactly W?
- Subset sum problem is NP-Complete
- Subset Sum problem can be solved in O(nW) time

Subset sum problem

- The reduction to show Subset Sum is NP-complete involves numbers with n digits
- In that case, the O(nW) algorithm is an exponential time and space algorithm

What is NP?

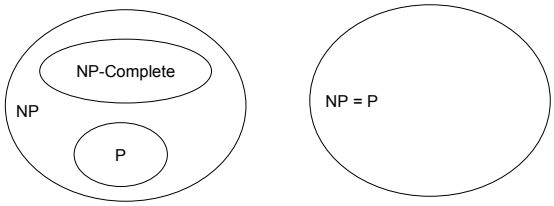
- Problems where 'yes' instances can be efficiently verified
  - Hamiltonian Circuit
  - 3-Coloring
  - 3-SAT
- Succinct certificate property

What about 'negative instances'

- How do you show that a graph does not have a Hamiltonian Circuit
- How do you show that a formula is not satisfiable?

What we don't know

- P vs. NP



Course summary
What did we cover in the last 29 lectures?

- Stable Matching (2)
- Models of computation and efficiency (1)
- Basic graph algorithms
  - BFS, Bipartiteness, SCC, Cycles, Topological Sort (2)
- Greedy Algorithms (2)
  - Interval Scheduling, HW Scheduling
  - Correctness proofs
- Dijkstra's Algorithm (1)
- Minimum Spanning Trees (2)
- Recurrences (2)
- Divide and Conquer Algorithms (3)
  - Closest Pair, FFT
- Dynamic Programming (7)
  - Weighted interval scheduling, subset sum, knapsack, longest common subsequence, shortest paths
- Network Flow (5)
  - Ford Fulkerson, Maxflow/mincut, Applications
- NP-Completeness (3)

Number of lectures on each topic in parentheses