CSE 421: Introduction to Algorithms

I: Overview

Fall 2011

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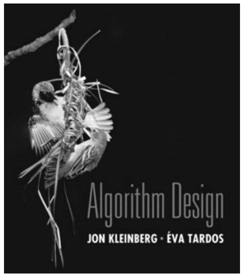
Administrivia

People:
- Anna Karlin
- Johnny Yan

All relevant course information:
http://www.cs.washington.edu/421
- Office hours, Wednesday 4-5, CSE 216



We will cover a good part of chapters 1-8.
Slides by combination of Larry Ruzzo, Kevin Wayne and others.

Administrivia

- Weekly homework, due Thursday ~ 40%
- Take home midterm, out Nov 10, due Nov 17 ~25%
- In-class, open book, open notes final ~35%

- Working on homework sets:
  - Collaboration on formulation of ideas allowed.
  - Writing up solutions – can submit jointly with one other person.
  - You may not consult written materials other than the course materials.
  - We prefer that homework solutions be typed.
  - Please indicate on your homework all people that you discussed the problems with, and indicate any and all sources you used.
  - See grading guidelines handout.

What the course is about

Design of Algorithms
- design methods
- common or important types of problems
- analysis of algorithms - efficiency
- correctness proofs

What the course is about

- Complexity, NP-completeness and intractability
  - solving problems in principle is not enough
    - algorithms must be *efficient*
  - some problems have *no efficient solution*
  - NP-complete problems
    - important & useful class of problems whose solutions (seemingly) cannot be found efficiently, but *can* be checked easily

Very Rough Division of Time

- Algorithms (7 weeks)
  - Analysis of Algorithms
  - Basic Algorithmic Design Techniques
  - Graph Algorithms
- Complexity & NP-completeness (2 weeks)

Check online
calendar page for
(evolving) details

Complexity Example

- Cryptography (e.g. RSA, SSL in browsers)
  - Secret: p,q prime, say 512 bits each
  - Public: n which equals p x q, 1024 bits
- In principle
  - *there is an algorithm* that given n will find p and q: try all $2^{512} > 1.3\text{x}10^{154}$ possible p's: kinda slow…
- In practice
  - *no fast algorithm* known for this problem (on non-quantum computers)
  - security of RSA depends on this fact
  - ("quantum computing": strongly driven by possibility of changing this)

Algorithms versus Machines

We all know about Moore's Law and the exponential improvements in hardware...

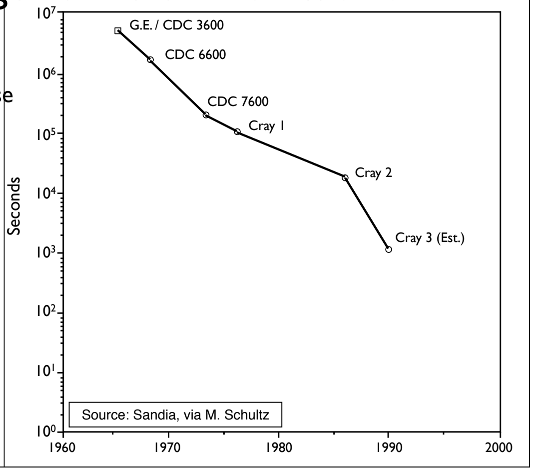
Ex: sparse linear equations over 25 years

10 orders of magnitude improvement!

Algorithms or Hardware?

25 years progress solving sparse linear systems

hardware: 4 orders of magnitude



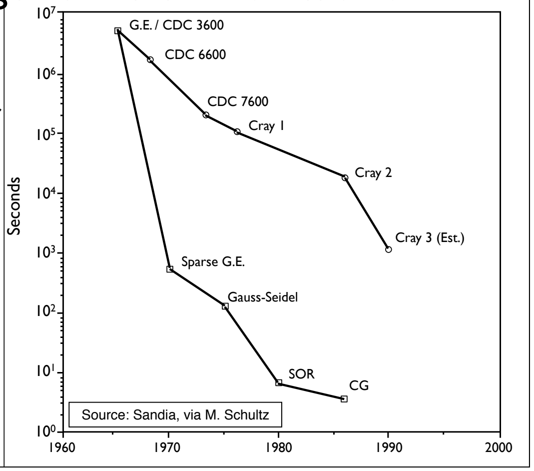
Source: Sandia, via M. Schultz

Algorithms or Hardware?

25 years progress solving sparse linear systems

hardware: 4 orders of magnitude

software: 6 orders of magnitude

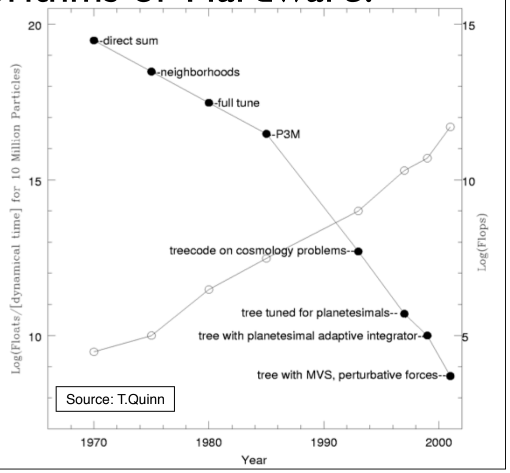


Source: Sandia, via M. Schultz

Algorithms or Hardware?

The N-Body Problem:

in 30 years
10^7 hardware
10^{10} software



Source: T.Quinn

Algorithm: definition

Procedure to accomplish a task or solve a well-specified problem

- Well-specified: know what all possible inputs look like and what output looks like given them
- “accomplish” via simple, well-defined steps
- Ex: sorting names (via comparison)
- Ex: checking for primality (via +, -, *, /, ≤)

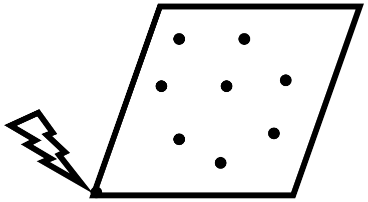
Algorithms: a sample problem

Printed circuit-board company has a robot arm that solders components to the board

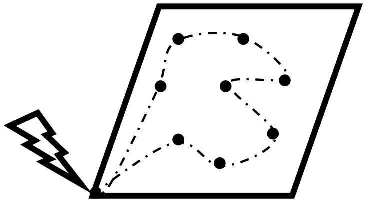
Time: proportional to total distance the arm must move from initial rest position around the board and back to the initial position

For each board design, find best order to do the soldering

Printed Circuit Board



Printed Circuit Board



A Well-defined Problem

Input: Given a set *S* of *n* points in the plane

Output: The shortest cycle tour that visits each point in the set *S*.

Better known as “TSP”

How might you solve it?

Nearest Neighbor Heuristic

heuristic:
A rule of thumb, simplification, or educated guess that reduces or limits the search for solutions in domains that are difficult and poorly understood. May be good, but usually *not* guaranteed to give the best or fastest solution.

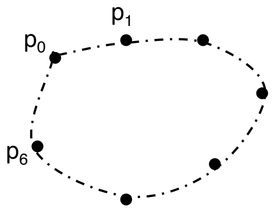
Start at some point p_0

Walk first to its nearest neighbor p_1

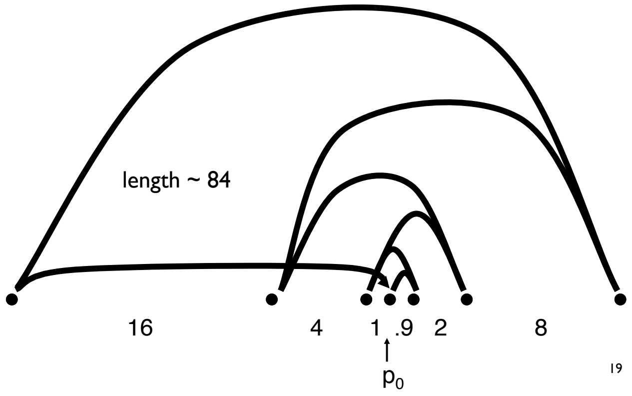
Repeatedly walk to the nearest unvisited neighbor p_2, then p_3,… until all points have been visited

Then walk back to p_0

Nearest Neighbor Heuristic

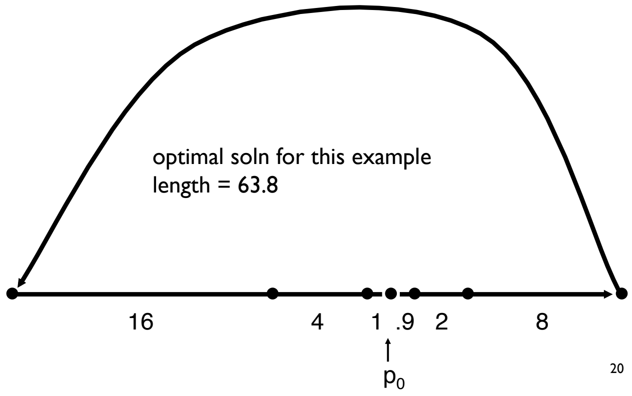


An input where it works badly



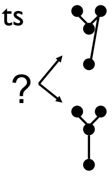
length ~ 84

An input where it works badly



optimal soln for this example
length = 63.8

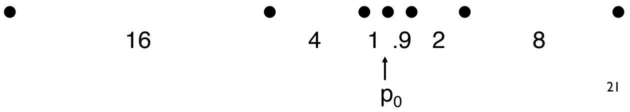
Revised idea - Closest pairs first



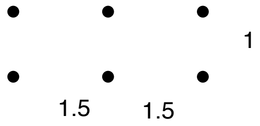
Repeatedly join the closest pair of points

(s.t. result can still be part of a single loop in the end. I.e., join endpoints, but not points in middle, of path segments already created.)

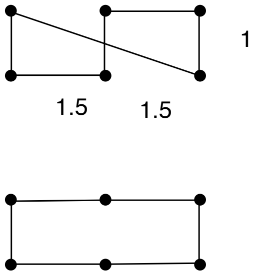
How does this work on our bad example?



Another bad example



Another bad example



6+√10 = 9.16

vs

8

Something that works

“Brute Force Search”:

For each of the n! = n(n-1)(n-2)…1 orderings of the points, check the length of the cycle you get

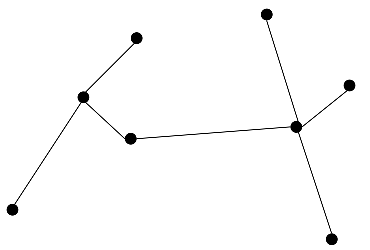
Keep the best one

Two Notes

- The two *in*correct algorithms were greedy
  - Often very natural & tempting ideas
  - They make choices that look great "locally" (and never reconsider them)
  - When greed works, the algorithms are typically efficient
  - BUT: often does not work - you get boxed in
- Our correct alg avoids this, but is incredibly slow
  - 20! is so large that checking one billion orderings per second would take 2.4 billion seconds (around 70 years!)
  - And *growing*: $n! \sim \sqrt{2 \pi n} \cdot (n/e)^n \sim 2^{O(n \log n)}$

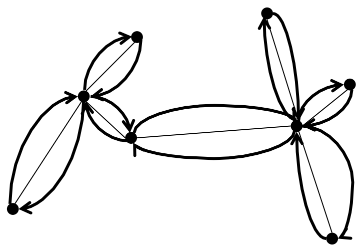
Something that "works" (differently)

1. Find Min Spanning Tree



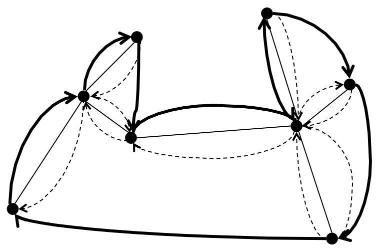
Something that "works" (differently)

2. Walk around it



Something that "works" (differently)

3. Take shortcuts (instead of revisiting)



Something that “works” (differently): Guaranteed Approximation

Does it seem wacky?

Maybe, but it’s *always* within a factor of 2 of the best tour!

- deleting one edge from best tour gives *a* spanning tree, so *Min* spanning tree < best tour
- best tour ≤ wacky tour ≤ 2 * MST < 2 * best

triangle inequality

The Morals of the Story

- Algorithms are important
  - Many performance gains outstrip Moore’s law
- Simple problems can be hard
  - Factoring, TSP
- Simple ideas don’t always work
  - Nearest neighbor, closest pair heuristics
- Simple algorithms can be very slow
  - Brute-force factoring, TSP
- Changing your objective can be good
  - Guaranteed approximation for TSP
- And: for some problems, even the *best* algorithms are slow