CSE421: Design and Analysis of Algorithms May 27th, 2021

Homework 8

Shayan Oveis Gharan Due: June 4th, 2021 at 23:59 PM

P1) In this exercise we give a polynomial time algorithm to find the minimum vertex cover in a bipartite graph $G = (X, Y, E)$. Following these steps:

a) Construct a flow network H from the given G just as in the maximum matching algorithm.

b) Given a min cut $s - t$ cut (A, B) in H, construct a vertex cover $S \subseteq X \cup Y$ of G such that $|S| = cap(A, B)$

c) Conversely, given a min vertex cover $S \subseteq X \cup Y$ of G, construct a $s - t$ cut (A, B) in H such that $cap(A, B) = |S|$.

d) Write down the algorithm and use the above argument to prove that it correctly finds the min vertex cover of G.

P2) Alice, the head of the US Olympic Volleyball team is supposed to select the team from n athletes. To get the strongest team, any two persons in the team must satisfy at least one of the following 3 criteria:

- They are born in the same year
- They are from the same city
- Their first names start with one of the first 13 letters of the alphabet or one of the last 13 letters of the alphabet.

Given the birthdate, city and the name of every athlete, help Alice by designing an algorithm that runs in polynomial time and outputs the largest possible number of individuals to form the Volleyball team.

P3) In the **Number Partition** problem we are given a collection of non-negative integers $x_1, \ldots, x_n$ and we want see whether it is possible to partition these numbers into two groups so that the sum in each group is the same.

In the **Number Sum** problem we given a collection of non-negative integers $x_1, \ldots, x_n$ and an integer m and we want to see whether there is subset S of these numbers such that $\sum_{i \in S} x_i = m$. Prove that **NumberPartition** $\leq_P$ **NumberSum**

P4) **Extra Credit:** Prove that the Hamiltonian cycle problem in directed graphs is NP-Complete. You may use the fact that 3SAT is NP-Complete.