1 Asymptotics

Some properties of asymptotics:

- If $f \leq O(g)$ and $g \leq O(h)$ then $f \leq O(h)$.

- If $f \geq \Omega(g)$ and $g \geq \Omega(h)$ then $f \geq \Omega(h)$.

- If $f = \Theta(g)$ and $g = \Theta(h)$ then $f = \Theta(h)$.

- If $f = O(h)$, $g = O(h)$ then $f + g = O(h)$.

Some common running times:

- Polynomial: $O(n^d)$. Exponential $2^{O(n)}$, Logarithmic $O(\log n)$.

- For every positive ϵ (no matter how small), $\log n \leq O(n^\epsilon)$. For every positive d (no matter how large), $n^d \leq O(2^n)$.

2 In class exercise

Arrange in increasing order of asymptotic growth. All logs are in base 2.

a) $n^{5/3}$

b) $2^{\sqrt{\log n}}$

c) $\sqrt{n^n}$

d) $\frac{n^2}{\log n}$

e) 2^n.

Hint: Recall rules of logarithm

- $\log(a \cdot b) = \log a + \log b$,

- $\log(a/b) = \log a - \log b$.

- $\log a^b = b \log a$.

Always keep in mind $n = 2^{\log_2 n}$. For example, $n^{1.5} = 2^{1.5 \log_2 n}$. Also recall and that $(2^a)^b = 2^{a \cdot b}$.