CSE 421 Lecture 5

1 Interval Scheduling

Theorem 1. *For the interval scheduling problem, greedy methods (ordered by finish time) gives an optimal solution.*

Proof:
Let $i_1, i_2, i_3, \cdots$ be the k jobs picked by greedy method (order by finish time).
Let $j_1, j_2, j_3, \cdots$ be any valid solution with the m jobs (order by finish time).
We show that "$k \geq r$ and $f(i_r) \leq f(j_r)$" by induction on $r \in \{1, 2, \cdots, m\}$.
Base base $r = 1$:
Greedy picks the job according to min finish time. So, $k \geq 1$ and $f(i_1) \leq f(j_1)$.
Induction:
By hypothesis, $f(i_{r-1}) \leq f(j_{r-1})$.
Since j_{r-1}, j_r does not overlap, $f(j_{r-1}) \leq s(j_r)$.
So, job j_r is one of the candidate greedy method looks at.
Since greedy picks the job finishes earliest among the candidates, $k \geq j$ and $f(i_r) \leq f(j_r)$.

2 Interval Partitioning

Theorem 2. *For the interval partitioning problem, greedy methods (ordered by start time) gives an optimal solution.*

Proof:
Let d is the number of classroom greedy method uses.

Classroom d is opened because some lecture j is not compatible with another $d-1$ lectures. Look at that time, there are d lectures at the same time.

So, the depth is $\geq d$.

So, any solution requires at least d classrooms.