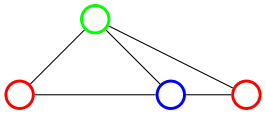
CSE421: Design and Analysis of Algorithms April 10, 2022

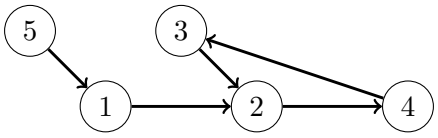
Homework 3

Shayan Oveis Gharan Due: April 17, 2022 at 23:59 PM

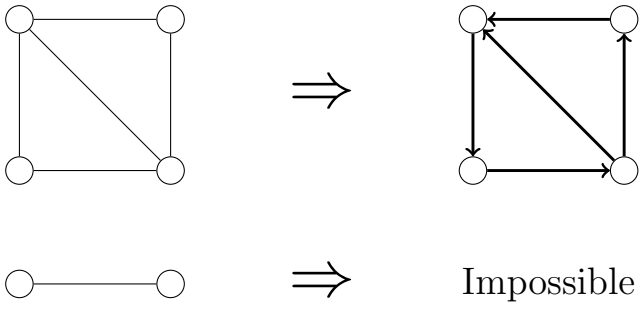
P1) Given a connected graph $G = (V, E)$ with n vertices such that the degree of every vertex of G is at most k. Furthermore, assume that G has a vertex of degree smaller than k. Design a polynomial time algorithm to color vertices of G with k colors such that the endpoints of every edge of G have distinct colors (note that you don't have to use all of the k colors). See the following graph for an example (here $k = 3$)



P2) Given a directed graph G with n vertices $V = \{1, 2, \ldots, n\}$ and m edges. We say that a vertex j is reachable from i if there is a directed path from i to j. Design an $O(m+n)$-time algorithm that for any vertex i outputs the smallest label reachable from i. For example, given the following graph you should output $1, 2, 2, 2, 1$ corresponding to the smallest indices reachable from vertices $1, 2, 3, 4, 5$ respectively.



P3) Given a connected undirected graph $G = (V, E)$, design a polynomial time algorithm to find an orientation of edges of G such that there is no source. If no such orientation exists output "Impossible". For example, given the graph on the left in the right we illustrated an orietnation with no source vertex.



P4) In a stock market, there is a product with an infinite number of shares. A genie tells you the price of this stock the next n days, $p_0, \ldots, p_{n-1}$ where p_i is the price on day i. In this market you can buy at most c_i many shares on day i. Suppose that your budget is B dollars. Design

a polynomial time algorithm that outputs the maximum number of stocks you can buy. For example, say $p_0 = 3, p_1 = 2$, $c_0 = 2, c_1 = 2$ and $B = 7$. Then, you will buy 1 on day 0 and 2 on day 1. So you should output 3.

P5) **Extra Credit:** Suppose G is a 3-colorable graph with n vertices, i.e., it is possible to color the vertices of G with 3 colors such that the endpoints of every edge have distinct colors. Design a polynomial time algorithm that colors vertices of G with $O(\sqrt{n})$ many colors.