CSE421: Design and Analysis of Algorithms	May 1th, 2024
Homework 5	
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P1) Let G be a graph such that $deg(v) \leq k$ for all v. A set $S \subseteq V$ of vertices of G form an independent set if there is no edges between vertices of S, i.e., for any $u, v \in S$, u, v are not connected by an edge. Design a polynomial time $O(k)$ approximation algorithm for the maximum independent set problem on G. Your algorithm needs to output an independent set S whose size is at least $\Omega(1/k)$ of the maximum independent set of G.

P2) Draw the dynamic programming table of the following instance of the knapsack problem: You are given 5 items with weight $1, 3, 5, 7, 9$ and value $1, 2, 4, 5, 7$ respectively and the size of your knapsack is 14.

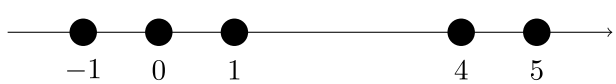
P3) You are a cashier at a Grocery store in a country with coins of values $v_1, v_2, \ldots, v_n$ dollars (you can assume $v_1, \ldots, v_n$ are positive integers). Furthermore, assume you have an infinite supply of each coin. A customer comes and you need to make a change for k dollars. Design an algorithm that runs in time polynomial in n, k and outputs the minimum number of coins you can use to make a change for k dollars.

P4) Given 3 integers $n \geq 1$ and $k_A, k_B \geq 1$, design an algorithm that runs in time polynomial in n, k_A, k_B and outputs the number of length n strings composed of copies of A, B such that no more than k_A copies of A are placed consecutively and no more than k_B copies of B are placed consecutively. For example, if $n = 3, k_A = k_B = 2$ you should output 6 corresponding to the following sequences:

$$AAB, BBA, ABB, BAA, ABA, BAB$$

P5) Interstate highway 5 is a straight highway from Washington all the way to California. There are n villages alongside this highway. Think about the highway as an integer axis, and the position of village i is an integer x_i along this axis. Assume that there are no two villages in the same position, i.e., $x_i \neq x_j$ for $i \neq j$. The distance between two villages x_i, x_j is simply $|x_i - x_j|$.

USPS is interested in building k post offices in some, but not necessarily all of the villages along highway 5, for some $1 \leq k \leq n$. A village and the post office in it have the same position. We want to choose the positions of these post offices so that the *sum* of the distances from each village to its *nearest* post office is minimized. Design an algorithm that runs in time polynomial in n and outputs the minimum possible sum of distances to the optimal location for post offices. For example, given the following location of 5 cities if $k = 2$ then the optimal location for post offices are at village 0 and 4 and your algorithm should output 3 as the *sum* of distance to nearest post offices.



P6) **Extra Credit:** A k-hypergraph is composed of a set V of vertices and a set of hyperedges where every hyperedge is a subset of V of size at least 2 and at most k, i.e., S is a hyperedge if $S \subseteq V$ and $2 \leq |S| \leq k$. Note that 2-hypergraph is the same as a graph. Given a k-hypergraph $G = (V, E)$ with n vertices where for some $k \geq 2$ design a k-approximation algorithm for the vertex cover problem: Find the minimum set W of vertices of G such that every hyperedge $S \in E$ has at least one vertex of W.