CSE421: Design and Analysis of Algorithms	March 28, 2024
Shayan Oveis Gharan	Section 1

P1) Consider the following stable matching instance:

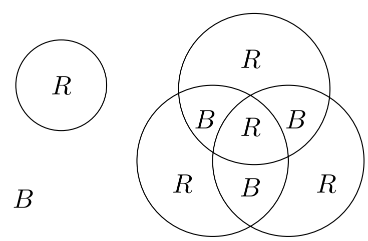
$$c_1 : a_3 > a_1 > a_2 > a_4 \qquad a_1 : c_4 > c_1 > c_3 > c_2$$
$$c_2 : a_2 > a_1 > a_4 > a_3 \qquad a_2 : c_1 > c_3 > c_2 > c_4$$
$$c_3 : a_2 > a_3 > a_1 > a_4 \qquad a_3 : c_1 > c_3 > c_4 > c_2$$
$$c_4 : a_3 > a_4 > a_1 > a_2 \qquad a_4 : c_3 > c_1 > c_2 > c_4$$

a) Run the Gale-Shapley Algorithm with companies proposing on the instance above. When choosing which free company to propose next, always choose the one with the smallest index (e.g., if c_1 and c_2 are both free, always choose c_1).

b) Now run the algorithm with applicants proposing, breaking ties by taking the free applicant with the **smallest** index. Do you get the same result?

P2) Show that an instance of the stable matching problem has exactly one stable matching if and only if the company optimal matching is equal to the applicant optimal matching.

P3) Suppose we have drawn n circles on the plane. Show that we can color the regions with 2 colors (R/B) such that any two neighboring regions are colored with distinct colors. Two regions are neighbors if the share a line segment. See the following example:



a) First explain what is wrong with the following inductive proof: We prove by induction that any n circles drawn on the plane can be colored with R/B such that any two neighboring regions have distinct colors.

The claim obviously holds for $n = 1$ we have a single circle and we color inside R and outside B.

Suppose have colored the regions with $n-1$ circles. Now, we add the n-th circle in such a way that it doesn't cross any of the previous $n-1$ circles. and we color inside of it the opposite of the outside region.

b) Now, solve the problem with a correct inductive proof.