CSE 421
Introduction to Algorithms

Richard Anderson

Winter 2024

Lecture 1

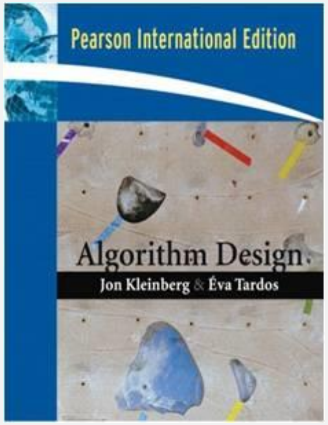
CSE 421 Course Introduction

- CSE 421, Introductions to Algorithms
  - MWF 1:30-2:20 PM, CSE2 G10
  - Thursday Section
- Instructor
  - Richard Anderson, anderson@cs.washington.edu
  - Office hours:
    - Office hours: TBD, CSE2 344
- Teaching Assistants
  - Raymond Gao, Sophie Robertson, Aman Thukral, Kaiyuan Liu, Albert Weng, Tom Zhaoyang Tian

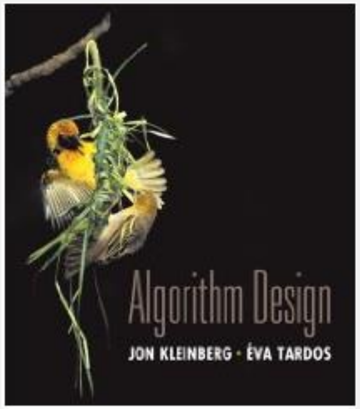
Announcements

- It's on the course website
  - https://courses.cs.washington.edu/courses/cse421/24wi/
- Homework weekly
  - Due Wednesdays
  - HW 1, Due Wednesday, January 10, 2024.
  - It's on the website
- Homework is to be submitted electronically
  - Due at 11:59 pm, Fridays. Five late days.
- Edstem Discussion Board
- Panopto Videos

Textbook



- Algorithm Design
- Jon Kleinberg, Eva Tardos
  - Only one edition
- Read Chapters 1 & 2
- Expected coverage:
  - Chapter 1 through 7
- Book available at:
  - UW Bookstore ($197.50/$79.99)
  - Ebay ($11.27 to $192.70)
  - Amazon ($156.95/$28.76)
  - Electronic ($10.99 per month)
  - PDF



Course Mechanics

- Homework
  - Due Wednesdays
  - Mix of written problems and programming
  - Target: 1-week turnaround on grading
- Exams
  - Midterm, Friday, February 9
  - Final, Monday, March 11, 2:30-4:20 PM
  - Approximate grade weighting:
    - HW: 50, MT: 15, Final: 35
- Course web
  - Slides, Handouts, Discussion Board
- Canvas
  - Panopto videos
- Section on Thursdays
  - Recent addition for CSE421

All of Computer Science is the Study of Algorithms

How to study algorithms

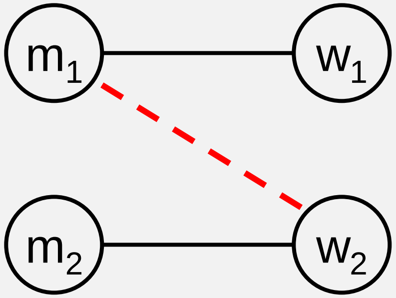
- Zoology
- Mine is faster than yours is
- Algorithmic ideas
  - Where algorithms apply
  - What makes an algorithm work
  - Algorithmic thinking
- Algorithm practice

Introductory Problem: Stable Matching

- Setting:
  - Assign TAs to Instructors
  - Avoid having TAs and Instructors wanting changes
    - E.g., Prof A. would rather have student X than her current TA, and student X would rather work for Prof A. than his current instructor.

Formal notions

- Perfect matching
- Ranked preference lists
- Stability



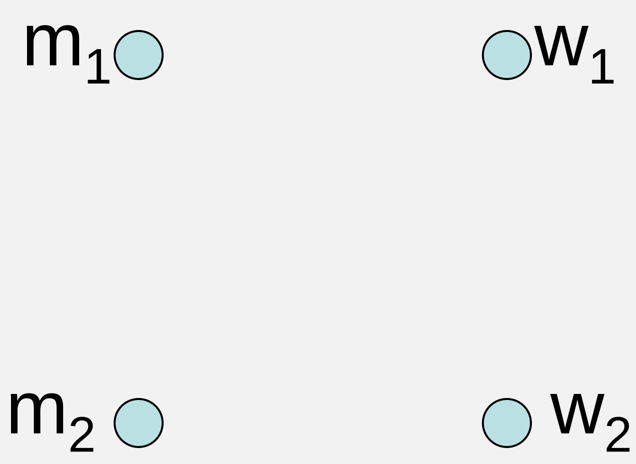
Example (1 of 3)

m_1: w_1 w_2

m_2: w_2 w_1

w_1: m_1 m_2

w_2: m_2 m_1



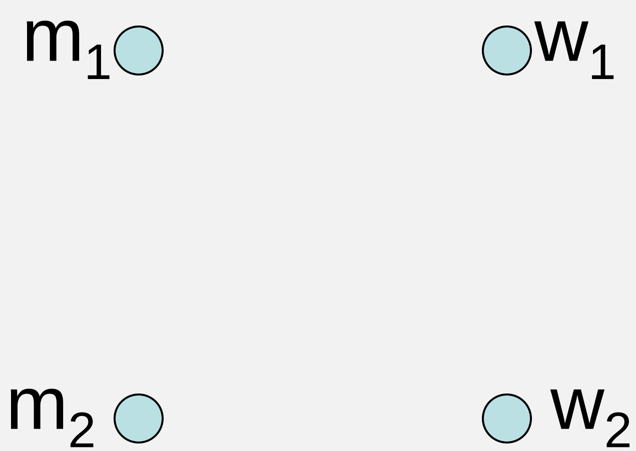
Example (2 of 3)

m_1: w_1 w_2

m_2: w_1 w_2

w_1: m_1 m_2

w_2: m_1 m_2



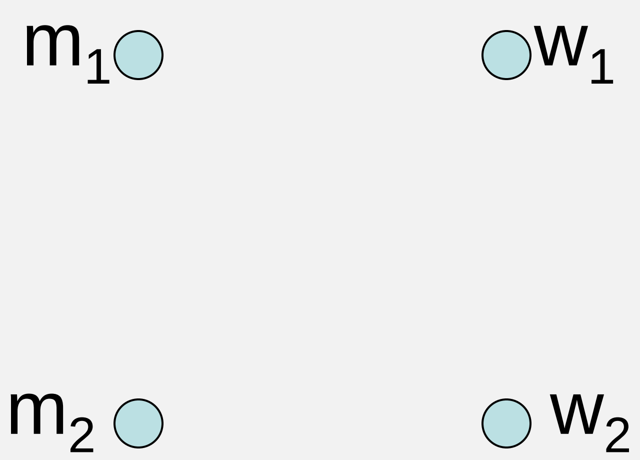
Example (3 of 3)

m_1: w_1 w_2

m_2: w_2 w_1

w_1: m_2 m_1

w_2: m_1 m_2



Formal Problem

- Input
  - Preference lists for $m_1, m_2, \ldots, m_n$
  - Preference lists for $w_1, w_2, \ldots, w_n$
- Output
  - Perfect matching M satisfying stability property:

If (m', w') $\in$ M and (m'', w'') $\in$ M then
(m' prefers w' to w'') or (w'' prefers m'' to m')

Idea for an Algorithm

m proposes to w

- If w is unmatched, w accepts
- If w is matched to m_2
  - If w prefers m to m_2 w accepts m, dumping m_2
  - If w prefers m_2 to m, w rejects m

Unmatched m proposes to the highest w on its preference list that it has not already proposed to

Algorithm

Initially all m in M and w in W are free
While there is a free m
    w highest on m's list that m has not proposed to
    if w is free, then match (m, w)
    else
        suppose (m_2, w) is matched
        if w prefers m to m_2
            unmatch (m_2, w)
            match (m, w)

Example

m_1: w_1 w_2 w_3

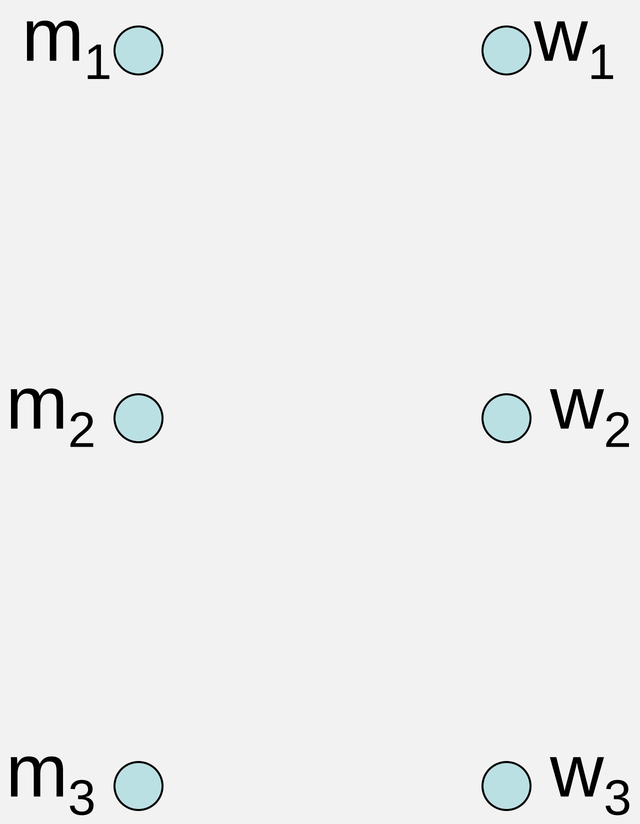
m_2: w_1 w_3 w_2

m_3: w_1 w_2 w_3

w_1: m_2 m_3 m_1

w_2: m_3 m_1 m_2

w_3: m_3 m_1 m_2



Does this work?

- Does it terminate?
- Is the result a stable matching?

- Begin by identifying invariants and measures of progress
  – m’s proposals get worse (have higher m-rank)
  – Once w is matched, w stays matched
  – w’s partners get better (have lower w-rank)

Claim: If an m reaches the end of its list, then all the w's are matched

Claim: The algorithm stops in at most n^2 steps

When the algorithms halts, every w is matched

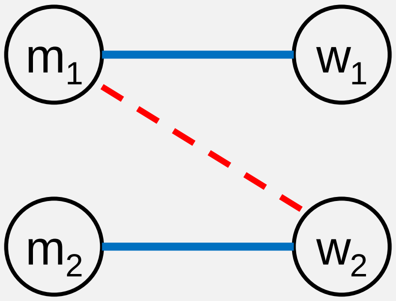
Why?

Hence, the algorithm finds a perfect matching

The resulting matching is stable

Suppose

- $(m_1, w_1) \in M$, $(m_2, w_2) \in M$
- m_1 prefers w_2 to w_1



How could this happen?

Result

- Simple, $O(n^2)$ algorithm to compute a stable matching
- Corollary
  - A stable matching always exists