Lecture 20:
Query Execution: Relational Algebra

Wednesday, May 16, 2007

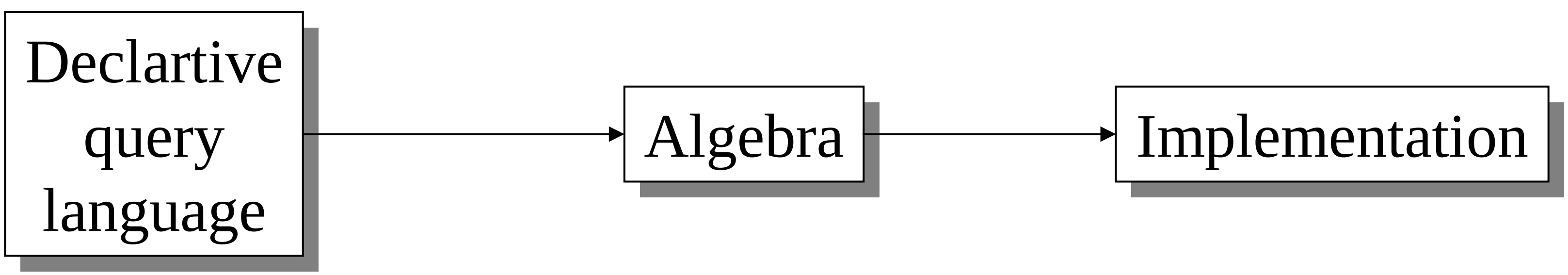
DBMS Architecture

How does a SQL engine work ?

- SQL query $\rightarrow$ relational algebra plan
- Relational algebra plan $\rightarrow$ Optimized plan
- Execute each operator of the plan

Relational Algebra

- Formalism for creating new relations from existing ones
- Its place in the big picture:



SQL,
relational calculus

Relational algebra
Relational bag algebra

Relational Algebra

- Five operators:
  - Union: $\cup$
  - Difference: -
  - Selection: σ
  - Projection: Π
  - Cartesian Product: $\times$
- Derived or auxiliary operators:
  - Intersection, complement
  - Joins (natural,equi-join, theta join, semi-join)
  - Renaming: ρ

1. Union and 2. Difference

- R1 $\cup$ R2
- Example:
  – ActiveEmployees $\cup$ RetiredEmployees

- R1 – R2
- Example:
  – AllEmployees -- RetiredEmployees

What about Intersection ?

- It is a derived operator
- $R1 \cap R2 = R1 - (R1 - R2)$
- Also expressed as a join (will see later)
- Example
  - UnionizedEmployees $\cap$ RetiredEmployees

3. Selection

- Returns all tuples which satisfy a condition
- Notation: $\sigma_c(R)$
- Examples
  - $\sigma_{Salary > 40000}$ (Employee)
  - $\sigma_{name = \text{“Smith”}}$ (Employee)
- The condition c can be $=, <, \leq, >, \geq, <>$

SSN	Name	Salary
1234545	John	200000
5423341	Smith	600000
4352342	Fred	500000

$\sigma_{Salary > 40000}$ (Employee)

SSN	Name	Salary
5423341	Smith	600000
4352342	Fred	500000

4. Projection

- Eliminates columns, then removes duplicates
- Notation: $\Pi_{A1,\ldots,An}(R)$
- Example: project social-security number and names:
  - $\Pi_{SSN,\ Name}$ (Employee)
  - Output schema: Answer(SSN, Name)

SSN	Name	Salary
1234545	John	200000
5423341	John	600000
4352342	John	200000

$\Pi_{Name,Salary}$ (Employee)

Name	Salary
John	20000
John	60000

5. Cartesian Product

- Each tuple in R1 with each tuple in R2
- Notation: R1 $\times$ R2
- Example:
  - Employee $\times$ Dependents
- Very rare in practice; mainly used to express joins

Cartesian Product Example

Employee

Name	SSN
John	999999999
Tony	777777777

Dependents

EmployeeSSN	Dname
999999999	Emily
777777777	Joe

Employee x Dependents

Name	SSN	EmployeeSSN	Dname
John	999999999	999999999	Emily
John	999999999	777777777	Joe
Tony	777777777	999999999	Emily
Tony	777777777	777777777	Joe

Relational Algebra

- Five operators:
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Renaming

- Changes the schema, not the instance
- Notation: $\rho_{B1,\ldots,Bn}$ (R)
- Example:
  - $\rho_{LastName,\ SocSocNo}$ (Employee)
  - Output schema: Answer(LastName, SocSocNo)

Renaming Example

Employee

Name	SSN
John	999999999
Tony	777777777

$$\rho_{LastName,\ SocSocNo}\ (\textbf{Employee})$$

LastName	SocSocNo
John	999999999
Tony	777777777

Natural Join

- Notation: $R1 \mid\times\mid R2$
- Meaning: $R1 \mid\times\mid R2 = \Pi_A(\sigma_C(R1 \times R2))$
- Where:
  - The selection σ_C checks equality of all common attributes
  - The projection eliminates the duplicate common attributes

Natural Join Example

Employee

Name	SSN
John	999999999
Tony	777777777

Dependents

SSN	Dname
999999999	Emily
777777777	Joe

Employee $\bowtie$ Dependents =

$$\Pi_{Name,\ SSN,\ Dname}(\sigma_{SSN=SSN2}(Employee \times \rho_{SSN2,\ Dname}(Dependents))$$

Name	SSN	Dname
John	999999999	Emily
Tony	777777777	Joe

Natural Join

- R=

A	B
X	Y
X	Z
Y	Z
Z	V

S=

B	C
Z	U
V	W
Z	V

- R |×| S=

A	B	C
X	Z	U
X	Z	V
Y	Z	U
Y	Z	V
Z	V	W

Natural Join

- Given the schemas R(A, B, C, D), S(A, C, E), what is the schema of R |×| S ?

- Given R(A, B, C), S(D, E), what is R |×| S ?

- Given R(A, B), S(A, B), what is R |×| S ?

Theta Join

- A join that involves a predicate
- $R1 \,|\times|_{\theta}\, R2 \;=\; \sigma_{\theta}(R1 \times R2)$
- Here θ can be any condition

Eq-join

- A theta join where θ is an equality
- $R1 \,|\times|_{A=B}\, R2 \;=\; \sigma_{A=B} (R1 \times R2)$
- Example:
  - Employee $|\times|_{SSN=SSN}$ Dependents

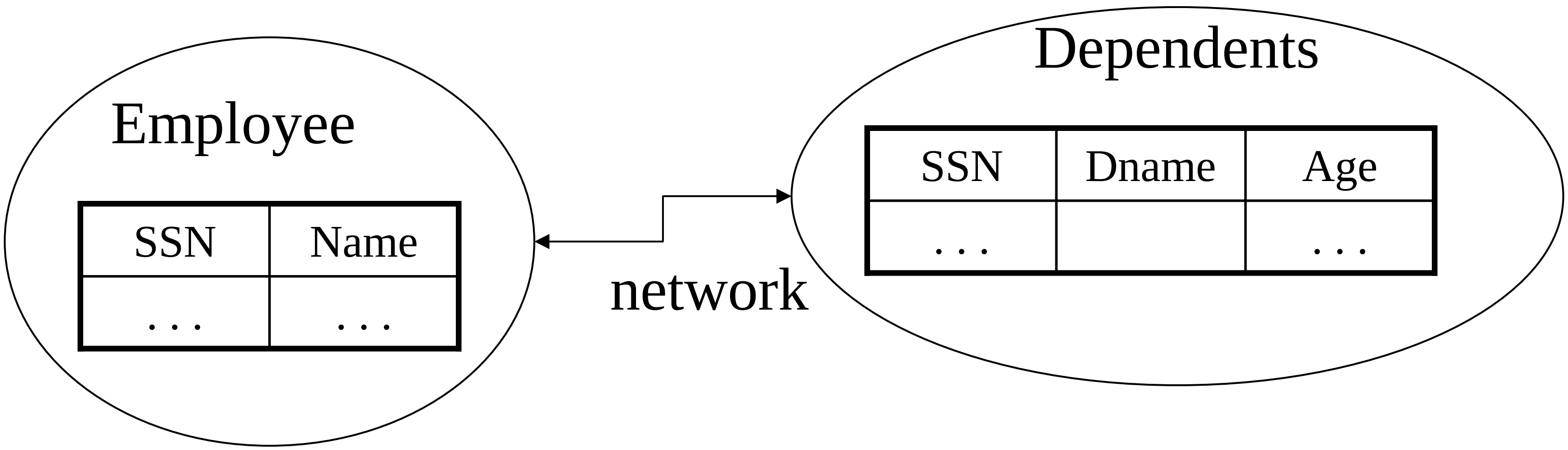
- Most useful join in practice

Semijoin

- $R \ltimes S = \Pi_{A1,\ldots,An} (R \bowtie S)$
- Where $A_1, \ldots, A_n$ are the attributes in R
- Example:
  - Employee $\ltimes$ Dependents

Semijoins in Distributed Databases

- Semijoins are used in distributed databases



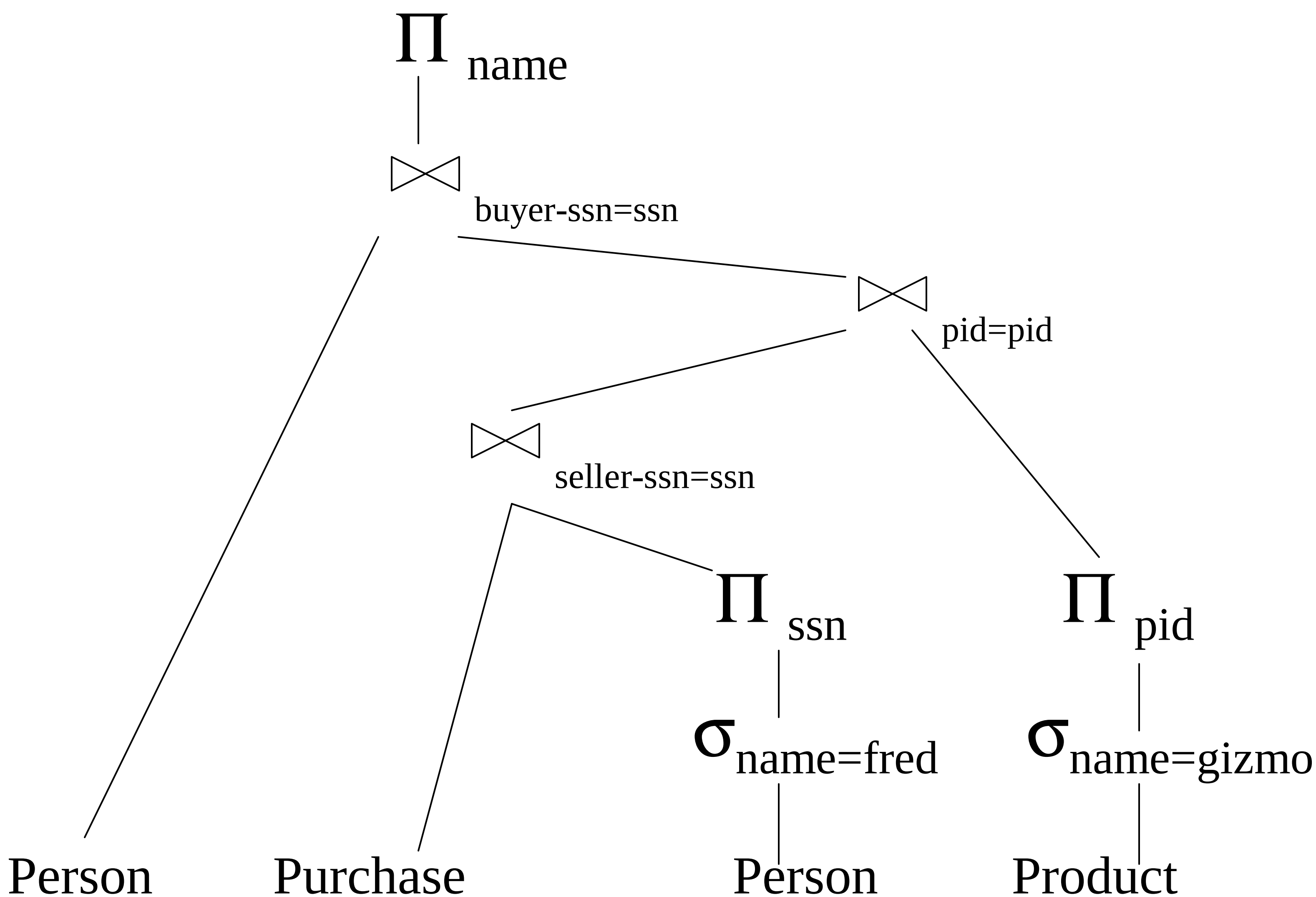
$$\text{Employee} \bowtie_{ssn=ssn} (\sigma_{age>71}(\text{Dependents}))$$

$T = \Pi_{SSN}\, \sigma_{age>71}(\text{Dependents})$

$R = \text{Employee} \ltimes T$

$\text{Answer} = R \bowtie \text{Dependents}$

Complex RA Expressions



Operations on Bags

A **bag** = a set with repeated elements

All operations need to be defined carefully on bags

- {a,b,b,c}$\cup${a,b,b,b,e,f,f}={a,a,b,b,b,b,b,c,e,f,f}
- {a,b,b,b,c,c} – {b,c,c,c,d} = {a,b,b,d}
- σ_C(R): preserve the number of occurrences
- Π_A(R): no duplicate elimination
- Cartesian product, join: no duplicate elimination

Important ! Relational Engines work on bags, not sets !

Reading assignment: 5.3 – 5.4

Note: RA has Limitations !

- Cannot compute “transitive closure”

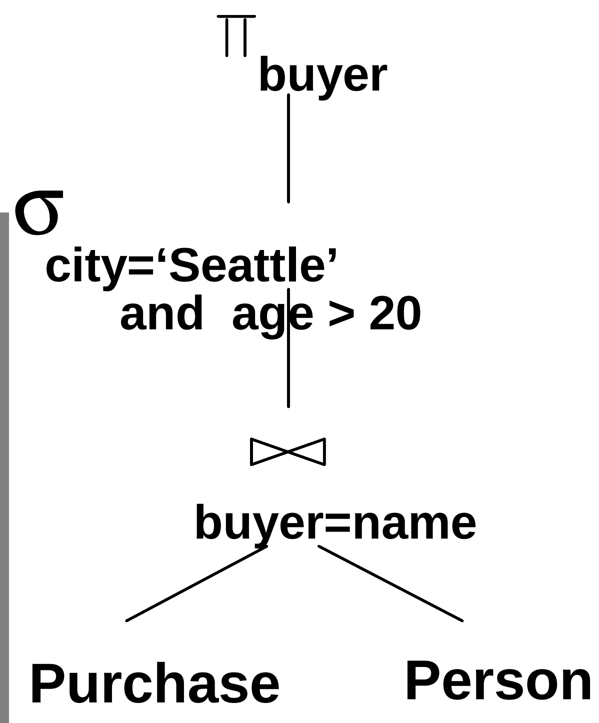
Name1	Name2	Relationship
Fred	Mary	Father
Mary	Joe	Cousin
Mary	Bill	Spouse
Nancy	Lou	Sister

- Find all direct and indirect relatives of Fred
- Cannot express in RA !!! Need to write C program

From SQL to RA

Purchase(buyer, product, city)
Person(name, age)

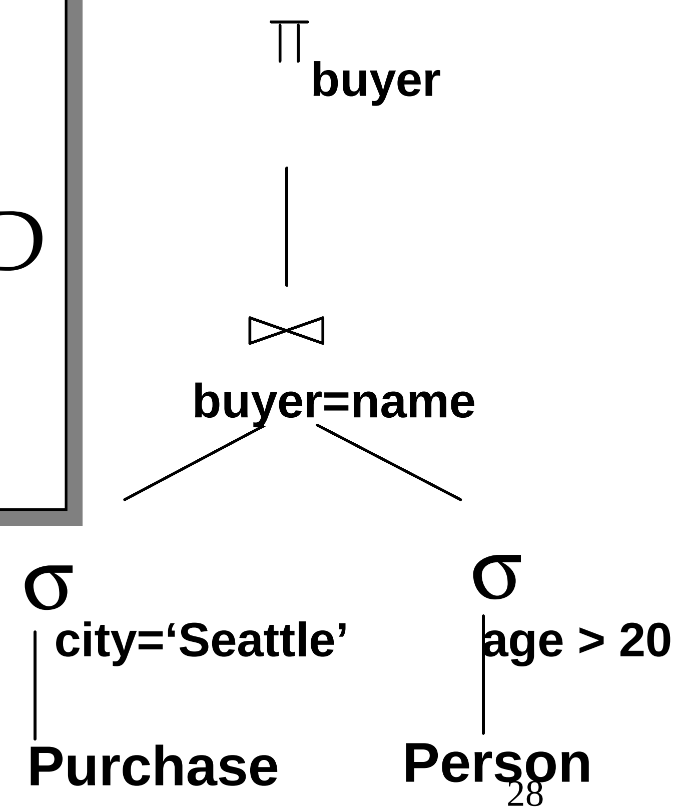
```
SELECT DISTINCT P.buyer
FROM Purchase P, Person Q
WHERE P.buyer=Q.name AND
      P.city='Seattle' AND
     Q.age > 20
```



Also…

Purchase(buyer, product, city)
Person(name, age)

```
SELECT DISTINCT P.buyer
FROM Purchase P, Person Q
WHERE P.buyer=Q.name AND
      P.city='Seattle' AND
    Q.age > 20
```



Non-monontone Queries (in class)

Purchase(buyer, product, city)

Person(name, age)

```
SELECT DISTINCT P.product
FROM Purchase P
WHERE P.city='Seattle' AND
   not exists (select *
               from Purchase P2, Person Q
               where P2.product = P.product
                  and P2.buyer = Q.name
                  and Q.age > 20)
```

Extended Logical Algebra Operators (operate on Bags, not Sets)

- Union, intersection, difference
- Selection σ
- Projection Π
- Join |x|
- Duplicate elimination δ
- Grouping γ
- Sorting τ

Logical Query Plan

```
SELECT city, count(*)
FROM sales
GROUP BY city
HAVING sum(price) > 100
```

T3(city, c)

$\Pi_{\text{city, c}}$

T2(city,p,c)

$\sigma_{p > 100}$

T1(city,p,c)

$\gamma_{\text{city, sum(price)} \rightarrow \text{p, count(*)} \rightarrow \text{c}}$

sales(product, city, price)

T1, T2, T3 = temporary tables

Logical v.s. Physical Algebra

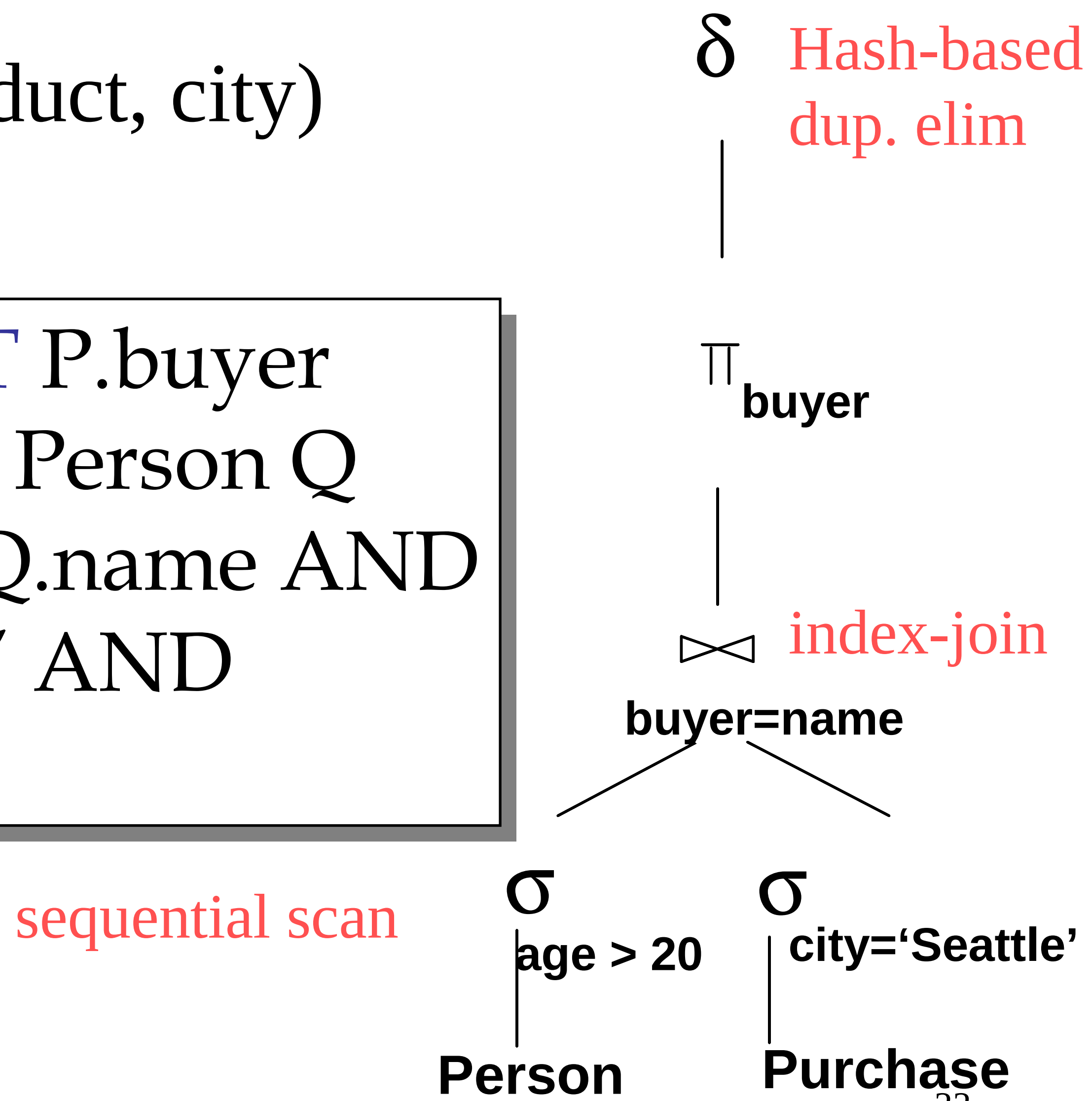
- We have seen the logical algebra so far:
  - Five basic operators, plus group-by, plus sort

- The Physical algebra refines each operator into a concrete algorithm

Physical Plan

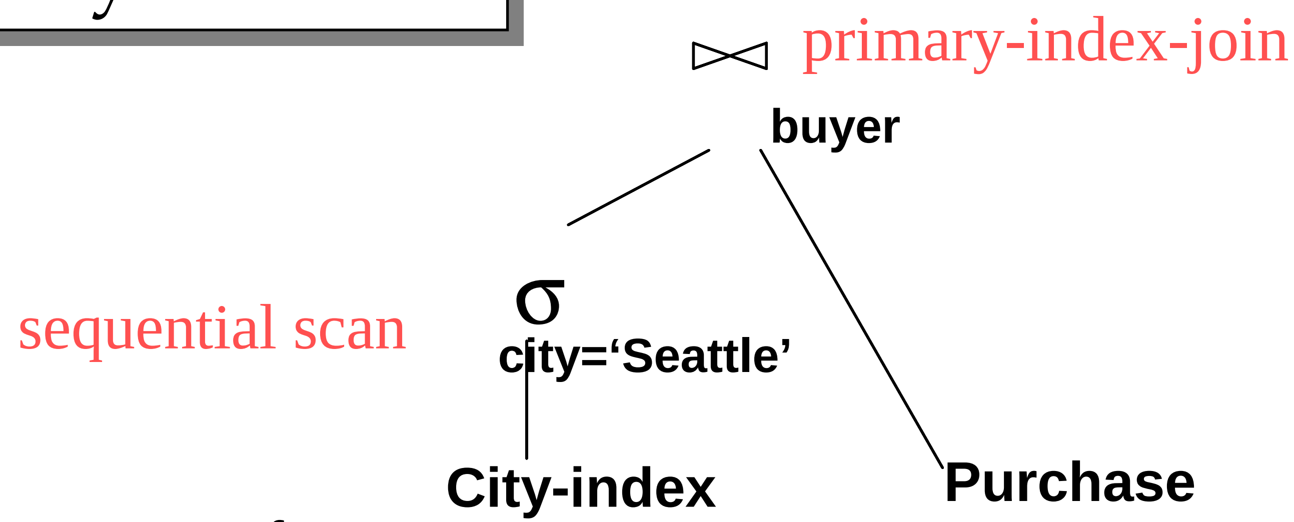
Purchase(buyer, product, city)
Person(name, age)

SELECT DISTINCT P.buyer
FROM Purchase P, Person Q
WHERE P.buyer=Q.name AND
P.city='Seattle' AND
Q.age > 20



Physical Plans Can Be Subtle

```
SELECT *
FROM Purchase P
WHERE P.city='Seattle'
```



Where did the join come from ?