# Lecture 22: <br> Query Optimization 

Wednesday, May 23, 2007 to
Wednesday, May 30, 2007

## Outline

- An example
- Query optimization: algebraic laws 16.2
- Cost-based optimization 16.5, 16.6
- Cost estimation: 16.4


## Example

Product(pname, maker), Company(cname, city)

> Select Product.pname
> From Product, Company
> Where Product.maker=Company.cname and Company.city $=$ "Seattle"

- How do we execute this query?


## Example

Product(pname, maker), Company(cname, city)

Assume:

Clustered index: Product.pname, Company.cname
Unclustered index: Product.maker, Company.city

## Logical Plan:



## Physical plan 1:



## Physical plans 2a and 2b:

## Which one is better ??

Merge-join



```
Total cost:
(2a): 3B(Product) + B(Company)
(2b): T(Product) + B(Company)
```

Physical plans 2a and 2b:


# Plan 1: $\mathrm{T}($ Company $) / \mathrm{V}($ Company, city $) \times$ T(Product)/V(Product,maker) <br> Plan 2a: B(Company) + 3B(Product) Plan 2b: B(Company) +T (Product) 

## Which one is better ??

It depends on the data !!

## Example

$$
\begin{aligned}
& \mathrm{T}(\text { Company })=5,000 \\
& \mathrm{~T}(\text { Product })=100,000 \quad \mathrm{~B}(\text { Company })=500 \quad \mathrm{M}=100 \\
& \\
& \text { We may assume } \mathrm{V}(\text { Product, maker }) \approx \mathrm{T}(\text { Company }) \text { (why ?) }
\end{aligned}
$$

- Case $1: \mathrm{V}($ Company, city $) \approx \mathrm{T}($ Company $)$

$$
\mathrm{V}(\text { Company,city })=2,000
$$

- Case 2: V(Company, city) << T(Company)

$$
\mathrm{V}(\text { Company,city })=20
$$

## Which Plan is Best?

```
Plan 1: T(Company)/V(Company,city) }\times\textrm{T}(\mathrm{ Product )/V(Product,maker)
Plan 2a: B(Company) + 3B(Product)
Plan 2b: B(Company) + T(Product)
```

Case 1:

Case 2:

## Lessons

- Need to consider several physical plan
- even for one, simple logical plan
- No magic "best" plan: depends on the data
- In order to make the right choice
- need to have statistics over the data
- the B's, the T's, the V's


## Query Optimzation

- Have a SQL query Q
- Create a plan P

- Find equivalent plans $\mathrm{P}=\mathrm{P}^{\prime}=\mathrm{P}^{\prime}{ }^{\prime}=\ldots$
- Choose the "cheapest".


## Logical Query Plan

## SELECT P.buyer <br> FROM Purchase P, Person Q <br> WHERE P.buyer=Q.name AND P.city='seattle' AND <br> Q.phone > '5430000'



Purchasse(buyer, city)
Person(name, phone)


## Logical Query Plan

$\mathrm{Q}=$
SELECT city, sum(quantity) FROM sales GROUP BY city HAVING sum(quantity) < 100

find a "better" plan P'

# The three components of an optimizer 

We need three things in an optimizer:

- Algebraic laws
- An optimization algorithm
- A cost estimator


## Algebraic Laws (incomplete list)

- Commutative and Associative Laws
$R \cup S=S \cup R, R \cup(S \cup T)=(R \cup S) \cup T$ $R|x| S=S|x| R, R|x|(S|x| T)=(R|x| S)|x| T$
- Distributive Laws

$$
R|x|(S \cup T)=(R|x| S) \cup(R|x| T)
$$

## Algebraic Laws (incomplete list)

- Laws involving selection:

$$
\begin{aligned}
& \sigma_{\mathrm{C} \text { AND } \mathrm{C}^{\prime}}(\mathrm{R})=\sigma_{\mathrm{C}}\left(\sigma_{\mathrm{C}^{\prime}}(\mathrm{R})\right) \\
& \sigma_{\mathrm{C} \mathrm{OR} \mathrm{C}},(\mathrm{R})=\sigma_{\mathrm{C}}(\mathrm{R}) \cup \sigma_{\mathrm{C}^{\prime}}(\mathrm{R})
\end{aligned}
$$

- When C involves only attributes of R

$$
\begin{aligned}
& \sigma_{\mathrm{C}}(\mathrm{R}|\times| S)=\sigma_{\mathrm{C}}(\mathrm{R})|\times| S \\
& \sigma_{\mathrm{C}}(\mathrm{R}-\mathrm{S})=\sigma_{\mathrm{C}}(\mathrm{R})-\mathrm{S} \\
& \sigma_{\mathrm{C}}(\mathrm{R}|\times| S)=\sigma_{\mathrm{C}}(\mathrm{R})|\times| S
\end{aligned}
$$

## Algebraic Laws

- Example: R(A, B, C, D), S(E, F, G)

$$
\sigma_{\mathrm{F}=3}\left(\mathrm{R}|\times|_{\mathrm{D}=\mathrm{E}} \mathrm{~S}\right)=
$$

$$
\sigma_{\mathrm{A}=5 \mathrm{AND} \mathrm{G}=9}\left(\mathrm{R}|\times|_{\mathrm{D}=\mathrm{E}} \mathrm{~S}\right)=?
$$

## Algebraic Laws

- Laws involving projections

$$
\begin{aligned}
& \Pi_{\mathrm{M}}(\mathrm{R}|\times| \mathrm{S})=\Pi_{\mathrm{M}}\left(\Pi_{\mathrm{P}}(\mathrm{R})|\times| \Pi_{\mathrm{Q}}(\mathrm{~S})\right) \\
& \Pi_{\mathrm{M}}\left(\Pi_{\mathrm{N}}(\mathrm{R})\right)=\Pi_{\mathrm{M}, \mathrm{~V}}(\mathrm{R})
\end{aligned}
$$

- Example R(A,B,C,D), S(E, F, G)
$\Pi_{\mathrm{A}, \mathrm{B}, \mathrm{G}}\left(\mathrm{R}|\times|_{\mathrm{D}=\mathrm{E}} \mathrm{S}\right)=\Pi_{?}\left(\Pi_{?}(\mathrm{R})|\times|_{\mathrm{D}=\mathrm{E}} \Pi_{?}(\mathrm{~S})\right)$


## Algebraic Laws

- Laws involving grouping and aggregation:

$$
\begin{aligned}
& \delta\left(\gamma_{\mathrm{A}, \operatorname{agg}(\mathrm{~B})}(\mathrm{R})\right)=\gamma_{\mathrm{A}, \operatorname{agg}(\mathrm{~B})}(\mathrm{R}) \\
& \gamma_{\mathrm{A}, \operatorname{agg}(\mathrm{~B})}(\delta(\mathrm{R}))=\gamma_{\mathrm{A}, \operatorname{agg}(\mathrm{~B})}(\mathrm{R}) \text { if agg is "duplicate insensitive" }
\end{aligned}
$$

- Which of the following are "duplicate insensitive" ? sum, count, avg, min, max

$$
\begin{aligned}
& \gamma_{\mathrm{A}, \operatorname{agg}(\mathrm{D})}\left(\mathrm{R}(\mathrm{~A}, \mathrm{~B})|\times|_{\mathrm{B}=\mathrm{C}} \mathrm{~S}(\mathrm{C}, \mathrm{D})\right)= \\
& \quad \gamma_{\mathrm{A}, \operatorname{agg}(\mathrm{D})}\left(\mathrm{R}(\mathrm{~A}, \mathrm{~B})|\times|_{\mathrm{B}=\mathrm{C}}\left(\gamma_{\mathrm{C}, \operatorname{agg}(\mathrm{D})} \mathrm{S}(\mathrm{C}, \mathrm{D})\right)\right)
\end{aligned}
$$

## Cost-based Optimizations

- Main idea: apply algebraic laws, until estimated cost is minimal
- Practically: start from partial plans, introduce operators one by one
- Will see in a few slides
- Problem: there are too many ways to apply the laws, hence too many (partial) plans


## Cost-based Optimizations

Approaches:

- Top-down: the partial plan is a top fragment of the logical plan
- Bottom up: the partial plan is a bottom fragment of the logical plan


## Dynamic Programming

Originally proposed in System R (the first research prototype for a relational database system -- late 70s)

- Only handles single block queries:

```
SELECT list
FROM list
WHERE cond \({ }_{1}\) AND cond 2 AND . . . AND cond \(_{\mathrm{k}}\)
```

- Heuristics: selections down, projections up
- Dynamic programming: join reordering


## Join Trees

- R1 $1 \times \mid$ R2 $|\times|\ldots .|\times|$ Rn
- Join tree:

- A plan = a join tree
- A partial plan $=$ a subtree of a join tree


## Types of Join Trees

- Left deep:



## Types of Join Trees

- Bushy:



## Types of Join Trees

- Right deep:



## Dynamic Programming

- Given: a query R1 $1 \times \mid$ R2 $|x| \ldots|x| \mathrm{Rn}$
- Assume we have a function cost() that gives us the cost of every join tree
- Find the best join tree for the query


## Dynamic Programming

- Idea: for each subset of $\{\mathrm{R} 1, \ldots, \mathrm{Rn}\}$, compute the best plan for that subset
- In increasing order of set cardinality:
- Step 1: for $\{R 1\},\{R 2\}, \ldots,\{R n\}$
- Step 2: for $\{R 1, R 2\},\{R 1, R 3\}, \ldots,\{R n-1, R n\}$
- ...
- Step n: for $\{$ R1, ..., Rn $\}$
- It is a bottom-up strategy
- A subset of $\{\mathrm{R} 1, \ldots, \mathrm{Rn}\}$ is also called a subquery


## Dynamic Programming

- For each subquery $\mathrm{Q} \subseteq\{\mathrm{R} 1, \ldots, \mathrm{Rn}\}$ compute the following:
- Size(Q)
- A best plan for Q: Plan(Q)
- The cost of that plan: $\operatorname{Cost}(\mathrm{Q})$


## Dynamic Programming

- Step 1: For each $\left\{\mathrm{R}_{\mathrm{i}}\right\}$ do:
$-\operatorname{Size}\left(\left\{R_{i}\right\}\right)=B\left(R_{i}\right)$
$-\operatorname{Plan}\left(\left\{R_{i}\right\}\right)=R_{i}$
$-\operatorname{Cost}\left(\left\{\mathrm{R}_{\mathrm{i}}\right\}\right)=\left(\right.$ cost of scanning $\left.\mathrm{R}_{\mathrm{i}}\right)$


## Dynamic Programming

- Step i: For each $\mathrm{Q} \subseteq\left\{\mathrm{R}_{1}, \ldots, \mathrm{R}_{\mathrm{n}}\right\}$ of cardinality i do:
- Compute Size(Q) (later...)
- For every pair of subqueries Q', Q"'
s.t. $\mathrm{Q}=\mathrm{Q}^{\prime} \cup \mathrm{Q}^{\prime \prime}$
compute $\operatorname{cost}\left(\operatorname{Plan}\left(\mathrm{Q}^{\prime}\right)|\times| \operatorname{Plan}\left(\mathrm{Q}^{\prime \prime}\right)\right)$
$-\operatorname{Cost}(\mathrm{Q})=$ the smallest such cost
$-\mathrm{Plan}(\mathrm{Q})=$ the corresponding plan


## Dynamic Programming

- Return $\operatorname{Plan}\left(\left\{\mathrm{R}_{1}, \ldots, \mathrm{R}_{\mathrm{n}}\right\}\right)$


## Dynamic Programming

To illustrate, we will make the following simplifications:

- $\operatorname{Cost}\left(\mathrm{P}_{1}|x| \mathrm{P}_{2}\right)=\operatorname{Cost}\left(\mathrm{P}_{1}\right)+\operatorname{Cost}\left(\mathrm{P}_{2}\right)+$ size(intermediate result( s ))
- Intermediate results:
- If $\mathrm{P}_{1}=$ a join, then the size of the intermediate result is $\operatorname{size}\left(\mathrm{P}_{1}\right)$, otherwise the size is 0
- Similarly for $\mathrm{P}_{2}$
- Cost of a scan $=0$


## Dynamic Programming

- Example:
- $\operatorname{Cost}(\mathrm{R} 5|\times| \mathrm{R} 7)=0 \quad$ (no intermediate results)
- $\operatorname{Cost}((\mathrm{R} 2|\times| \mathrm{R} 1)|\times| \mathrm{R} 7)$

$$
\begin{aligned}
& =\operatorname{Cost}(\mathrm{R} 2|\times| \mathrm{R} 1)+\operatorname{Cost}(\mathrm{R} 7)+\operatorname{size}(\mathrm{R} 2|\times| \mathrm{R} 1) \\
& =\operatorname{size}(\mathrm{R} 2|\times| \mathrm{R} 1)
\end{aligned}
$$

## Dynamic Programming

- Relations: R, S, T, U
- Number of tuples: 2000, 5000, 3000, 1000
- Size estimation: $\mathrm{T}(\mathrm{A}|\times| \mathrm{B})=0.01 * \mathrm{~T}(\mathrm{~A}) * \mathrm{~T}(\mathrm{~B})$

| Subquery | Size | Cost | Plan |
| :---: | :--- | :--- | :--- |
| RS |  |  |  |
| RT |  |  |  |
| RU |  |  |  |
| ST |  |  |  |
| SU |  |  |  |
| RST |  |  |  |
| RSU |  |  |  |
| RTU |  |  |  |
| RSTU |  |  |  |
| RTU |  |  |  |


| Subquery | Size | Cost | Plan |
| :---: | :---: | :---: | :---: |
| RS | 100k | 0 | RS |
| RT | 60k | 0 | RT |
| RU | 20k | 0 | RU |
| ST | 150k | 0 | ST |
| SU | 50k | 0 | SU |
| TU | 30k | 0 | TU |
| RST | 3 M | 60k | (RT)S |
| RSU | 1 M | 20k | (RU)S |
| RTU | 0.6 M | 20k | (RU)T |
| STU | 1.5 M | 30k | (TU)S |
| RSTU | 30M | $60 \mathrm{k}+50 \mathrm{k}=110 \mathrm{k}$ | (RT)(SU) |

## Reducing the Search Space

- Left-linear trees v.s. Bushy trees
- Trees without cartesian product

Example: $\mathrm{R}(\mathrm{A}, \mathrm{B})|\times|\mathrm{S}(\mathrm{B}, \mathrm{C})| \times| \mathrm{T}(\mathrm{C}, \mathrm{D})$

Plan: ( $\mathrm{R}(\mathrm{A}, \mathrm{B})|\times| \mathrm{T}(\mathrm{C}, \mathrm{D}))|\times| \mathrm{S}(\mathrm{B}, \mathrm{C})$ has a cartesian product most query optimizers will not consider it

## Dynamic Programming: <br> Summary

- Handles only join queries:
- Selections are pushed down (i.e. early)
- Projections are pulled up (i.e. late)
- Takes exponential time in general, BUT:
- Left linear joins may reduce time
- Non-cartesian products may reduce time further


## Rule-Based Optimizers

- Extensible collection of rules

Rule $=$ Algebraic law with a direction

- Algorithm for firing these rules

Generate many alternative plans, in some order Prune by cost

- Volcano (later SQL Sever)
- Starburst (later DB2)


## Completing the Physical Query Plan

- Choose algorithm to implement each operator
- Need to account for more than cost:
- How much memory do we have ?
- Are the input operand(s) sorted ?
- Decide for each intermediate result:
- To materialize
- To pipeline


## Materialize Intermediate Results Between Operators



## Materialize Intermediate Results Between Operators

Question in class

Given $\mathrm{B}(\mathrm{R}), \mathrm{B}(\mathrm{S}), \mathrm{B}(\mathrm{T}), \mathrm{B}(\mathrm{U})$

- What is the total cost of the plan?
- Cost =
- How much main memory do we need ?
- $\mathrm{M}=$


## Pipeline Between Operators



## Pipeline Between Operators

Question in class

Given $B(R), B(S), B(T), B(U)$

- What is the total cost of the plan?
- Cost =
- How much main memory do we need ?
- $\mathrm{M}=$


## Pipeline in Bushy Trees



## Example

- Logical plan is:

- Main memory $\mathrm{M}=101$ buffers


## Example

$$
\mathrm{M}=101
$$



Naïve evaluation:

- 2 partitioned hash-joins
- $\operatorname{Cost} 3 \mathrm{~B}(\mathrm{R})+3 \mathrm{~B}(\mathrm{~S})+4 \mathrm{k}+3 \mathrm{~B}(\mathrm{U})=75000+4 \mathrm{k}$


## Example

$$
\mathrm{M}=101
$$



Smarter:

- Step 1: hash R on $x$ into 100 buckets, each of 50 blocks; to disk
- Step 2: hash $S$ on $x$ into 100 buckets; to disk
- Step 3: read each $\mathrm{R}_{\mathrm{i}}$ in memory ( 50 buffer) join with $\mathrm{S}_{\mathrm{i}}$ (1 buffer); hash result on y into 50 buckets ( 50 buffers) -- here we pipeline
- Cost so far: $3 \mathrm{~B}(\mathrm{R})+3 \mathrm{~B}(\mathrm{~S})$


## Example

$$
\mathrm{M}=101
$$



Continuing:

- How large are the 50 buckets on y ? Answer: k/50.
- If $\mathrm{k}<=50$ then keep all 50 buckets in Step 3 in memory, then:
- Step 4: read U from disk, hash on y and join with memory
- Total cost: 3B(R) + 3B(S) + B(U) = 55,000


## Example

$$
\mathrm{M}=101
$$



Continuing:

- If $50<\mathrm{k}<=5000$ then send the 50 buckets in Step 3 to disk
- Each bucket has size $\mathrm{k} / 50<=100$
- Step 4: partition U into 50 buckets
- Step 5: read each partition and join in memory
- Total cost: $3 \mathrm{~B}(\mathrm{R})+3 \mathrm{~B}(\mathrm{~S})+2 \mathrm{k}+3 \mathrm{~B}(\mathrm{U})=75,000+2 \mathrm{k}$


## Example

$\mathrm{M}=101$


Continuing:

- If $\mathrm{k}>5000$ then materialize instead of pipeline
- 2 partitioned hash-joins
- $\operatorname{Cost} 3 \mathrm{~B}(\mathrm{R})+3 \mathrm{~B}(\mathrm{~S})+4 \mathrm{k}+3 \mathrm{~B}(\mathrm{U})=75000+4 \mathrm{k}$


## Example

Summary:

- If $\mathrm{k}<=50, \quad$ cost $=55,000$
- If $50<\mathrm{k}<=5000, \quad$ cost $=75,000+2 \mathrm{k}$
- If k > 5000,
$\operatorname{cost}=75,000+4 \mathrm{k}$


## Size Estimation

The problem: Given an expression E, compute $T(E)$ and $V(E, A)$

- This is hard without computing E
- Will 'estimate’ them instead


## Size Estimation

Estimating the size of a projection

- Easy: $\mathrm{T}\left(\Pi_{\mathrm{L}}(\mathrm{R})\right)=\mathrm{T}(\mathrm{R})$
- This is because a projection doesn't eliminate duplicates


## Size Estimation

Estimating the size of a selection

- $S=\sigma_{A=c}(R)$
- $\mathrm{T}(\mathrm{S})$ san be anything from 0 to $\mathrm{T}(\mathrm{R})-\mathrm{V}(\mathrm{R}, \mathrm{A})+1$
- Estimate: $T(S)=T(R) / V(R, A)$
- When $\mathrm{V}(\mathrm{R}, \mathrm{A})$ is not available, estimate $\mathrm{T}(\mathrm{S})=\mathrm{T}(\mathrm{R}) / 10$
- $\mathrm{S}=\sigma_{\mathrm{A}<c}(\mathrm{R})$
- $T(S)$ can be anything from 0 to $T(R)$
- Estimate: $\mathrm{T}(\mathrm{S})=(\mathrm{c}-\operatorname{Low}(\mathrm{R}, \mathrm{A})) /(\operatorname{High}(\mathrm{R}, \mathrm{A})-\operatorname{Low}(\mathrm{R}, \mathrm{A})) \mathrm{T}(\mathrm{R})$
- When Low, High unavailable, estimate $T(S)=T(R) / 3$


## Size Estimation

Estimating the size of a natural join, $\mathrm{R}|\times|_{\mathrm{A}} \mathrm{S}$

- When the set of A values are disjoint, then $T\left(R|\times|_{A} S\right)=0$
- When $A$ is a key in $S$ and a foreign key in $R$, then $T\left(R|x|_{A} S\right)=T(R)$
- When $A$ has a unique value, the same in $R$ and $S$, then $T\left(R|x|_{A} S\right)=T(R) T(S)$


## Size Estimation

Assumptions:

- Containment of values: if $\mathrm{V}(\mathrm{R}, \mathrm{A})<=\mathrm{V}(\mathrm{S}, \mathrm{A})$, then the set of A values of $R$ is included in the set of $A$ values of $S$
- Note: this indeed holds when A is a foreign key in R, and a key in S
- Preservation of values: for any other attribute B , $\mathrm{V}\left(\mathrm{R}|\times|_{\mathrm{A}} \mathrm{S}, \mathrm{B}\right)=\mathrm{V}(\mathrm{R}, \mathrm{B}) \quad(\operatorname{or} \mathrm{V}(\mathrm{S}, \mathrm{B}))$


## Size Estimation

## Assume $\mathrm{V}(\mathrm{R}, \mathrm{A})<=\mathrm{V}(\mathrm{S}, \mathrm{A})$

- Then each tuple t in R joins some tuple(s) in S
- How many?
- On average T(S)/V(S,A)
- $t$ will contribute $T(S) / V(S, A)$ tuples in $R|x|_{A} S$
- Hence $T\left(R|\times|_{\mathrm{A}} S\right)=T(R) T(S) / V(S, A)$

In general: $T\left(R|\times|_{A} S\right)=T(R) T(S) / \max (V(R, A), V(S, A))$

## Size Estimation

Example:

- $\mathrm{T}(\mathrm{R})=10000, \mathrm{~T}(\mathrm{~S})=20000$
- $\mathrm{V}(\mathrm{R}, \mathrm{A})=100, \mathrm{~V}(\mathrm{~S}, \mathrm{~A})=200$
- How large is $R|x|_{A} S$ ?

Answer: $T\left(\mathrm{R}|\times|_{\mathrm{A}} \mathrm{S}\right)=1000020000 / 200=1 \mathrm{M}$

## Size Estimation

Joins on more than one attribute:

- $T\left(R|\times|_{A, B} S\right)=$

$$
\mathrm{T}(\mathrm{R}) \mathrm{T}(\mathrm{~S}) /(\max (\mathrm{V}(\mathrm{R}, \mathrm{~A}), \mathrm{V}(\mathrm{~S}, \mathrm{~A})) * \max (\mathrm{~V}(\mathrm{R}, \mathrm{~B}), \mathrm{V}(\mathrm{~S}, \mathrm{~B})))
$$

## Histograms

- Statistics on data maintained by the RDBMS
- Makes size estimation much more accurate (hence, cost estimations are more accurate)


## Histograms

Employee(ssn, name, salary, phone)

- Maintain a histogram on salary:

| Salary: | $0 . .20 \mathrm{k}$ | $20 \mathrm{k} . .40 \mathrm{k}$ | $40 \mathrm{k} . .60 \mathrm{k}$ | $60 \mathrm{k} . .80 \mathrm{k}$ | $80 \mathrm{k} . .100 \mathrm{k}$ | $>100 \mathrm{k}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Tuples | 200 | 800 | 5000 | 12000 | 6500 | 500 |

- $\mathrm{T}($ Employee $)=25000$, but now we know the distribution


## Histograms

## Ranks(rankName, salary)

- Estimate the size of Employee $|\times|_{\text {Salary }}$ Ranks

| Employee | $0 . .20 \mathrm{k}$ | $20 \mathrm{k} . .40 \mathrm{k}$ | $40 \mathrm{k} . .60 \mathrm{k}$ | $60 \mathrm{k} . .80 \mathrm{k}$ | $80 \mathrm{k} . .100 \mathrm{k}$ | $>100 \mathrm{k}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 200 | 800 | 5000 | 12000 | 6500 | 500 |


| Ranks | $0 . .20 \mathrm{k}$ | $20 \mathrm{k} . .40 \mathrm{k}$ | $40 \mathrm{k} . .60 \mathrm{k}$ | $60 \mathrm{k} . .80 \mathrm{k}$ | $80 \mathrm{k} . .100 \mathrm{k}$ | $>100 \mathrm{k}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 8 | 20 | 40 | 80 | 100 | 2 |

## Histograms

- Eqwidth

| $0 . .20$ | $20 . .40$ | $40 . .60$ | $60 . .80$ | $80 . .100$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 104 | 9739 | 152 | 3 |

- Eqdepth

| $0 . .44$ | $44 . .48$ | $48 . .50$ | $50 . .56$ | $55 . .100$ |
| :---: | :---: | :---: | :---: | :---: |
| 2000 | 2000 | 2000 | 2000 | 2000 |

