

## Schema Refinements = Normal Forms

- 1st Normal Form = all tables are flat
- 2nd Normal Form = obsolete
- Boyce Codd Normal Form = will study
- 3rd Normal Form = see book

First Normal Form (1NF)

- A database schema is in First Normal Form if all tables are flat

Student
Student


| Name | GPA |
| :---: | :---: |
| Alice | 3.8 |
| Bob | 3.7 |
| Carol | 3.9 |

Takes

| Student | Course |
| :--- | :--- |
| Alice | Math |
| Carol | Math |
| Alice | DB |
| Bob | DB |
| Alice | OS |
| Carol | OS |



| Data Anomalies |  |
| :---: | :---: |
| When a database is poorly designed we get anomalies: |  |
| Redundancy: data is repeated |  |
| Updated anomalies: need to change in several places |  |
| Delete anomalies: may lose data when we don't want |  |
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## Relational Schema Design

Recall set attributes (persons with several phones):

| Name | $\underline{\text { SSN }}$ | PhoneNumber | City |
| :--- | :--- | :--- | :--- |
| Fred | $123-45-6789$ | $206-555-1234$ | Seattle |
| Fred | $123-45-6789$ | $206-555-6543$ | Seattle |
| Joe | $987-65-4321$ | $908-555-2121$ | Westfield |

One person may have multiple phones, but lives in only one city
Primary key is thus (SSN,PhoneNumber)
The above is in 1NF, but was is the problem with this schema? Magda Balazinska - CSE 444, Fall 2010

## Relational Schema Design

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Anomalies:

- Redundancy = repeat data
- Update anomalies = what if Fred moves to "Bellevue"?
- Deletion anomalies $=$ what if Joe deletes his phone number? (what if Joe had only one phone \#)
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| Relation Decomposition |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Break the relation into two: |  |  |  |  |
|  |  | SSN | PhoneNumber | City |
|  |  | 123-45-6789 | 206-555-1234 | Seattle |
|  |  | 123-45-6789 | 206-555-6543 | Seattle |
|  |  | 987-65-4321 | 908-555-2121 | Westfield |
| Name | SSN | City | SSN | PhoneNumber |
| Fred | 123-45-6789 | Seattle | 123-45-6789 | 206-555-1234 |
| Joe | 987-65-4321 | Westfield | 123-45-6789 | 206-555-6543 |
| Anomalies have gone: <br> - No more repeated data <br> - Easy to move Fred to "Bellevue" (how?) <br> - Easy to delete all Joe's phone numbers (how ?) |  |  |  | 908-555-2121 |
|  |  |  |  | ?) 9 |

## Relational Schema Design (or Logical Design)

Main idea:

- Start with some relational schema
- Find out its functional dependencies
- They come from the application domain knowledge!
- Use them to design a better relational schema


## Functional Dependencies

- A form of constraint
- Hence, part of the schema
- Finding them is part of the database design
- Use them to normalize the relations


## Functional Dependencies (FDs)

Definition:
If two tuples agree on the attributes
$A_{1}, A_{2}, \ldots, A_{n}$
then they must also agree on the attributes

$$
\mathrm{B}_{1}, \mathrm{~B}_{2}, \ldots, \mathrm{~B}_{\mathrm{m}}
$$

Formally:

$$
A_{1}, A_{2}, \ldots, A_{n} \rightarrow B_{1}, B_{2}, \ldots, B_{m}
$$

## When Does an FD Hold

Definition: $A_{1}, \ldots, A_{m} \rightarrow B_{1}, \ldots, B_{n}$ holds in $R$ if:
$\forall t, t^{\prime} \in R$,
$\left(t . A_{1}=t^{\prime} . A_{1} \wedge \ldots \wedge t . A_{m}=t^{\prime} . A_{m} \Rightarrow t . B_{1}=t^{\prime} . B_{1} \wedge \ldots \wedge t . B_{n}=t^{\prime} . B_{n}\right)$

|  | $A_{1}$ | $\ldots$ | $A_{m}$ |  | $B_{1}$ | $\ldots$ | $n_{m}$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  | $t_{\text {if } t, t}$ ' agree here |  |  |  |  |  |  |  |  |
| $\mathrm{t}^{\prime}$ |  |  |  |  |  |  |  |  |  |
| then $\mathrm{t}, \mathrm{t}$ ' agree here |  |  |  |  |  |  |  |  |  |


| Example |  |  |  |
| :--- | :--- | :--- | :---: |
| An FD holds, or does not hold on an instance: |  |  |  |
| EmpID Name Phone Position <br> E0045 Smith 1234 Clerk <br> E3542 Mike 9876 Salesrep <br> E1111 Smith 9876 Salesrep <br> E9999 Mary 1234 Lawyer |  |  |  |
| EmpID $\rightarrow$ Name, Phone, Position <br> Position $\rightarrow$ Phone <br> but not Phone $\rightarrow$ Position <br> Magda Balazinska - CSE 444, Fall 2010 |  |  |  |


| Example |  |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
| $\qquad$EmpID Name Phone Position <br> E0045 Smith 1234 Clerk <br> E3542 Mike 9876 $\leftarrow$ <br> Salesrep    <br> E1111 Smith 9876 $\leftarrow$ <br> E9999 Mary 1234 Lawyer |  |  |  |  |


| Example |  |  |  |
| :--- | :--- | :--- | :--- |
| EmpID Name Phone Position <br> E0045 Smith 1234 $\rightarrow$ <br> Clerk    <br> E3542 Mike 9876 Salesrep <br> E1111 Smith 9876 Salesrep <br> E9999 Mary 1234 $\rightarrow$ Lawyer |  |  |  |


| Example |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| FD's are constraints: <br> - On some instances they hold <br> - On others they don't |  |  | $\begin{array}{\|l} \text { name } \rightarrow \text { color } \\ \text { category } \rightarrow \text { department } \\ \text { color, category } \rightarrow \text { price } \\ \hline \end{array}$ |  |
| name | category | color | department | price |
| Gizmo | Gadget | Green | Toys | 49 |
| Tweaker | Gadget | Green | Toys | 99 |
| Does this instance satisfy all the FDs ? |  |  |  |  |




## Goal: Find ALL Functional Dependencies

- Anomalies occur when certain "bad" FDs hold
- We know some of the FDs
- Need to find all FDs
- Then look for the bad ones




## Armstrong's Rules (3/3)

Transitive Rule
If $\quad A_{1}, A_{2}, \ldots, A_{n} \rightarrow B_{1}, B_{2}, \ldots, B_{m}$
and $\quad B_{1}, B_{2}, \ldots, B_{m} \rightarrow C_{1}, C_{2}, \ldots, C_{p}$
then $\quad A_{1}, A_{2}, \ldots, A_{n} \rightarrow C_{1}, C_{2}, \ldots, C_{p}$
Why?
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## Armstrong's Rules (3/3)

Illustration


|  | Example (continued) |
| :--- | :--- |
| Start from the following FDs: <br> 1. name $\rightarrow$ color <br> 2. category $\rightarrow$ department <br> 3. color, category $\rightarrow$ price |  |
| Infer the following FDs: Which Rule <br> did we apply ? <br> 4. name, category $\rightarrow$ name  <br> 5. name, category $\rightarrow$ color  <br> 6. name, category $\rightarrow$ category  <br> 7. name, category $\rightarrow$ color, category  <br> 8. name, category $\rightarrow$ price  |  |


| Example (continued) |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: |
| 1. name $\rightarrow$ color <br> 2. category $\rightarrow$ department <br> 3. color, category $\rightarrow$ price |  |  |  |  |  |



## Closure Algorithm


\{name, category ${ }^{+}=$
\{ name, category, color, department, price \}
Hence: name, category $\rightarrow$ color, department, price
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## Example

In class:

$R(A, B, C, D, E, F) \quad$| $A, B$ | $\rightarrow$ | $C$ |
| :--- | :--- | :--- | :--- |
| $A$, | $\rightarrow$ | $E$ |
| $B$ | $\rightarrow$ | $D$ |
| $A$, | $\rightarrow$ | $B$ |

Compute $\{A, B\}^{+} \quad X=\{A, B, C, D, E\}$
Compute $\{\mathrm{A}, \mathrm{F}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{F}, \quad\}$

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| Example |  |  |
| :---: | :---: | :---: |
| In class: |  |  |
| R(A,B,C,D,E,F) | $\|$A, $B$ $\rightarrow$ $C$  <br> $A$, $\rightarrow$ $E$  <br> $B$  $\rightarrow$ $D$ <br> $A$, $F$ $\rightarrow$ $B$ |  |
| Compute $\{\mathrm{A}, \mathrm{B}\}^{+}$ | B, C, D, E \} |  |
| Compute $\{\mathrm{A}, \mathrm{F}\}^{+}$ | F, B, C, D, E \} |  |
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## Why Do We Need Closure

- With closure we can find all FD's easily
- To check if $X \rightarrow A$
- Compute $\mathrm{X}^{+}$
- Check if $\mathrm{A} \in \mathrm{X}^{+}$


## Using Closure to Infer ALL FDs

Example: | A, | $\rightarrow$ | C |
| :--- | :--- | :--- |
| A, D | $\rightarrow$ | B |
| B | $\rightarrow$ | D |

Step 1: Compute $\mathrm{X}^{+}$, for every X :
$\mathrm{A}+=\mathrm{A}, \mathrm{B}+=\mathrm{BD}, \mathrm{C}+=\mathrm{C}, \mathrm{D}+=\mathrm{D}$
$\mathrm{AB}+=\mathrm{ABCD}, \mathrm{AC}+=\mathrm{AC}, \mathrm{AD}+=\mathrm{ABCD}$,
$\mathrm{BC}+=\mathrm{BCD}, \mathrm{BD}+=\mathrm{BD}, \mathrm{CD}+=\mathrm{CD}$

## Another Example

- Enrollment(student, major, course, room, time) student $\rightarrow$ major
major, course $\rightarrow$ room
course $\rightarrow$ time
$\mathrm{ABC}+=\mathrm{ABD}+=\mathrm{ACD}+=\mathrm{ABCD}$ (no need to compute - why ?)
$\mathrm{BCD}^{+}=\mathrm{BCD}, \quad \mathrm{ABCD}+=\mathrm{ABCD}$
Step 2: Enumerate all FD's $\mathrm{X} \rightarrow \mathrm{Y}$, s.t. $\mathrm{Y} \subseteq \mathrm{X}^{+}$and $\mathrm{X} \cap \mathrm{Y}=\varnothing$ :
$\mathrm{AB} \rightarrow \mathrm{CD}, \mathrm{AD} \rightarrow \mathrm{BC}, \mathrm{BC} \rightarrow \mathrm{D}, \mathrm{ABC} \rightarrow \mathrm{D}, \mathrm{ABD} \rightarrow \mathrm{C}, \mathrm{ACD} \rightarrow \mathrm{B}$


## Keys

- A superkey is a set of attributes $A_{1}, \ldots, A_{n}$ s.t. for Computing (Super)Keys
- Compute $\mathrm{X}^{+}$for all sets X
- If $X^{+}=$all attributes, then $X$ is a superkey
- List only the minimal $X$ 's to get the keys
- A key is a minimal superkey
- I.e. set of attributes which is a superkey and for which no subset is a superkey



## Example

Product(name, price, category, color)
name, category $\rightarrow$ price category $\rightarrow$ color

What is the key?
(name, category) + = \{ name, category, price, color \}
Hence (name, category) is a key

## Examples of Keys

Enrollment(student, address, course, room, time)

```
student }->\mathrm{ address
room, time }->\mathrm{ course
student, course }->\mathrm{ room, time
(find keys at home)

\section*{Eliminating Anomalies}

Main idea:
- \(X \rightarrow A\) is \(O K\) if \(X\) is a (super)key
- \(X \rightarrow A\) is not OK otherwise
\begin{tabular}{l}
\begin{tabular}{l} 
Example \\
\begin{tabular}{|l|l|l|l|}
\hline Name & SSN & PhoneNumber & City \\
\hline Fred & \(123-45-6789\) & \(206-555-1234\) & Seattle \\
\hline Fred & \(123-45-6789\) & \(206-555-6543\) & Seattle \\
\hline Joe & \(987-65-4321\) & \(908-555-2121\) & Westfield \\
\hline Joe & \(987-65-4321\) & \(908-555-1234\) & Westfield \\
\hline
\end{tabular} \\
\begin{tabular}{|l|l|l}
\hline SSN \(\rightarrow\) Name, City \\
\hline
\end{tabular} \\
What is the key? \\
\{SSN, PhoneNumber\}
\end{tabular} \begin{tabular}{l} 
Mence SSN \(\rightarrow\) Name, City \\
is a "bad" dependency
\end{tabular} \\
\hline
\end{tabular}

\section*{Key or Keys ?}

Can we have more than one key ?

Given \(R(A, B, C)\) define FD's s.t. there are two or more keys

\section*{Key or Keys?}

Can we have more than one key?

Given \(R(A, B, C)\) define FD's s.t. there are two or more keys

what are the keys here?
Can you design FDs such that there are three keys?

\section*{Boyce-Codd Normal Form}

A simple condition for removing anomalies from relations:
A relation \(R\) is in BCNF if:
If \(A_{1}, \ldots, A_{n} \rightarrow B\) is a non-trivial dependency in \(R\),
then \(\left\{A_{1}, \ldots, A_{n}\right\}\) is a superkey for \(R\)
In other words: there are no "bad" FDs
Equivalently:
for all X , either \(\left(\mathrm{X}^{+}=\mathrm{X}\right) \quad\) or \(\quad\left(\mathrm{X}^{+}=\right.\)all attributes \()\)
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\section*{Example Decomposition}

Person(name, SSN, age, hairColor, phoneNumber)
FD1: SSN \(\rightarrow\) name, age
FD2: age \(\rightarrow\) hairColor
Decompose in BCNF (in class):

\section*{Example Decomposition}

Person(name, SSN, age, hairColor, phoneNumber)
FD1: SSN \(\rightarrow\) name, age
FD2: age \(\rightarrow\) hairColor
Decompose in BCNF (in class): What is the key?
\{SSN, phoneNumber\}
But how to decompose?
Person(SSN, name, age)
Phone(SSN, hairColor, phoneNumber)
Or
Person(SSN, name, age, hairColor)
Phone(SSN, phoneNumber)
Or ....

BCNF Decomposition Algorithm
BCNF_Decompose(R)
find X s.t.: \(\mathrm{X} \neq \mathrm{X}^{+} \neq[\)all attributes \(]\)
if (not found) then " \(R\) is in BCNF"
let \(\mathrm{Y}=\mathrm{X}^{+}-\mathrm{X}\)
let \(\mathrm{Z}=\) [all attributes \(]-\mathrm{X}^{+}\)
decompose R into \(\mathrm{R} 1(\mathrm{X} \cup \mathrm{Y})\) and \(\mathrm{R} 2(\mathrm{X} \cup \mathrm{Z})\)
continue to decompose recursively R1 and R2
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\section*{Theory of Decomposition}
- Sometimes it is correct:


Lossless decomposition

\section*{Incorrect Decomposition}

\section*{- Sometimes it is not:}
\begin{tabular}{|c|c|c|c|}
\hline Name & Price & Category \\
\hline Gizmo & 19.99 & Gadget \\
\hline OneClick & 24.99 & Camera \\
\hline Gizmo & 19.99 & Camera \\
\hline
\end{tabular}


\section*{Optional}
- The following four slides are optional
- The content will not be on any exam
- But please take a look because they motivate the need for 3NF
- It's good to know at least why 3NF exists

\section*{General Decomposition Goals}
1. Elimination of anomalies
2. Recoverability of information
- Can we get the original relation back?
3. Preservation of dependencies
- Want to enforce FDs without performing joins

Sometimes cannot decomposed into BCNF without losing ability to check some FDs

\section*{BCNF and Dependencies}
\begin{tabular}{|c|c|c|c|c|c|}
\hline & \multicolumn{5}{|l|}{Or} \\
\hline
\end{tabular}

FD's: Unit \(\rightarrow\) Company; Company, Product \(\rightarrow\) Unit So, there is a BCNF violation, and we decompose.

\section*{BCNF and Dependencies}
\begin{tabular}{|l|l|l|}
\hline Unit & Company & Product \\
\hline
\end{tabular}

FD's: Unit \(\rightarrow\) Company; Company, Product \(\rightarrow\) Unit So, there is a BCNF violation, and we decompose.


Unit \(\rightarrow\) Company


No FDs
In BCNF we lose the FD: Company, Product \(\rightarrow\) Unit Magda Balazinska - CSE 444, Fall 2010
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