## Lectures 6-7: Database Design

Friday, April 9 and Monday April 12, 2010

## Outline

- Design theory: 3.1-3.4


## Schema Refinements = Normal Forms

- 1st Normal Form = all tables are flat
- 2nd Normal Form = obsolete
- Boyce Codd Normal Form = will study
- 3rd Normal Form = see book


## First Normal Form (1NF)

- A database schema is in First Normal Form if all tables are flat


## Student

| Name | GPA | Courses |
| :---: | :---: | :---: |
| Alice | 3.8 | Math <br> DB <br> os <br> Bob <br> 3.7 <br> Carol <br> 3.9 <br> os |
| Math |  |  |

Student

| Name | GPA |
| :---: | :---: |
| Alice | 3.8 |
| Bob | 3.7 |
| Carol | 3.9 |

Takes

| Student | Course |
| :--- | :--- |
| Alice | Course |
| Carol | Math |
| Alice | DB |
| Bob | DB |
| Alice | OS |
| Carol | OS |$\quad$| Course |
| :--- |
| Math |
| DB |
| OS |

## Relational Schema Design

Conceptual Model:


Relational Model: plus FD's


Normalization:
Eliminates anomalies


## Data Anomalies

When a database is poorly designed we get anomalies:

Redundancy: data is repeated

Updated anomalies: need to change in several places

Delete anomalies: may lose data when we don't want

## Relational Schema Design

Recall set attributes (persons with several phones):

| Name | SSN | PhoneNumber | City |
| :--- | :--- | :--- | :--- |
| Fred | $123-45-6789$ | $206-555-1234$ | Seattle |
| Fred | $123-45-6789$ | $206-555-6543$ | Seattle |
| Joe | $987-65-4321$ | $908-555-2121$ | Westfield |

One person may have multiple phones, but lives in only one city

## Anomalies:

- Redundancy = repeat data
- Update anomalies = Fred moves to "Bellevue"
- Deletion anomalies = Joe deletes his phone number: what is his city?


## Relation Decomposition

## Break the relation into two:

|  |  | Name |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: |
| Fred | SSN |  |  |  |  |
|  |  |  |  | Fred | $123-45-6789$ |
|  |  |  |  | Joe | $987-65-4321$ |
| Name | $\underline{S S N}$ | City |  |  |  |
| Fred | $123-45-6789$ | Seattle |  |  |  |
| Joe | $987-65-4321$ | Westfield |  |  |  |

Anomalies have gone:

| SSN | PhoneNumber |
| :--- | :--- |
| $123-45-6789$ | $206-555-1234$ |
| $123-45-6789$ | $206-555-6543$ |
| $987-65-4321$ | $908-555-2121$ |

- No more repeated data
- Easy to move Fred to "Bellevue" (how ?)
- Easy to delete all Joe's phone number (how ?)


## Relational Schema Design (or Logical Design)

Main idea:

- Start with some relational schema
- Find out its functional dependencies
- Use them to design a better relational schema


## Functional Dependencies

- A form of constraint
- hence, part of the schema
- Finding them is part of the database design
- Also used in normalizing the relations


## Functional Dependencies

## Definition:

If two tuples agree on the attributes

$$
\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{n}}
$$

then they must also agree on the attributes

$$
\mathrm{B}_{1}, \mathrm{~B}_{2}, \ldots, \mathrm{~B}_{\mathrm{m}}
$$

Formally:

$$
\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{n}} \rightarrow \mathrm{~B}_{1}, \mathrm{~B}_{2}, \ldots, \mathrm{~B}_{\mathrm{m}}
$$

## When Does an FD Hold

Definition: $A_{1}, \ldots, A_{m} \rightarrow B_{1}, \ldots, B_{n}$ holds in $R$ if:
$\forall \mathrm{t}, \mathrm{t}^{\prime} \in \mathrm{R},\left(\mathrm{t} . \mathrm{A}_{1}=\mathrm{t}^{\prime} . \mathrm{A}_{1} \wedge \ldots \wedge \mathrm{t} . \mathrm{A}_{\mathrm{m}}=\mathrm{t}^{\prime} . \mathrm{A}_{\mathrm{m}} \Rightarrow \mathrm{t} . \mathrm{B}_{1}=\mathrm{t}^{\prime} . \mathrm{B}_{1} \wedge \ldots \wedge \mathrm{t} . \mathrm{B}_{\mathrm{n}}=\mathrm{t}^{\prime} . \mathrm{B}_{\mathrm{n}}\right)$ R


## Examples

An FD holds, or does not hold on an instance:

| EmpID | Name | Phone | Position |
| :--- | :--- | :--- | :--- |
| E0045 | Smith | 1234 | Clerk |
| E3542 | Mike | 9876 | Salesrep |
| E1111 | Smith | 9876 | Salesrep |
| E9999 | Mary | 1234 | Lawyer |

EmpID $\rightarrow$ Name, Phone, Position
Position $\rightarrow$ Phone
but not Phone $\rightarrow$ Position

## Example

| EmpID | Name | Phone | Position |
| :--- | :--- | :--- | :--- |
| E0045 | Smith | 1234 | Clerk |
| E3542 | Mike | $9876 \leftarrow$ | Salesrep |
| E1111 | Smith | $9876 \leftarrow$ | Salesrep |
| E9999 | Mary | 1234 | Lawyer |

Position $\rightarrow$ Phone

## Example

| EmpID | Name | Phone | Position |
| :--- | :--- | :--- | :--- |
| E0045 | Smith | $1234 \rightarrow$ | Clerk |
| E3542 | Mike | 9876 | Salesrep |
| E1111 | Smith | 9876 | Salesrep |
| E9999 | Mary | $1234 \rightarrow$ | Lawyer |

but not Phone $\rightarrow$ Position

## Example

FD's are constraints:

- On some instances they hold
- On others they don't

$$
\begin{aligned}
& \text { name } \rightarrow \text { color } \\
& \text { category } \rightarrow \text { department } \\
& \text { color, category } \rightarrow \text { price }
\end{aligned}
$$

| name | category | color | department | price |
| :---: | :---: | :---: | :---: | :---: |
| Gizmo | Gadget | Green | Toys | 49 |
| Tweaker | Gadget | Green | Toys | 99 |

Does this instance satisfy all the FDs ?

## Example

## name $\rightarrow$ color category $\rightarrow$ department color, category $\rightarrow$ price

| name | category | color | department | price |
| :---: | :---: | :---: | :---: | :---: |
| Gizmo | Gadget | Green | Toys | 49 |
| Tweaker | Gadget | Black | Toys | 99 |
| Gizmo | Stationary | Green | Office-supp. | 59 |

## An Interesting Observation

If all these FDs are true:

$$
\begin{aligned}
& \text { name } \rightarrow \text { color } \\
& \text { category } \rightarrow \text { department } \\
& \text { color, category } \rightarrow \text { price } \\
& \hline
\end{aligned}
$$

Then this FD also holds:

$$
\text { name, category } \rightarrow \text { price }
$$

## Goal: Find ALL Functional Dependencies

- Anomalies occur when certain "bad" FDs hold
- We know some of the FDs
- Need to find all FDs, then look for the bad ones


## Armstrong's Rules (1/3)

$$
\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{n}} \rightarrow \mathrm{~B}_{1}, \mathrm{~B}_{2}, \ldots, \mathrm{~B}_{\mathrm{m}}
$$

Is equivalent to
Splitting rule
and
Combing rule

$$
\begin{gathered}
\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{n}} \rightarrow \mathrm{~B}_{1} \\
\mathrm{~A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{n}} \rightarrow \mathrm{~B}_{2} \\
\ldots \ldots \\
\mathrm{~A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{n}} \rightarrow \mathrm{~B}_{\mathrm{m}}
\end{gathered}
$$



## Armstrong's Rules (2/3)

$$
\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{n}} \rightarrow \mathrm{~A}_{\mathrm{i}}
$$

## Trivial Rule

where $\mathrm{i}=1,2, \ldots, \mathrm{n}$

Why?


## Armstrong's Rules (3/3)

## Transitive Closure Rule

If

$$
\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{n}} \rightarrow \mathrm{~B}_{1}, \mathrm{~B}_{2}, \ldots, \mathrm{~B}_{\mathrm{m}}
$$

and

$$
\mathrm{B}_{1}, \mathrm{~B}_{2}, \ldots, \mathrm{~B}_{\mathrm{m}} \rightarrow \mathrm{C}_{1}, \mathrm{C}_{2}, \ldots, \mathrm{C}_{\mathrm{p}}
$$

then

$$
\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{n}} \rightarrow \mathrm{C}_{1}, \mathrm{C}_{2}, \ldots, \mathrm{C}_{\mathrm{p}}
$$

Why?

|  | $\mathrm{A}_{1}$ | $\ldots$ | $\mathrm{~A}_{\mathrm{m}}$ |  | $\mathrm{B}_{1}$ | $\ldots$ | $\mathrm{~B}_{\mathrm{m}}$ |  | $\mathrm{C}_{1}$ | $\ldots$ | $\mathrm{C}_{\mathrm{p}}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

## Example (continued)

Start from the following FDs:

1. name $\rightarrow$ color
2. category $\rightarrow$ department
3. color, category $\rightarrow$ price

Infer the following FDs:

| Inferred FD | Which Rule <br> did we apply ? |
| :--- | :--- |
| 4. name, category $\rightarrow$ name |  |
| 5. name, category $\rightarrow$ color |  |
| 6. name, category $\rightarrow$ category |  |
| 7. name, category $\rightarrow$ color, category |  |
| 8. name, category $\rightarrow$ price |  |

## Example (continued)

Answers:

1. name $\rightarrow$ color
2. category $\rightarrow$ department
3. color, category $\rightarrow$ price

| Inferred FD | Which Rule <br> did we apply ? |
| :--- | :--- |
| 4. name, category $\rightarrow$ name | Trivial rule |
| 5. name, category $\rightarrow$ color | Transitivity on 4, 1 |
| 6. name, category $\rightarrow$ category | Trivial rule |
| 7. name, category $\rightarrow$ color, category | Split/combine on 5, 6 |
| 8. name, category $\rightarrow$ price | Transitivity on 3, 7 |

THIS IS TOO HARD! Let's see an easier way.

## Closure of a set of Attributes

Given a set of attributes $A_{1}, \ldots, A_{n}$
The closure, $\left\{\mathrm{A}_{1}, \ldots, \mathrm{~A}_{\mathrm{n}}\right\}^{+}=$the set of attributes B

$$
\text { s.t. } A_{1}, \ldots, A_{n} \rightarrow B
$$

Example:

Closures:

$$
\begin{aligned}
& \text { name } \rightarrow \text { color } \\
& \text { category } \rightarrow \text { department } \\
& \text { color, category } \rightarrow \text { price } \\
& \hline
\end{aligned}
$$

```
name }\mp@subsup{}{}{+}={\mathrm{ name, color}
    {name, category }}\mp@subsup{}{}{+}={\mathrm{ name, category, color, department, price}
    color }\mp@subsup{}{}{+}={\mathrm{ color }}\quad\mathrm{ Dan Suciu -- 444 Spring 2010

\section*{Closure Algorithm}
\(X=\{A 1, \ldots, A n\}\).
Repeat until X doesn't change do:
if \(B_{1}, \ldots, B_{n} \rightarrow C\) is a FD and \(\mathrm{B}_{1}, \ldots, \mathrm{~B}_{\mathrm{n}}\) are all in X then add C to X .

Example:
\[
\begin{aligned}
& \text { name } \rightarrow \text { color } \\
& \text { category } \rightarrow \text { department } \\
& \text { color, category } \rightarrow \text { price }
\end{aligned}
\]
\(\{\text { name, category }\}^{+}=\)
\{ name, category, color, department, price \}
Hence: name, category \(\rightarrow\) color, department, price

\section*{Example}

In class:

R(A,B,C,D,E,F)
\[
\begin{array}{|lll|}
\hline \mathrm{A}, \mathrm{~B} & \rightarrow \mathrm{C} \\
\mathrm{~A}, \mathrm{D} & \rightarrow & \mathrm{E} \\
\mathrm{~B} & \rightarrow & \mathrm{D} \\
\mathrm{~A}, \mathrm{~F} & \rightarrow & \mathrm{~B} \\
\hline
\end{array}
\]

Compute \(\{\mathrm{A}, \mathrm{B}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{B}\),
Compute \(\{\mathrm{A}, \mathrm{F}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{F}\),

\section*{Why Do We Need Closure}
- With closure we can find all FD's easily
- To check if \(\mathrm{X} \rightarrow \mathrm{A}\)
- Compute \(\mathrm{X}^{+}\)
- Check if \(\mathrm{A} \in \mathrm{X}^{+}\)

\section*{Using Closure to Infer ALL FDs}

Example:
\[
\begin{array}{|lll|}
\hline \mathrm{A}, \mathrm{~B} & \rightarrow \mathrm{C} \\
\mathrm{~A}, \mathrm{D} & \rightarrow & \mathrm{~B} \\
\mathrm{~B} & \rightarrow & \mathrm{D} \\
\hline
\end{array}
\]

Step 1: Compute \(\mathrm{X}^{+}\), for every X :
\[
\begin{aligned}
& \mathrm{A}+=\mathrm{A}, \mathrm{~B}+=\mathrm{BD}, \mathrm{C}+=\mathrm{C}, \mathrm{D}+=\mathrm{D} \\
& \mathrm{AB}+=\mathrm{ABCD}, \mathrm{AC}+=\mathrm{AC}, \mathrm{AD}+=\mathrm{ABCD}, \\
& \mathrm{BC}+=\mathrm{BCD}, \mathrm{BD}+=\mathrm{BD}, \mathrm{CD}+=\mathrm{CD} \\
& \mathrm{ABC}+=\mathrm{ABD}+=\mathrm{ACD}+=\mathrm{ABCD} \text { (no need to compute }- \text { why } ? \text { ) } \\
& \mathrm{BCD}^{+}=\mathrm{BCD}, \mathrm{ABCD}+=\mathrm{ABCD}
\end{aligned}
\]

Step 2: Enumerate all FD's \(\mathrm{X} \rightarrow \mathrm{Y}\), s.t. \(\mathrm{Y} \subseteq \mathrm{X}^{+}\)and \(\mathrm{X} \cap \mathrm{Y}=\varnothing\) : \(\mathrm{AB} \rightarrow \mathrm{CD}, \mathrm{AD} \rightarrow \mathrm{BC}, \mathrm{ABC} \rightarrow \mathrm{D}, \mathrm{ABD} \rightarrow \mathrm{C}, \mathrm{ACD} \rightarrow \mathrm{B}\)

\section*{Another Example}
- Enrollment(student, major, course, room, time) student \(\rightarrow\) major
major, course \(\rightarrow\) room
course \(\rightarrow\) time

What else can we infer ? [in class, or at home]

\section*{Keys}
- A superkey is a set of attributes \(A_{1}, \ldots, A_{n}\) s.t. for any other attribute \(B\), we have \(A_{1}, \ldots, A_{n} \rightarrow B\)
- A key is a minimal superkey
- I.e. set of attributes which is a superkey and for which no subset is a superkey

\section*{Computing (Super)Keys}
- Compute \(\mathrm{X}^{+}\)for all sets X
- If \(\mathrm{X}^{+}=\)all attributes, then X is a key
- List only the minimal X's

\section*{Example}

\section*{Product(name, price, category, color)}
\[
\begin{aligned}
& \text { name, category } \rightarrow \text { price } \\
& \text { category } \rightarrow \text { color } \\
& \hline
\end{aligned}
\]

What is the key?

\section*{Example}

\section*{Product(name, price, category, color)}

\section*{name, category \(\rightarrow\) price category \(\rightarrow\) color}

What is the key?
(name, category) \(+=\) name, category, price, color
Hence (name, category) is a key

\section*{Examples of Keys}

Enrollment(student, address, course, room, time)
```

student }->\mathrm{ address
room, time }->\mathrm{ course
student, course }->\mathrm{ room, time

```
(find keys at home)

\section*{Eliminating Anomalies}

Main idea:
- \(X \rightarrow A\) is OK if \(X\) is a (super)key
- \(\mathrm{X} \rightarrow \mathrm{A}\) is not OK otherwise

\section*{Example}
\begin{tabular}{|l|l|l|l|}
\hline Name & SSN & PhoneNumber & City \\
\hline Fred & \(123-45-6789\) & \(206-555-1234\) & Seattle \\
\hline Fred & \(123-45-6789\) & \(206-555-6543\) & Seattle \\
\hline Joe & \(987-65-4321\) & \(908-555-2121\) & Westfield \\
\hline Joe & \(987-65-4321\) & \(908-555-1234\) & Westfield \\
\hline
\end{tabular}

SSN \(\rightarrow\) Name, City

What the key?
\{SSN, PhoneNumber\}
Hence SSN \(\rightarrow\) Name, City is a "bad" dependency

\section*{Key or Keys ?}

Can we have more than one key?

Given \(R(A, B, C)\) define FD's s.t. there are two or more keys

\section*{Key or Keys ?}

\section*{Can we have more than one key?}

Given \(R(A, B, C)\) define FD's s.t. there are two or more keys
\[
\begin{array}{|l|l}
\mathrm{AB} \rightarrow \mathrm{C} \\
\mathrm{BC} \rightarrow \mathrm{~A}
\end{array} \quad \text { or } \quad \begin{aligned}
& \mathrm{A} \rightarrow \mathrm{BC} \\
& \mathrm{~B} \rightarrow \mathrm{AC}
\end{aligned}
\]
what are the keys here ?
Can you design FDs such that there are three keys?

\section*{Boyce-Codd Normal Form}

A simple condition for removing anomalies from relations:

\section*{A relation R is in BCNF if:}

If \(A_{1}, \ldots, A_{n} \rightarrow B\) is a non-trivial dependency
in \(R\), then \(\left\{A_{1}, \ldots, A_{n}\right\}\) is a superkey for \(R\)

In other words: there are no "bad" FDs

Equivalently:
\(\forall \mathrm{X}\), either \(\left(\mathrm{X}^{+}=\mathrm{X}\right) \quad\) or \(\quad\left(\mathrm{X}^{+}=\right.\)all attributes \()\)

\section*{BCNF Decomposition Algorithm}

\section*{repeat}
choose \(A_{1}, \ldots, A_{m} \rightarrow B_{1}, \ldots, B_{n}\) that violates \(\operatorname{BNCF}\)
split \(R\) into \(R_{1}\left(A_{1}, \ldots, A_{m}, B_{1}, \ldots, B_{n}\right)\) and \(R_{2}\left(A_{1}, \ldots, A_{m}\right.\), [others]) continue with both \(\mathrm{R}_{1}\) and \(\mathrm{R}_{2}\)
until no more violations


Is there a
2-attribute
relation that is not in BCNF?

In practice, we have a better algorithm (coming \({ }^{42}\) up)

\section*{Example}
\begin{tabular}{|l|l|l|l|}
\hline Name & SSN & PhoneNumber & City \\
\hline Fred & \(123-45-6789\) & \(206-555-1234\) & Seattle \\
\hline Fred & \(123-45-6789\) & \(206-555-6543\) & Seattle \\
\hline Joe & \(987-65-4321\) & \(908-555-2121\) & Westfield \\
\hline Joe & \(987-65-4321\) & \(908-555-1234\) & Westfield \\
\hline
\end{tabular}

\section*{SSN \(\rightarrow\) Name, City}

What the key?
\(\{\) SSN, PhoneNumber \(\} \quad \begin{aligned} & \text { use SSN } \rightarrow \text { Name, City } \\ & \text { to split }\end{aligned}\)

\section*{Example}
\begin{tabular}{|l|l|l|}
\hline Name & \(\underline{\text { SSN }}\) & City \\
\hline \multirow{2}{*}{ SSN \(\rightarrow\) Name, City } \\
\hline Fred & \(123-45-6789\) & Seattle \\
\hline Joe & \(987-65-4321\) & Westfield \\
\hline
\end{tabular}
\begin{tabular}{|l|l|}
\hline SSN & PhoneNumber \\
\hline \(123-45-6789\) & \(206-555-1234\) \\
\hline \(123-45-6789\) & \(206-555-6543\) \\
\hline \(987-65-4321\) & \(908-555-2121\) \\
\hline \(987-65-4321\) & \(908-555-1234\) \\
\hline
\end{tabular}

Let's check anomalies:
- Redundancy?
- Update?
-Delete ?

\section*{Example Decomposition}

Person(name, SSN, age, hairColor, phoneNumber) SSN \(\rightarrow\) name, age age \(\rightarrow\) hairColor

Decompose in BCNF (in class):

\section*{BCNF Decomposition Algorithm}

\section*{BCNF_Decompose(R)}
find X s.t.: \(\mathrm{X} \neq \mathrm{X}^{+} \neq[\)all attributes \(]\)
if (not found) then " \(R\) is in BCNF"
let \(\mathrm{Y}=\mathrm{X}^{+}-\mathrm{X}\)
let \(\mathrm{Z}=\) [all attributes \(]-\mathrm{X}^{+}\)
decompose R into \(\mathrm{R} 1(\mathrm{X} \cup \mathrm{Y})\) and \(\mathrm{R} 2(\mathrm{X} \cup \mathrm{Z})\) continue to decompose recursively R1 and R2

Find X s.t.: \(\mathrm{X} \neq \mathrm{X}^{+} \neq[\)all attributes \(]\)

\section*{Example BCNF Decomposition}

Person(name, SSN, age, hairColor, phoneNumber)
SSN \(\rightarrow\) name, age
age \(\rightarrow\) hairColor
Iteration 1: Person
SSN+ = SSN, name, age, hairColor
Decompose into: P(SSN, name, age, hairColor) Phone(SSN, phoneNumber)

Iteration 2: P
age \(+=\) age, hairColor
Decompose: People(SSN, name, age)
Hair(age, hairColor)
Phone(SSN, phoneNumber)

What are the keys?

R(A,B,C,D)

\section*{Example}
\[
\begin{aligned}
& \mathrm{A} \rightarrow \mathrm{~B} \\
& \mathrm{~B} \rightarrow \mathrm{C}
\end{aligned}
\]


What happens if in R we first pick \(\mathrm{B}^{+}\)? \(\mathrm{Or} \mathrm{AB}^{+}{ }_{48}\) ?

\section*{Decompositions in General}

\(\mathrm{R}_{1}=\) projection of R on \(\mathrm{A}_{1}, \ldots, \mathrm{~A}_{\mathrm{n}}, \mathrm{B}_{1}, \ldots, \mathrm{~B}_{\mathrm{m}}\) \(\mathrm{R}_{2}=\) projection of R on \(\mathrm{A}_{1}, \ldots, \mathrm{~A}_{\mathrm{n}}, \mathrm{C}_{1}, \ldots, \mathrm{C}_{\mathrm{p}}\)

\section*{Theory of Decomposition}
- Sometimes it is correct:
\begin{tabular}{|c|c|c|}
\hline Name & Price & Category \\
\hline Gizmo & 19.99 & Gadget \\
\hline OneClick & 24.99 & Camera \\
\hline Gizmo & 19.99 & Camera \\
\hline
\end{tabular}


Lossless decomposition

\section*{Incorrect Decomposition}
- Sometimes it is not:


\section*{Decompositions in General}

\[
\text { If } \mathrm{A}_{1}, \ldots, \mathrm{~A}_{\mathrm{n}} \rightarrow \mathrm{~B}_{1}, \ldots, \mathrm{~B}_{\mathrm{m}}
\]

Then the decomposition is lossless
Note: don't need \(A_{1}, \ldots, A_{n} \rightarrow C_{1}, \ldots, C_{p}\)```

