## TA Section 3

## Solution for Project 1

Exercise 3.1.2
Consider a relation representing the present position of molecules in a closed container. The attributes are an

ID for the molecule,
the $x, y$, and $z$ coordinates of the molecule, and
its velocity in the $x, y$, and $z$ dimensions.
What FD's would you expect to hold? What are the keys?

Exercise 3.1.3
Suppose $R$ is a relation with attributes $A 1, A 2$, ..., An. As a function of $n$, tell how many superkeys $R$ has, if:
a) The only key is A1.
b) The only keys are A1 and A2.
c) The only keys are $\{\mathrm{A} 1, \mathrm{~A} 2\}$ and \{A3, A4\}
d) The only keys are $\{\mathrm{A} 1, \mathrm{~A} 2\}$ and \{A1, A3\}

Exercise 3.2.1
Consider a relation with schema R(A,B,C,D) and FD's
$A B \rightarrow C, C \rightarrow D$, and $D \rightarrow A$.
a) What are all the nontrivial FD's that follow from the given FD's? Restrict to 1 attr on right side.
b) What are all the keys of R?
c) What are all the superkeys of $R$ that are not keys?

Exercise 3.2.2 i)
Consider a relation with schema $S(A, B, C, D)$ and FD's
$A \rightarrow B, B \rightarrow C$, and $B \rightarrow D$.
a) What are all the nontrivial FD's that follow from the given FD's? Restrict to 1 attr on right side.
b) What are all the keys of $S$ ?
c) What are all the superkeys of $S$ that are not keys?

Exercise 3.2.2 ii)
Consider a relation with schema $T(A, B, C, D)$ and FD's
$A B \rightarrow C, B C \rightarrow D, C D \rightarrow A$, and $A D \rightarrow B$.
a) What are all the nontrivial FD's that follow from the given FD's? Restrict to 1 attr on right side.
b) What are all the keys of $T$ ?
c) What are all the superkeys of $T$ that are not keys?

Exercise 3.2.2 iii)
Consider a relation with schema $U(A, B, C, D)$ and FD's
$A \rightarrow B, B \rightarrow C, C \rightarrow D$, and $D \rightarrow A$.
a) What are all the nontrivial FD's that follow from the given FD's? Restrict to 1 attr on right side.
b) What are all the keys of U?
c) What are all the superkeys of $U$ that are not keys?

Exercise 3.2.4
Show that each of the following are not valid rules about FD's by giving example relations that satisfy the given FD's (following the "if") but not the FD that allegedly follows (after the "then").
a) If $A \rightarrow B$ then $B \rightarrow A$.
b) If $A B \rightarrow C$ and $A \rightarrow C$, then $B \rightarrow C$.
c) If $A B \rightarrow C$, then $A \rightarrow C$ or $B \rightarrow C$.

Exercise 3.2.10
Suppose we have relation $R(A, B, C, D, E)$, with some set of FD's, and we wish to project those FD's onto relation $S(A, B, C)$. Give the FD's that hold in S if the FD's for R are:
a) $A B \rightarrow D E, C \rightarrow E, D \rightarrow C$, and $E \rightarrow A$.
b) $\mathrm{A} \rightarrow \mathrm{D}, \mathrm{BD} \rightarrow \mathrm{E}, \mathrm{AC} \rightarrow \mathrm{E}$, and $\mathrm{DE} \rightarrow \mathrm{B}$.
c) $\mathrm{AB} \rightarrow \mathrm{D}, \mathrm{AC} \rightarrow \mathrm{E}, \mathrm{BC} \rightarrow \mathrm{D}, \mathrm{D} \rightarrow \mathrm{A}$, and $\mathrm{E} \rightarrow \mathrm{B}$.
d) $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow E$, and $E \rightarrow A$.

In each case, it is sufficient to give a minimal basis for the full set of FD's of $S$.

## Exercise 3.3.1

For each of the following relation schemas and sets of FD's:
a) $R(A, B, C, D)$ with $F D$ 's $A B \rightarrow C, C \rightarrow D$, and $D \rightarrow A$.
b) $\quad R(A, B, C, D)$ with $F D$ 's $B \rightarrow C$ and $B \rightarrow D$.
c) $R(A, B, C, D)$ with $F D ' s A B \rightarrow C, B C \rightarrow D, C D \rightarrow A$, and $A D \rightarrow B$.
d) $R(A, B, C, D)$ with $F D$ 's $A \rightarrow B, B \rightarrow C, C \rightarrow D$, and $D \rightarrow A$.
e) (skipped)
f) $R(A, B, C, D, E)$ with $F D$ 's $A B \rightarrow C, C \rightarrow D, D \rightarrow B$, and $D \rightarrow E$.
do the following:
i) Indicate all the BCNF violations. Do not forget to consider FD's that are not in the given set, but follow from them. (Not necessary: violations that have more than one attribute on the right side.)
ii) Decompose the relations, as necessary, into collections of relations that are in BCNF.

List all FD's which hold in this table.

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| a | b1 | c1 | d |
| a | b1 | c | d2 |
| a3 | b | c | d |

Find all closures.

