# Introduction to Database Systems CSE 444 

Lectures 6-7: Database Design

## Outline

- Design theory: 3.1-3.4


## Schema Refinements = Normal Forms

- 1st Normal Form = all tables are flat
- 2nd Normal Form = obsolete
- Boyce-Codd Normal Form = will study
- 3rd Normal Form = see book


## First Normal Form (1NF)

- A database schema is in First Normal Form if all tables are flat

Student

| Name | GPA | Courses |
| :---: | :---: | :---: |
| Alice | 3.8 | Math <br> DB <br> Os <br> Bob <br> Carol <br> 3.7 <br> DB <br> Os |
|  |  |  |
|  |  |  |
| Math |  |  |

Student

| Name | GPA |
| :---: | :---: |
| Alice | 3.8 |
| Bob | 3.7 |
| Carol | 3.9 |

Takes

| Student | Course |
| :--- | :--- |
| Alice | Course |
| Carol | Math |
| Alice | Math |
| Bob | DB |
| Alice | OS |
| Carol | OS |
|  | Math |
| DB |  |
| OS |  |

## Relational Schema Design

Conceptual Model:


Relational Model: plus FD's

Normalization:
Eliminates anomalies


## Data Anomalies

When a database is poorly designed we get anomalies:

Redundancy: data is repeated

Updated anomalies: need to change in several places

Delete anomalies: may lose data when we don't want

## Relational Schema Design

Recall set attributes (persons with several phones):

| Name | $\underline{\text { SSN }}$ | $\underline{\text { PhoneNumber }}$ | City |
| :--- | :--- | :--- | :--- |
| Fred | $123-45-6789$ | $206-555-1234$ | Seattle |
| Fred | $123-45-6789$ | $206-555-6543$ | Seattle |
| Joe | $987-65-4321$ | $908-555-2121$ | Westfield |

One person may have multiple phones, but lives in only one city
Primary key is thus (SSN,PhoneNumber)
The above is in 1NF, but was is the problem with this schema?

## Relational Schema Design

Recall set attributes (persons with several phones):

| Name | SSN | PhoneNumber | City |
| :--- | :--- | :--- | :--- |
| Fred | $123-45-6789$ | $206-555-1234$ | Seattle |
| Fred | $123-45-6789$ | $206-555-6543$ | Seattle |
| Joe | $987-65-4321$ | $908-555-2121$ | Westfield |

## Anomalies:

- Redundancy
= repeat data
- Update anomalies = what if Fred moves to "Bellevue"?
- Deletion anomalies = what if Joe deletes his phone number? (what is his city?)


## Relation Decomposition

## Break the relation into two:

|  | Name | SSN | PhoneNumber | City |
| :---: | :---: | :---: | :---: | :---: |
|  | Fred | 123-45-6789 | 206-555-1234 | Seattle |
|  | Fred | 123-45-6789 | 206-555-6543 | Seattle |
|  | Joe | 987-65-4321 | 908-555-2121 | Westfield |
| Name | SSN | City | SSN | PhoneNumber |
| Fred | 123-45-6789 | Seattle | 123-45-6789 | 206-555-1234 |
| Joe | 987-65-4321 | Westfield | 123-45-6789 | 206-555-6543 |
| Anomalies have gone: |  |  | 987-65-4321 | 908-555-2121 |

- No more repeated data
- Easy to move Fred to "Bellevue" (how ?)
- Easy to delete all Joe's phone numbers (how ?)


## Relational Schema Design (or Logical Design)

Main idea:

- Start with some relational schema
- Find out its functional dependencies
- Use them to design a better relational schema


## Functional Dependencies

- A form of constraint
- Hence, part of the schema
- Finding them is part of the database design
- Use them to normalize the relations


## Functional Dependencies (FDs)

## Definition:

If two tuples agree on the attributes

$$
A_{1}, A_{2}, \ldots, A_{n}
$$

then they must also agree on the attributes

$$
\mathrm{B}_{1}, \mathrm{~B}_{2}, \ldots, \mathrm{~B}_{\mathrm{m}}
$$

Formally:

$$
A_{1}, A_{2}, \ldots, A_{n} \rightarrow B_{1}, B_{2}, \ldots, B_{m}
$$

## When Does an FD Hold?

Definition: $\quad A_{1}, \ldots, A_{m} \rightarrow B_{1}, \ldots, B_{n}$ holds in $R$ if:
$\forall \mathrm{t}, \mathrm{t}^{\prime} \in \mathrm{R}$,
$\left(\mathrm{t} . \mathrm{A}_{1}=\mathrm{t}^{\prime} . \mathrm{A}_{1} \wedge \ldots \wedge \mathrm{t} . \mathrm{A}_{\mathrm{m}}=\mathrm{t}^{\prime} . \mathrm{A}_{\mathrm{m}} \Rightarrow \mathrm{t} . \mathrm{B}_{1}=\mathrm{t}^{\prime} . \mathrm{B}_{1} \wedge \ldots \wedge \mathrm{t} . \mathrm{B}_{\mathrm{n}}=\mathrm{t}^{\prime} . \mathrm{B}_{\mathrm{n}}\right)$

if $t, t^{\prime}$ agree here then $t, t^{\prime}$ agree here

## Example

An FD holds, or does not hold on an instance:

| EmpID | Name | Phone | Position |
| :--- | :--- | :--- | :--- |
| E0045 | Smith | 1234 | Clerk |
| E3542 | Mike | 9876 | Salesrep |
| E1111 | Smith | 9876 | Salesrep |
| E9999 | Mary | 1234 | Lawyer |

EmpID $\rightarrow$ Name, Phone, Position
Position $\rightarrow$ Phone
but not Phone $\rightarrow$ Position

## Example

| EmpID | Name | Phone | Position |
| :--- | :--- | :--- | :--- |
| E0045 | Smith | 1234 | Clerk |
| E3542 | Mike | $9876 \leftarrow$ | Salesrep |
| E1111 | Smith | $9876 \leftarrow$ | Salesrep |
| E9999 | Mary | 1234 | Lawyer |

Position $\rightarrow$ Phone

## Example

| EmpID | Name | Phone | Position |
| :--- | :--- | :--- | :--- |
| E0045 | Smith | $1234 \rightarrow$ | Clerk |
| E3542 | Mike | 9876 | Salesrep |
| E1111 | Smith | 9876 | Salesrep |
| E9999 | Mary | $1234 \rightarrow$ | Lawyer |

But not Phone $\rightarrow$ Position

## Example

FD's are constraints:

- On some instances they hold
- On others they don't
name $\rightarrow$ color category $\rightarrow$ department color, category $\rightarrow$ price

| name | category | color | department | price |
| :---: | :---: | :---: | :---: | :---: |
| Gizmo | Gadget | Green | Toys | 49 |
| Tweaker | Gadget | Green | Toys | 99 |

Does this instance satisfy all the FDs ?

## Example name $\rightarrow$ color category $\rightarrow$ department color, category $\rightarrow$ price

| name | category | color | department | price |
| :---: | :---: | :---: | :---: | :---: |
| Gizmo | Gadget | Green | Toys | 49 |
| Tweaker | Gadget | Black | Toys | 99 |
| Gizmo | Stationary | Green | Office- <br> supp. | 59 |

What about this one?

## When Does an FD Hold?

- If we can be sure that every instance of $R$ will be one in which a given FD is true, then we say that $R$ satisfies the FD.
- If we say that R satisfies an FD F, we are stating a constraint on R .


## An Interesting Observation

name $\rightarrow$ color<br>If all these FDs are true:<br>category $\rightarrow$ department color, category $\rightarrow$ price

Then this FD also holds: name, category $\rightarrow$ price

Why ??

## Goal: Find ALL Functional Dependencies

- Anomalies occur when certain "bad" FDs hold
- We know some of the FDs
- Need to find all FDs
- Then look for the bad ones


## Armstrong's Rules (1/3)

```
\(A_{1}, A_{2}, \ldots, A_{n} \rightarrow B_{1}, B_{2}, \ldots, B_{m}\)
```

Is equivalent to

## Splitting rule and <br> Combing rule

$$
\begin{gathered}
A_{1}, A_{2}, \ldots, A_{n} \rightarrow B_{1} \\
A_{1}, A_{2}, \ldots, A_{n} \rightarrow B_{2} \\
\ldots \\
A_{1}, A_{2}, \ldots, A_{n} \rightarrow B_{m}
\end{gathered}
$$



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## Armstrong's Rules (2/3)

$$
A_{1}, A_{2}, \ldots, A_{n} \rightarrow A_{i}
$$

Trivial Rule

where $\mathrm{i}=1,2, \ldots, \mathrm{n}$

Why?


## Armstrong's Rules (3/3)

## Transitive Rule

If

$$
A_{1}, A_{2}, \ldots, A_{n} \rightarrow B_{1}, B_{2}, \ldots, B_{m}
$$

and

$$
\mathrm{B}_{1}, \mathrm{~B}_{2}, \ldots, \mathrm{~B}_{\mathrm{m}} \rightarrow \mathrm{C}_{1}, \mathrm{C}_{2}, \ldots, \mathrm{C}_{\mathrm{p}}
$$

then

$$
\begin{aligned}
& A_{1}, A_{2}, \ldots, A_{n} \rightarrow C_{1}, C_{2}, \ldots, C_{p} \\
& \quad \text { Why ? } \\
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\end{aligned}
$$

## Armstrong's Rules (3/3)

## Illustration

|  | $\mathrm{A}_{1}$ | $\ldots$ | $\mathrm{~A}_{\mathrm{m}}$ |  | $\mathrm{B}_{1}$ | $\ldots$ | $\mathrm{~B}_{\mathrm{m}}$ |  | $\mathrm{C}_{1}$ | $\ldots$ | $\mathrm{C}_{\mathrm{p}}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
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## Example (continued)

Start from the following FDs:
Infer the following FDs:

| Inferred FD | Which Rule <br> did we apply ? |
| :--- | :--- |
| 4. name, category $\rightarrow$ name |  |
| 5. name, category $\rightarrow$ color |  |
| 6. name, category $\rightarrow$ category |  |
| 7. name, category $\rightarrow$ color, category |  |
| 8. name, category $\rightarrow$ price |  |

THIS IS TOO HARD! Let's see an easier way.

## Closure of a set of Attributes

Given a set of attributes $A_{1}, \ldots, A_{n}$
The closure, $\left\{A_{1}, \ldots, A_{n}\right\}^{+}=$the set of attributes $B$ s.t. $A_{1}, \ldots, A_{n} \rightarrow B$

| Example: | $\begin{array}{l}\text { name } \rightarrow \text { color } \\ \text { category } \rightarrow \text { department } \\ \text { color, category } \rightarrow \text { price }\end{array}$ |
| :--- | :--- |

Closures:
name ${ }^{+}=$\{name, color\}
\{name, category\} ${ }^{+}=\{$name, category, color, department, price $\}$ color $^{+}=$\{color $\}$

## Closure Algorithm

$X=\{A 1, \ldots, A n\}$.
Repeat until X doesn't change do: if $\mathrm{B}_{1}, \ldots, \mathrm{~B}_{\mathrm{n}} \rightarrow \mathrm{C}$ is a FD and $B_{1}, \ldots, B_{n}$ are all in $X$
then $\operatorname{add} \mathrm{C}$ to X .

Example:
name $\rightarrow$ color
category $\rightarrow$ department color, category $\rightarrow$ price
\{name, category\} ${ }^{+}=$
\{ name, category, color, department, price \}
Hence: name, category $\rightarrow$ color, department, price

## Example

In class:
$R(A, B, C, D, E, F)$

$$
\left.\begin{array}{l}
A, B \rightarrow \\
A, D \\
B \\
B
\end{array}\right] \quad \text { D }
$$

Compute $\{\mathrm{A}, \mathrm{B}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{B}$,
Compute $\{\mathrm{A}, \mathrm{F}\}^{+} \quad \mathrm{X}=\{\mathrm{A}, \mathrm{F}, \quad\}$

## Why Do We Need Closure

- With closure we can find all FD's easily
- To check if $X \rightarrow A$
- Compute $\mathrm{X}^{+}$
- Check if $A \in X^{+}$


## Using Closure to Infer ALL FDs

Example: $\left[\begin{array}{lll}\mathrm{A}, \mathrm{B} & \rightarrow & \mathrm{C} \\ \mathrm{A}, \mathrm{D} & \rightarrow & \mathrm{B} \\ \mathrm{B} & \rightarrow & \mathrm{D}\end{array}\right.$
Step 1: Compute $\mathrm{X}^{+}$, for every X :

$$
\begin{aligned}
& \mathrm{A}+=\mathrm{A}, \mathrm{~B}+=\mathrm{BD}, \mathrm{C}+=\mathrm{C}, \mathrm{D}+=\mathrm{D} \\
& \mathrm{AB}+=\mathrm{ABCD}, \mathrm{AC}+=\mathrm{AC}, \mathrm{AD}+=\mathrm{ABCD}, \\
& \mathrm{BC}+=\mathrm{BCD}, \mathrm{BD}+=\mathrm{BD}, \mathrm{CD}+=\mathrm{CD} \\
& \mathrm{ABC}+=\mathrm{ABD}+=\mathrm{ACD}^{+}=\mathrm{ABCD} \text { (no need to compute- why ?) } \\
& \mathrm{BCD}^{+}=\mathrm{BCD}, \quad \mathrm{ABCD}+=\mathrm{ABCD}
\end{aligned}
$$

Step 2: Enumerate all FD's $\mathrm{X} \rightarrow \mathrm{Y}$, s.t. $\mathrm{Y} \subseteq \mathrm{X}^{+}$and $\mathrm{X} \cap \mathrm{Y}=\varnothing$ :

$$
\mathrm{AB} \rightarrow \mathrm{CD}, \mathrm{AD} \rightarrow \mathrm{BC}, \mathrm{BC} \rightarrow \mathrm{D}, \mathrm{ABC} \rightarrow \mathrm{D}, \mathrm{ABD} \rightarrow \mathrm{C}, \mathrm{ACD} \rightarrow \mathrm{~B}
$$

## Another Example

- Enrollment(student, major, course, room, time)
student $\rightarrow$ major
major, course $\rightarrow$ room
course $\rightarrow$ time

What else can we infer ? [in class, or at home]

## Keys

- A superkey is a set of attributes $A_{1}, \ldots, A_{n}$ s.t. for any other attribute $B$, we have $A_{1}, \ldots, A_{n} \rightarrow B$
- A key is a minimal superkey
- I.e. set of attributes which is a superkey and for which no subset is a superkey


## Computing (Super)Keys

- Compute $\mathrm{X}^{+}$for all sets X
- If $X^{+}=$all attributes, then $X$ is a superkey
- List only the minimal $X$ 's to get the keys


## Example

# Product(name, price, category, color) 

name, category $\rightarrow$ price category $\rightarrow$ color

What is the key?

## Examples of Keys

Enrollment(student, address, course, room, time)

student $\rightarrow$ address<br>room, time $\rightarrow$ course<br>student, course $\rightarrow$ room, time

(find keys at home or in class)

## Eliminating Anomalies

Main idea:

- $X \rightarrow A$ is OK if $X$ is a (super)key
- $X \rightarrow A$ is not OK otherwise


## Example

| Name | SSN | PhoneNumber | City |
| :--- | :--- | :--- | :--- |
| Fred | $123-45-6789$ | $206-555-1234$ | Seattle |
| Fred | $123-45-6789$ | $206-555-6543$ | Seattle |
| Joe | $987-65-4321$ | $908-555-2121$ | Westfield |
| Joe | $987-65-4321$ | $908-555-1234$ | Westfield |

## SSN $\rightarrow$ Name, City

What is the key?
\{SSN, PhoneNumber\} Hence SSN $\rightarrow$ Name, City is a "bad" dependency

## Key or Keys ?

Can we have more than one key ?

Given $R(A, B, C)$ define FD's s.t. there are two or more keys

## Key or Keys ?

Can we have more than one key ?

Given $R(A, B, C)$ define FD's s.t. there are two or more keys

$$
\begin{aligned}
& \mathrm{AB} \rightarrow \mathrm{C} \\
& \mathrm{BC} \rightarrow \mathrm{~A}
\end{aligned} \quad \text { or } \quad \begin{aligned}
& \mathrm{A} \rightarrow \mathrm{BC} \\
& \mathrm{~B} \rightarrow \mathrm{AC}
\end{aligned}
$$

what are the keys here ?
Can you design FDs such that there are three keys ?

## Boyce-Codd Normal Form

A simple condition for removing anomalies from relations:

## A relation $R$ is in BCNF if:

If $A_{1}, \ldots, A_{n} \rightarrow B$ is a non-trivial dependency in $R$, then $\left\{A_{1}, \ldots, A_{n}\right\}$ is a superkey for $R$

In other words: there are no "bad" FDs

Equivalently: for all $X$, either $\left(X^{+}=X\right) \quad$ or $\quad\left(X^{+}=\right.$all attributes $)$

## BCNF Decomposition Algorithm

## repeat

choose $A_{1}, \ldots, A_{m} \rightarrow B_{1}, \ldots, B_{n}$ that violates $B C N F$ split $R$ into $R_{1}\left(A_{1}, \ldots, A_{m}, B_{1}, \ldots, B_{n}\right)$ and $R_{2}\left(A_{1}, \ldots, A_{m}\right.$, [others]) continue with both $R_{1}$ and $R_{2}$
until no more violations


## Is there a <br> 2-attribute relation that is not in BCNF ?

In practice, we have a better algorithm (coming up)

## Example

| Name | SSN | PhoneNumber | City |
| :--- | :--- | :--- | :--- |
| Fred | $123-45-6789$ | $206-555-1234$ | Seattle |
| Fred | $123-45-6789$ | $206-555-6543$ | Seattle |
| Joe | $987-65-4321$ | $908-555-2121$ | Westfield |
| Joe | $987-65-4321$ | $908-555-1234$ | Westfield |

## SSN $\rightarrow$ Name, City

What is the key?
\{SSN, PhoneNumber\} use SSN $\rightarrow$ Name, City to split

## Example

| Name | $\underline{\text { SSN }}$ | City |
| :--- | :--- | :--- |
| SSN $\rightarrow$ Name, City |  |  |
|  | $123-45-6789$ | Seattle |
| Joe | $987-65-4321$ | Westfield |
|  |  |  |


| SSN | PhoneNumber |
| :--- | :--- |
| $123-45-6789$ | $206-555-1234$ |
| $123-45-6789$ | $206-555-6543$ |
| $987-65-4321$ | $908-555-2121$ |
| $987-65-4321$ | $908-555-1234$ |

Let's check anomalies:

- Redundancy?
- Update ?
- Delete ?


## Example Decomposition

Person(name, SSN, age, hairColor, phoneNumber) FD1: SSN $\rightarrow$ name, age FD2: age $\rightarrow$ hairColor
Decompose in BCNF (in class):
What is the key?
How to decompose?

## BCNF Decomposition Algorithm

BCNF_Decompose(R)
find X s.t.: $\mathrm{X} \neq \mathrm{X}^{+} \neq[$all attributes $]$
if (not found) then " $R$ is in BCNF"
let $\mathrm{Y}=\mathrm{X}^{+}-\mathrm{X}$
let $\mathrm{Z}=[$ all attributes $]-\mathrm{X}^{+}$ decompose R into $\mathrm{R} 1(\mathrm{X} \cup \mathrm{Y})$ and $\mathrm{R} 2(\mathrm{X} \cup \mathrm{Z})$ continue to decompose recursively R1 and R2

Find X s.t.: $\mathrm{X} \neq \mathrm{X}^{+} \neq[$all attributes $]$

## Example BCNF Decomposition

Person(name, SSN, age, hairColor, phoneNumber) SSN $\rightarrow$ name, age age $\rightarrow$ hairColor

> Iteration 1: Person
> SSN+ = SSN, name, age, hairColor
> Decompose into: P(SSN, name, age, hairColor) Phone(SSN, phoneNumber)

Iteration 2: P
age+ = age, hairColor
Decompose: People(SSN, name, age) Hair(age, hairColor)
Phone(SSN, phoneNumber)

What are the keys?

R(A,B,C,D)

## Example

$$
\begin{aligned}
& \mathrm{A} \rightarrow \mathrm{~B} \\
& \mathrm{~B} \rightarrow \mathrm{C}
\end{aligned}
$$



What happens if in R we first pick $\mathrm{B}^{+}$? Or $\mathrm{AB}^{+}$?

## Decompositions in General


$R_{1}=$ projection of $R$ on $A_{1}, \ldots, A_{n}, B_{1}, \ldots, B_{m}$
$R_{2}=$ projection of $R$ on $A_{1}, \ldots, A_{n}, C_{1}, \ldots, C_{p}$

## Theory of Decomposition

- Sometimes it is correct:


Lossless decomposition

## Incorrect Decomposition

- Sometimes it is not:



## Decompositions in General



$$
\text { If } A_{1}, \ldots, A_{n} \rightarrow B_{1}, \ldots, B_{m}
$$

Then the decomposition is lossless

Note: don't need $A_{1}, \ldots, A_{n} \rightarrow C_{1}, \ldots, C_{p}$
BCNF decomposition is always lossless. WHY?

## Ontin?

- The following four slides are optional
- The content will not be on any exam
- But please take a look because they motivate the need for 3NF
- It's good to know at least why 3NF exists


## General Decomposition Goals

1. Elimination of anomalies
2. Recoverability of information

- Can we get the original relation back?

3. Preservation of dependencies

- Want to enforce FDs without performing joins


## Sometimes cannot decomposed into BCNF without losing ability to check some FDs

## BCNF and Dependencies

| Unit | Company | Product |
| :---: | :---: | :---: |
|  |  |  |

FD's: Unit $\rightarrow$ Company; Company, Product $\rightarrow$ Unit So, there is a BCNF violation, and we decompose.

## BCNF and Dependencies

| Unit | Company | Product |
| :---: | :---: | :---: |
|  |  |  |

FD's: Unit $\rightarrow$ Company; Company, Product $\rightarrow$ Unit So, there is a BCNF violation, and we decompose.


Unit $\rightarrow$ Company


No FDs

In BCNF we lose the FD: Company, Product $\rightarrow$ Unit

## 3NF Motivation

A relation R is in 3rd normal form if :
Whenever there is a nontrivial dep. $A_{1}, A_{2}, \ldots, A_{n} \rightarrow B$ for $R$, then $\left\{A_{1}, A_{2}, \ldots, A_{n}\right\}$ is a super-key for $R$, or $B$ is part of a key.

Tradeoffs
BCNF = no anomalies, but may lose some FDs 3NF = keeps all FDs, but may have some anomalies

