Impossibility of Consensus in Asynchronous Environments

Ellis Michael

Consensus

n processes, all of which have an input value from some domain.

Processes output a value by calling *decide*(v).

Non-faulty processes continue correctly executing protocol steps forever.

We denote the number of faulty processes f.

- **Agreement:** No two correct processes decide different values.
- **Integrity:** Every correct process decides at most one value, and if a correct process decides a value v, some process had v as its input.
- **Termination:** Every correct process eventually decides a value.

Binary Consensus

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If you can solve consensus, you can solve binary consensus.

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Aside: Both safety and liveness properties are necessary to create a meaningful specification!

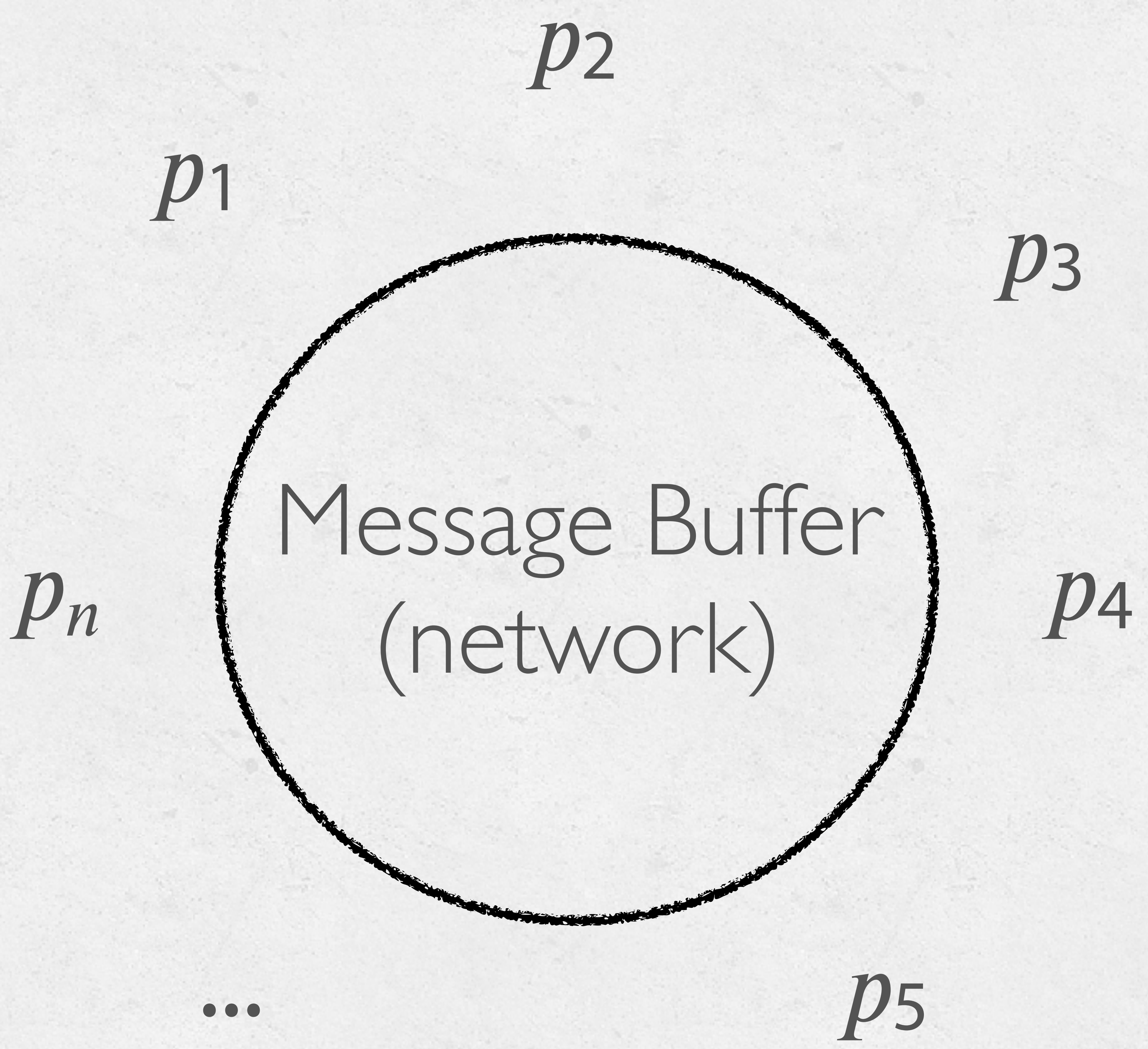
Theorem (FLP Impossibility Result): *In an asynchronous environment in which a single process can fail by crashing, there does not exist a protocol which solves binary consensus.*

Intuition

- In an asynchronous setting, failed processes are indistinguishable from slow processes.
- Waiting for failed processes will take forever.
- Not waiting for slow processes could violate safety.

Computation Model

- Processes are deterministic I/O automata (just like in your labs; timers are just messages sent from process to itself).



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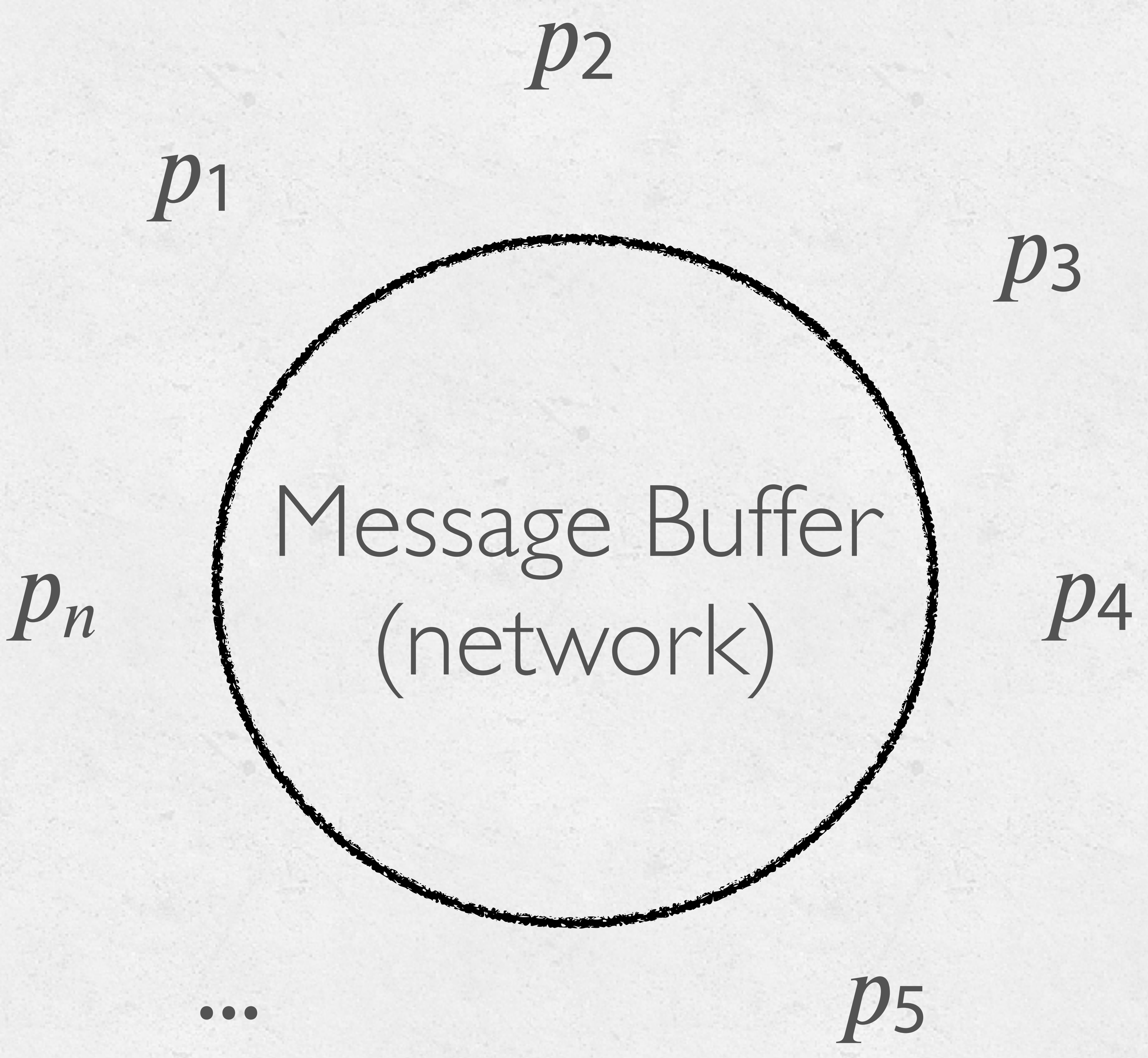

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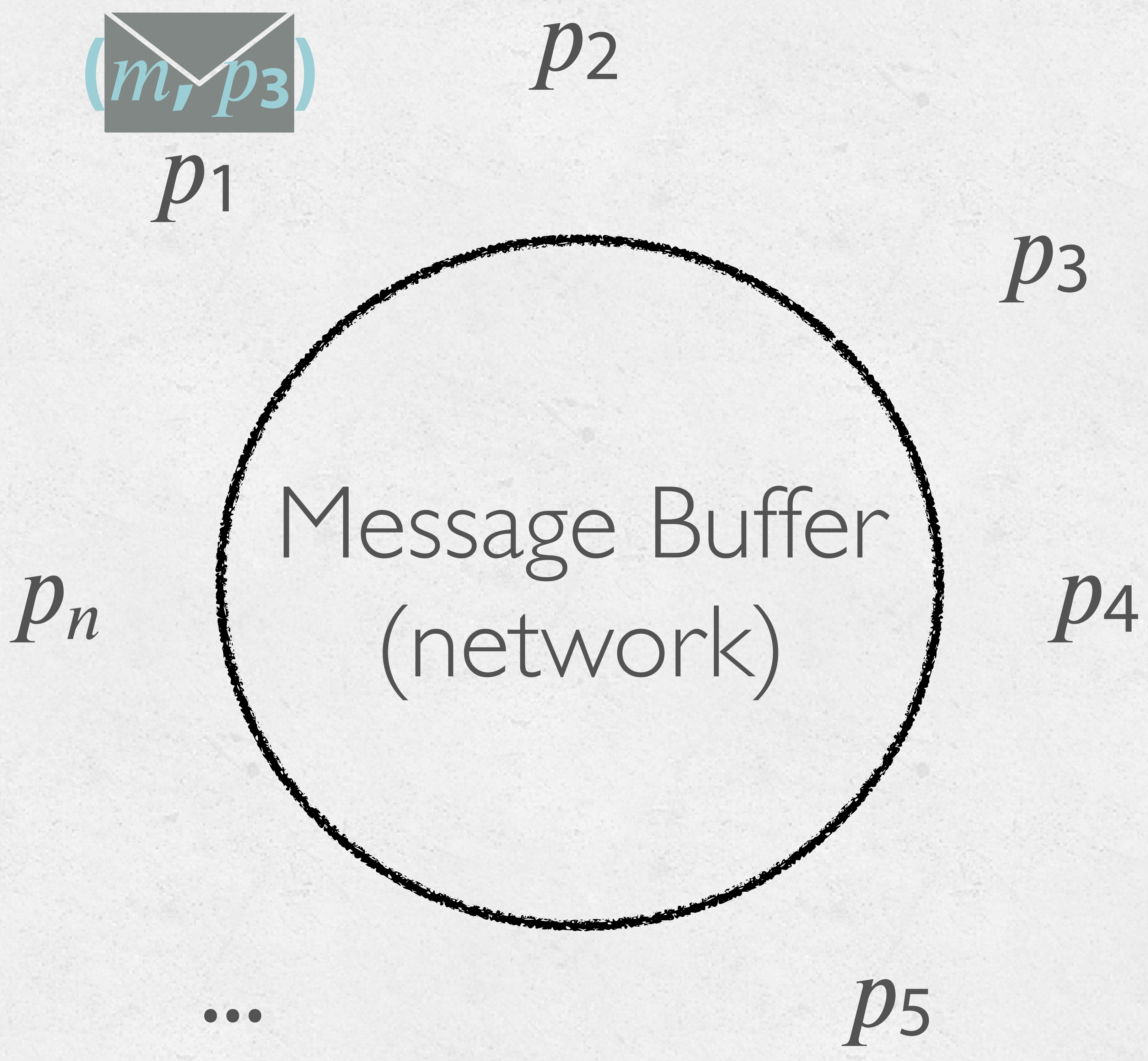

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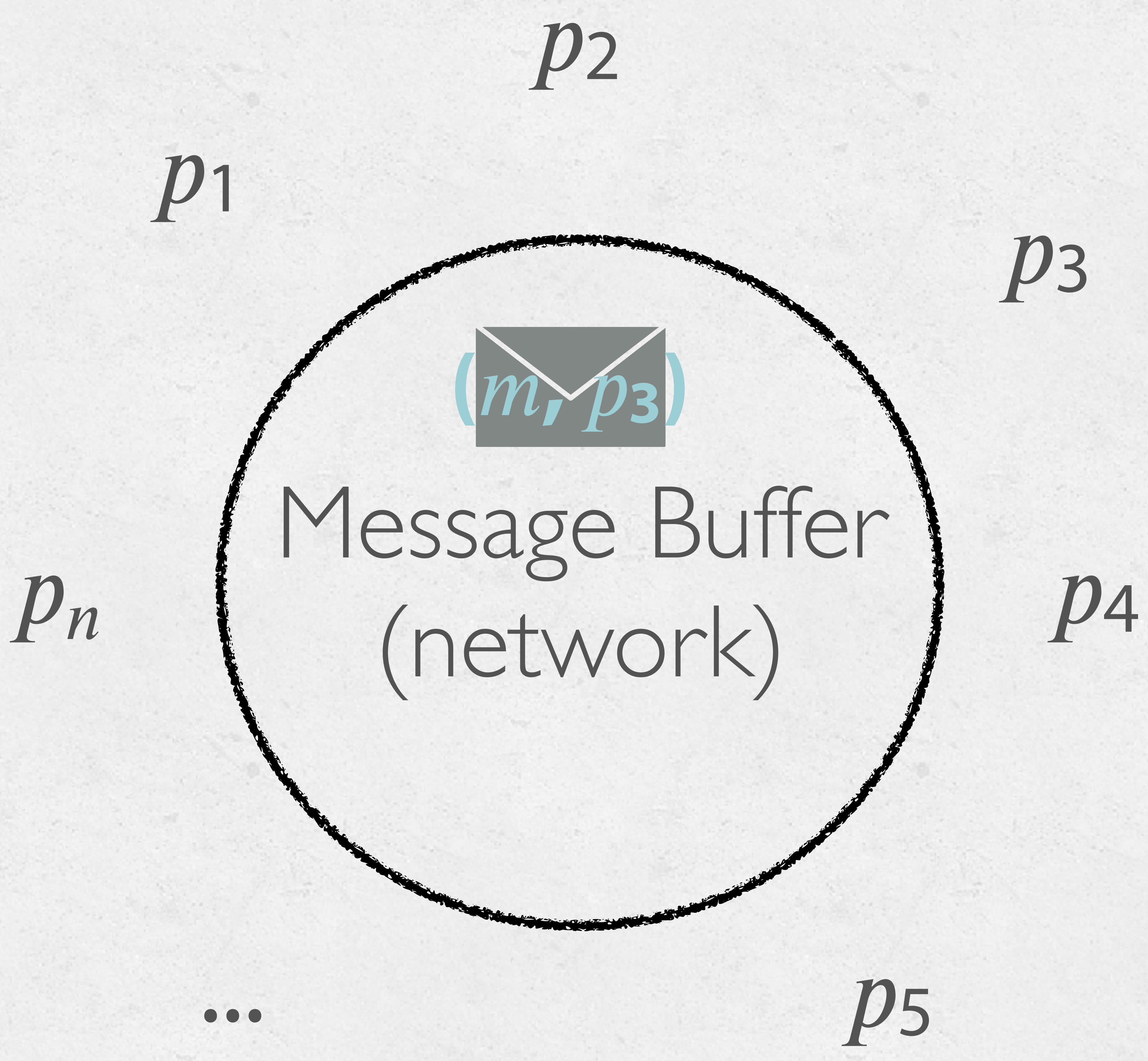

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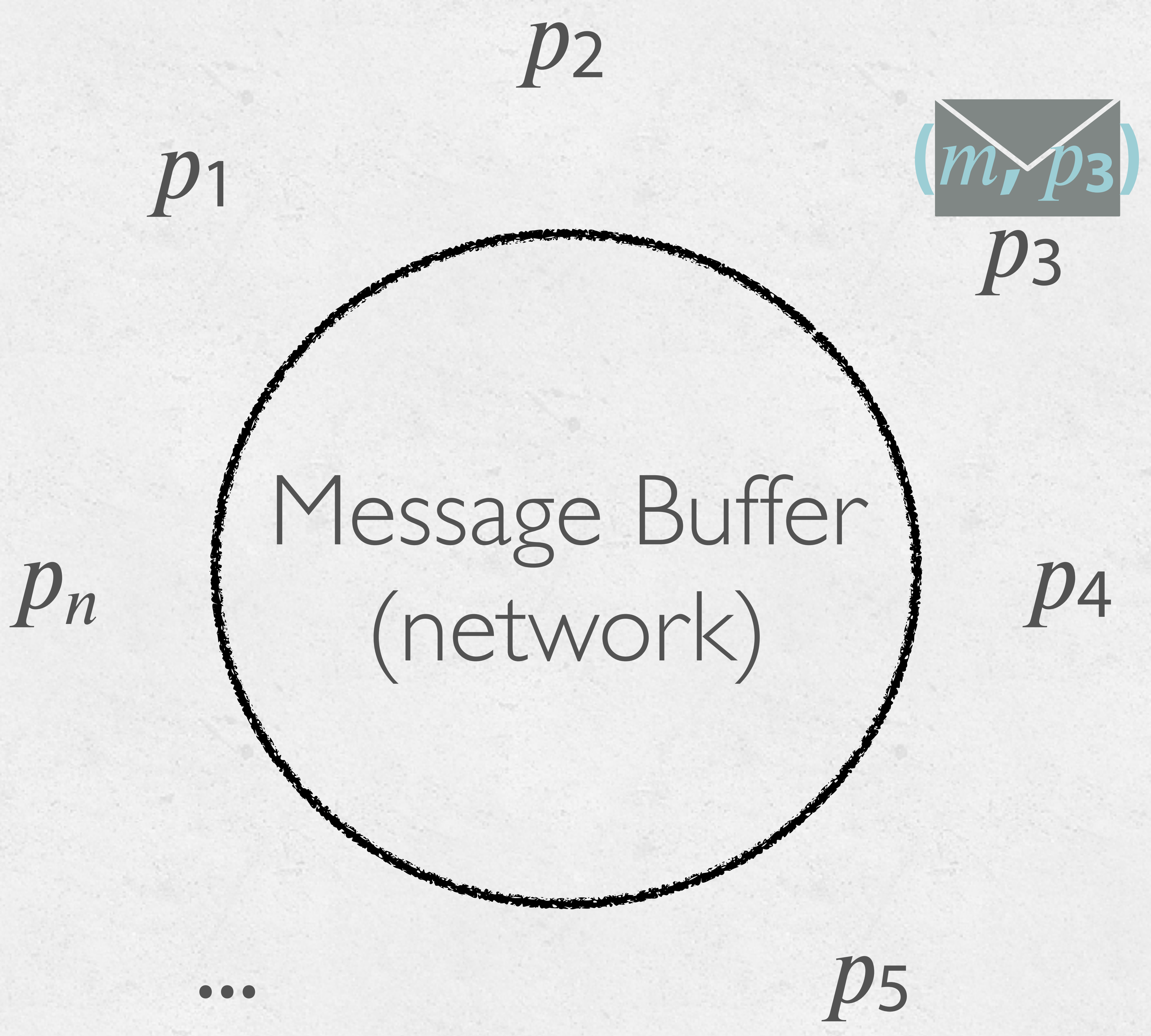

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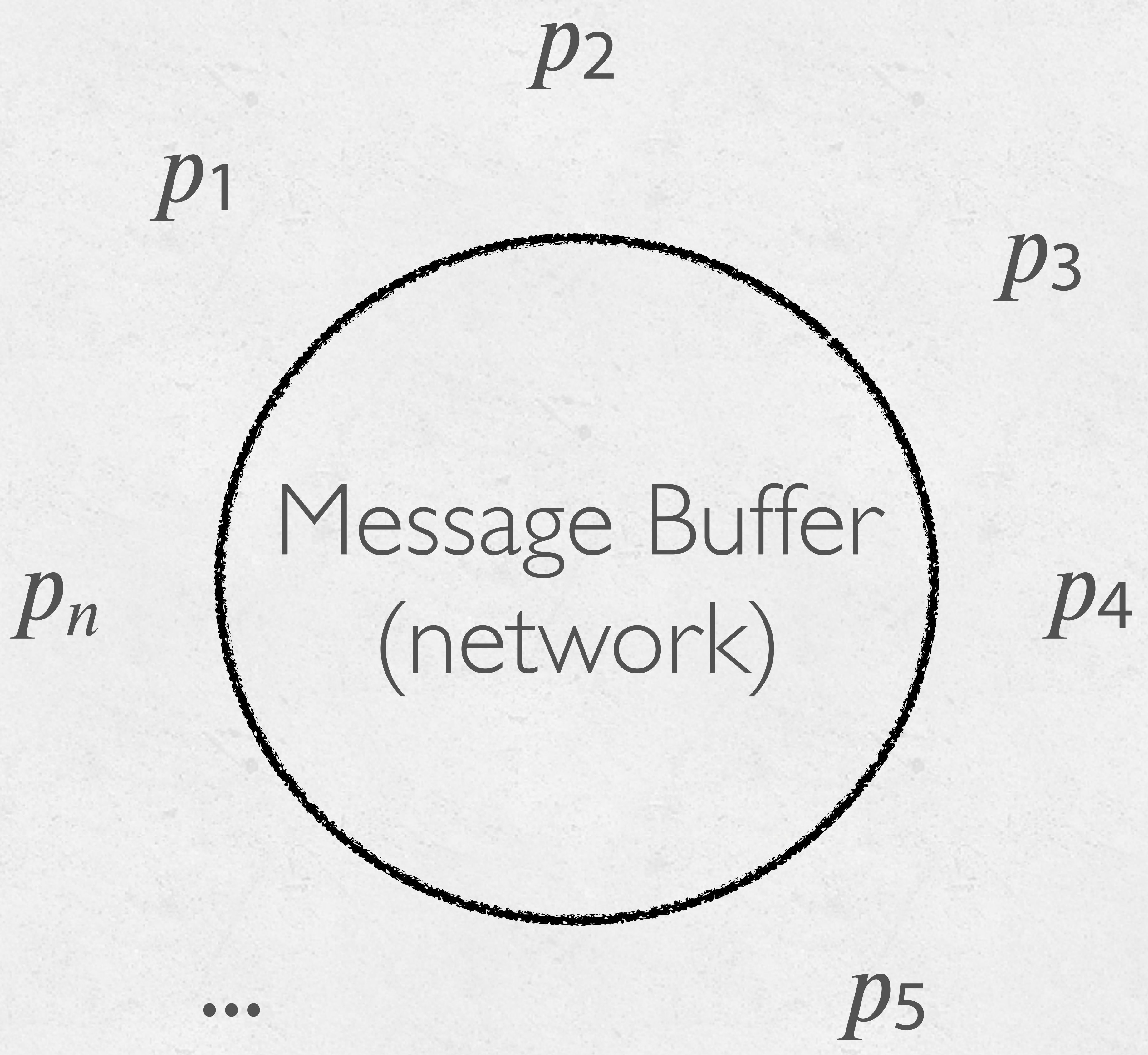

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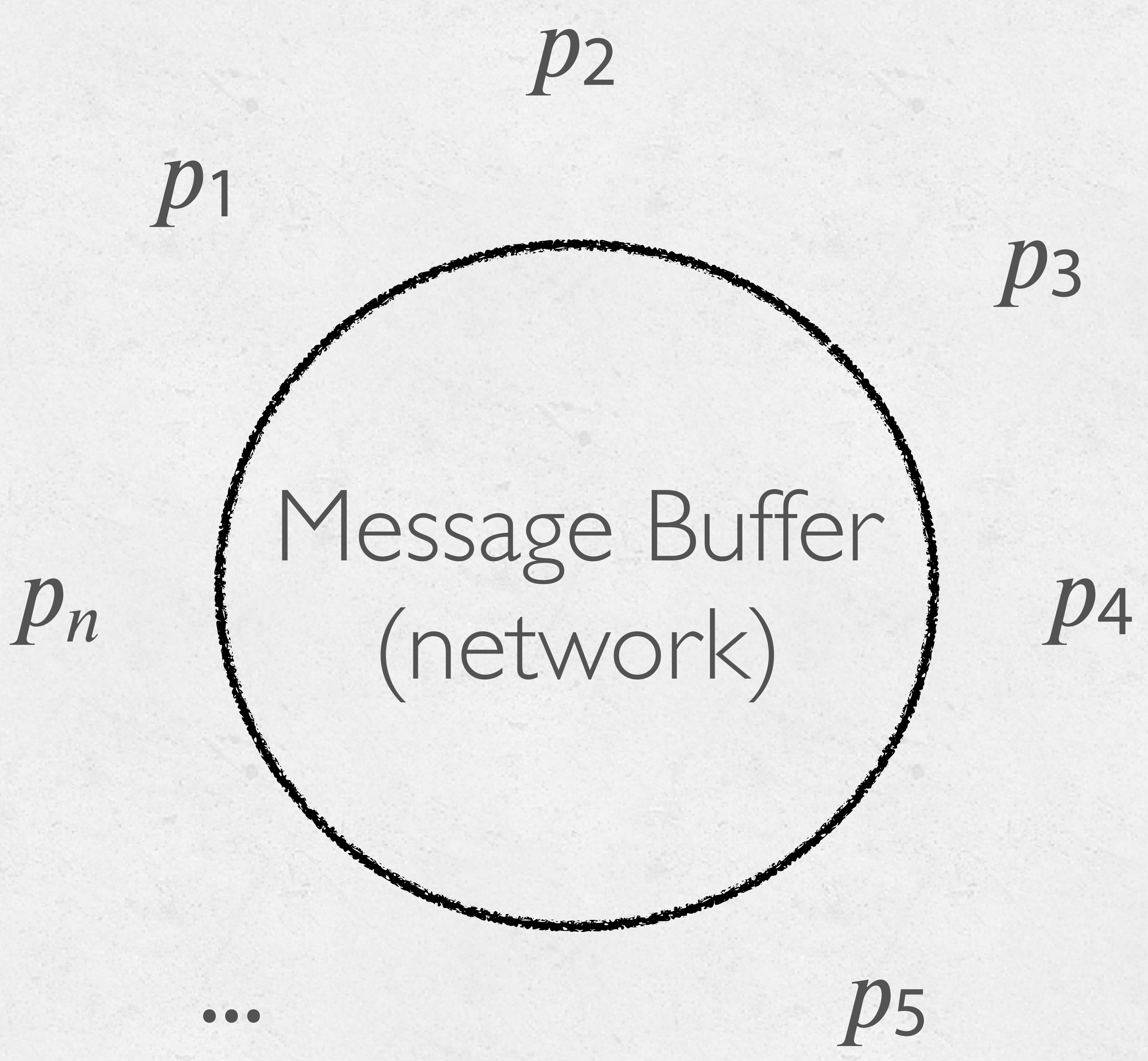

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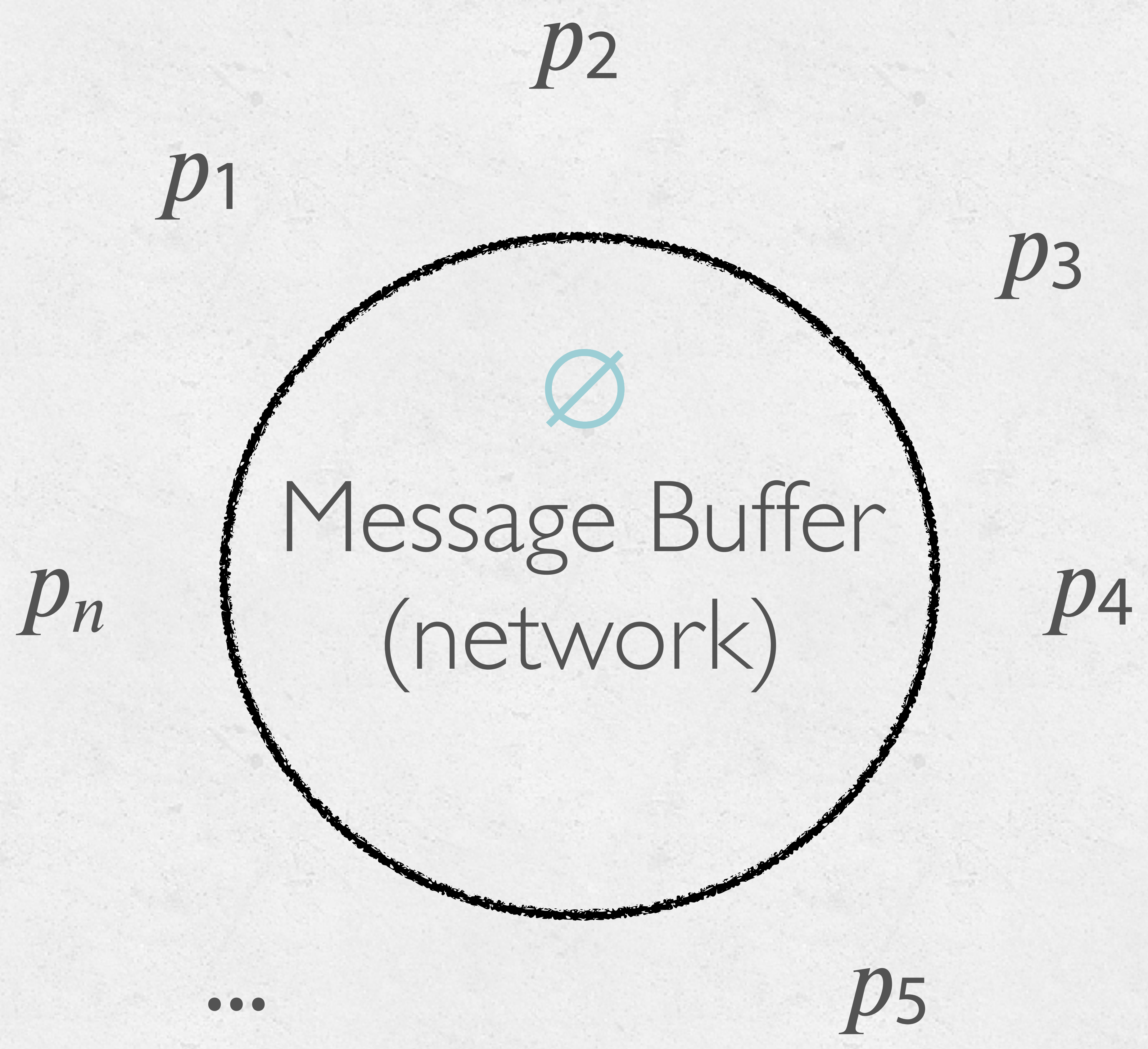

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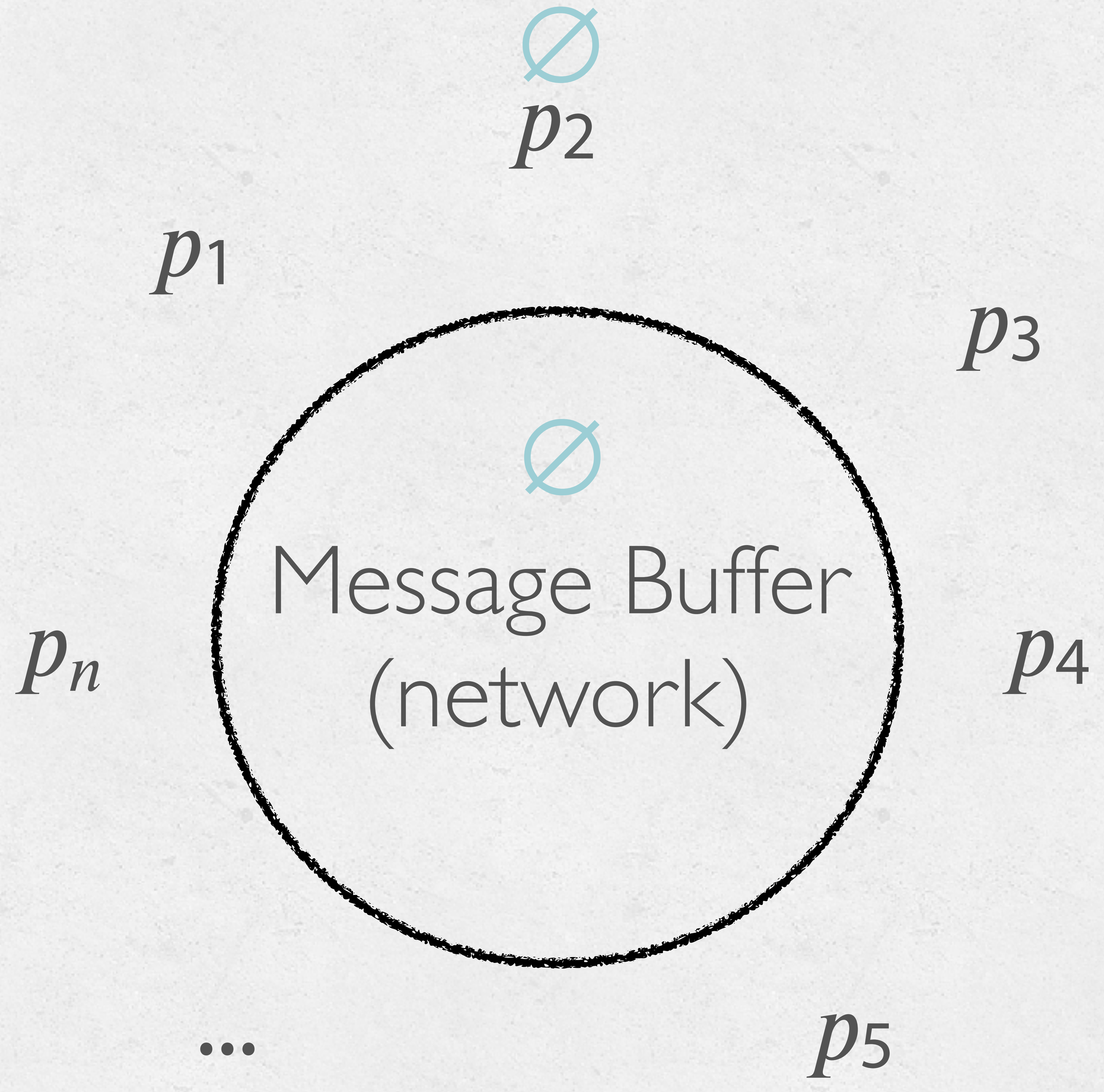

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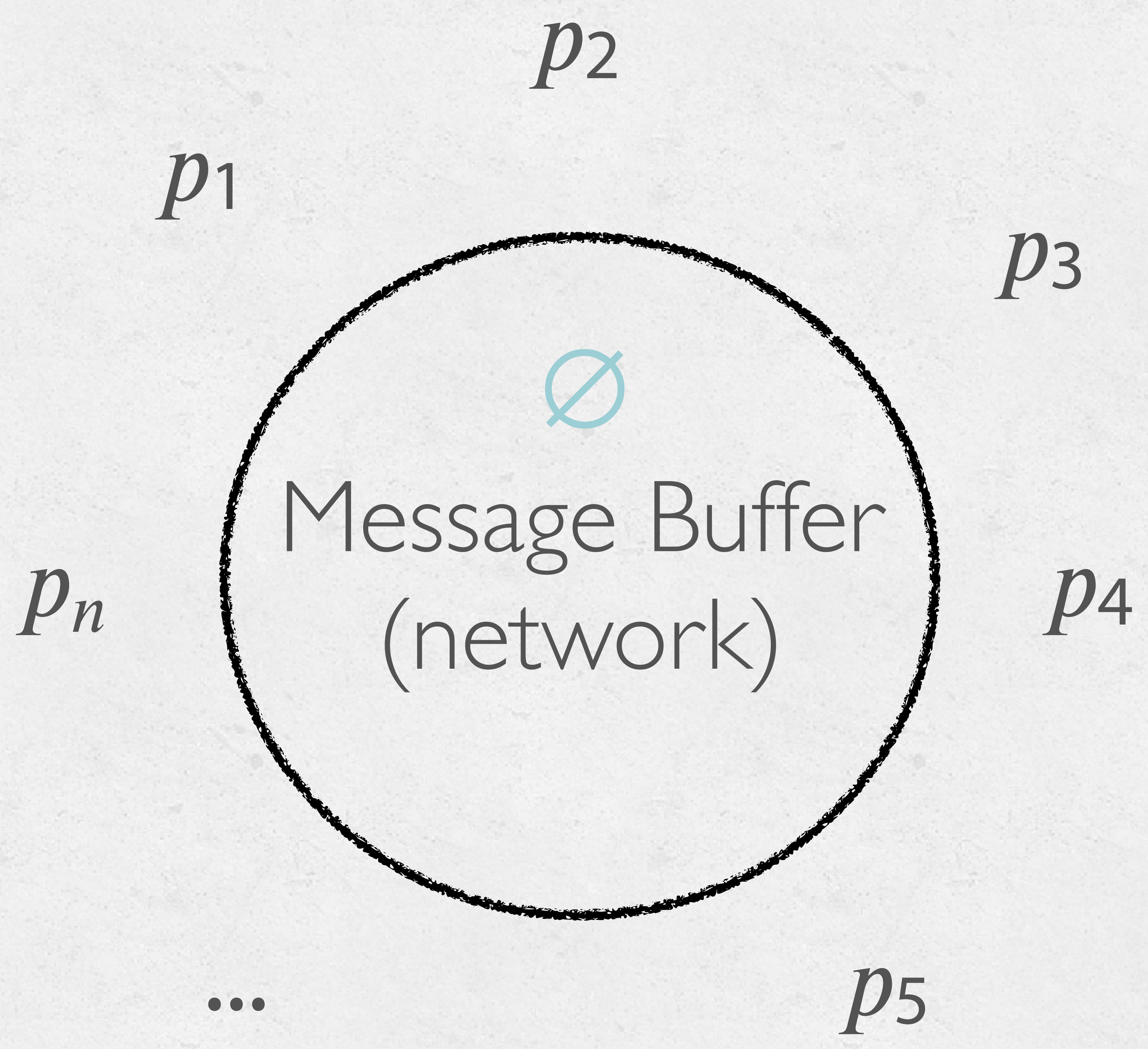

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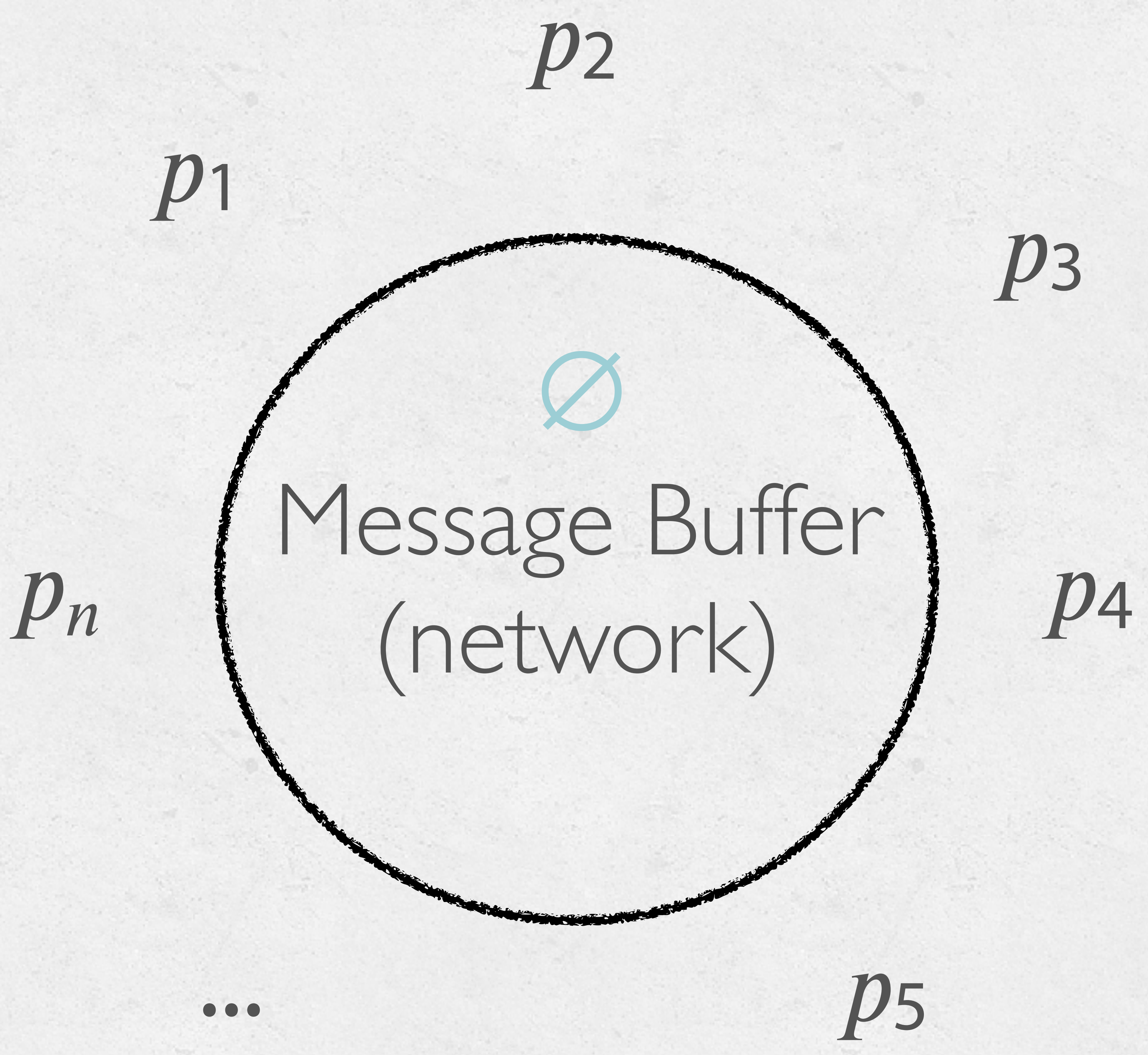

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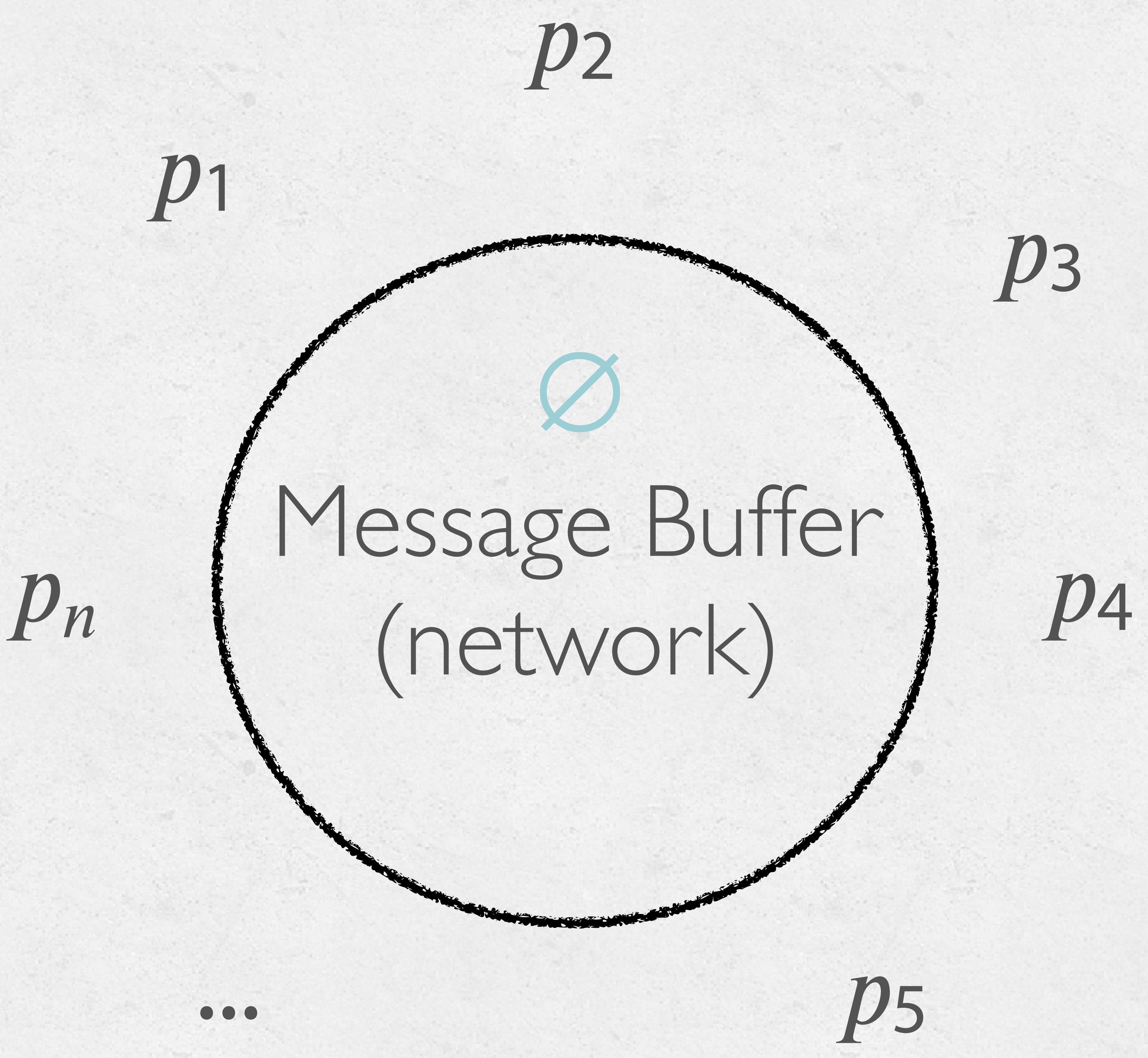
Makes the impossibility result is stronger!



Configurations

A **configuration** (usually denoted C) consists of the states of all processes and the state of the message buffer.

An **event** is the delivery of a single message (or $\varnothing$) to a process. An event is **applicable** to C if it is a $\varnothing$ or a message in C's message buffer.

A configuration C' is **reachable** from C if there is a (possibly empty) sequence of applicable events starting from C that results in C'.

Configuration C is **decided** if at least one process has decided in C.

Runs

A **run** is an infinite sequence of events starting from an initial configuration.

A process is **non-faulty** in a run if it takes infinitely many steps. It is faulty otherwise.

A run is **admissible** if at most one process is faulty and every message sent to a non-faulty process is eventually delivered.

In other words, the FLP theorem states that **any protocol** for binary consensus either doesn't satisfy safety or allows for an admissible run in which no value is ever decided (i.e., that it doesn't satisfy termination, the liveness property).

From now on, we'll consider a **safe** and **live** binary consensus protocol and show a contradiction.

Valency

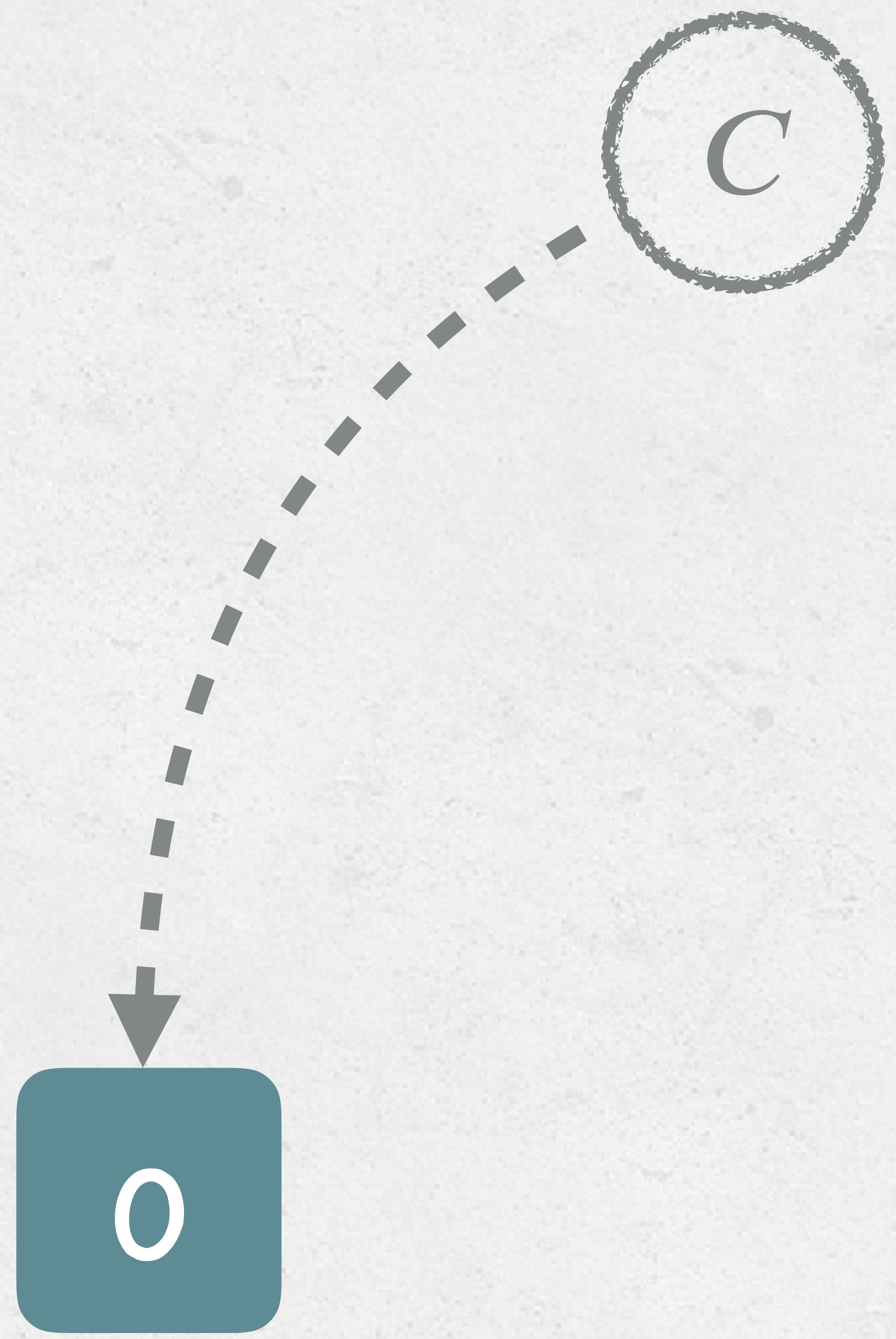
By assumption of safety, no configuration has processes deciding different values.

C is **0-valent** if there are decided configurations reachable from C that decide 0, but none that decide 1.

1-valency is defined in the analogous way.

C is **univalent** if it is 0-valent or 1-valent.

C is **bivalent** if both 0-deciding and 1-deciding are reachable from C.



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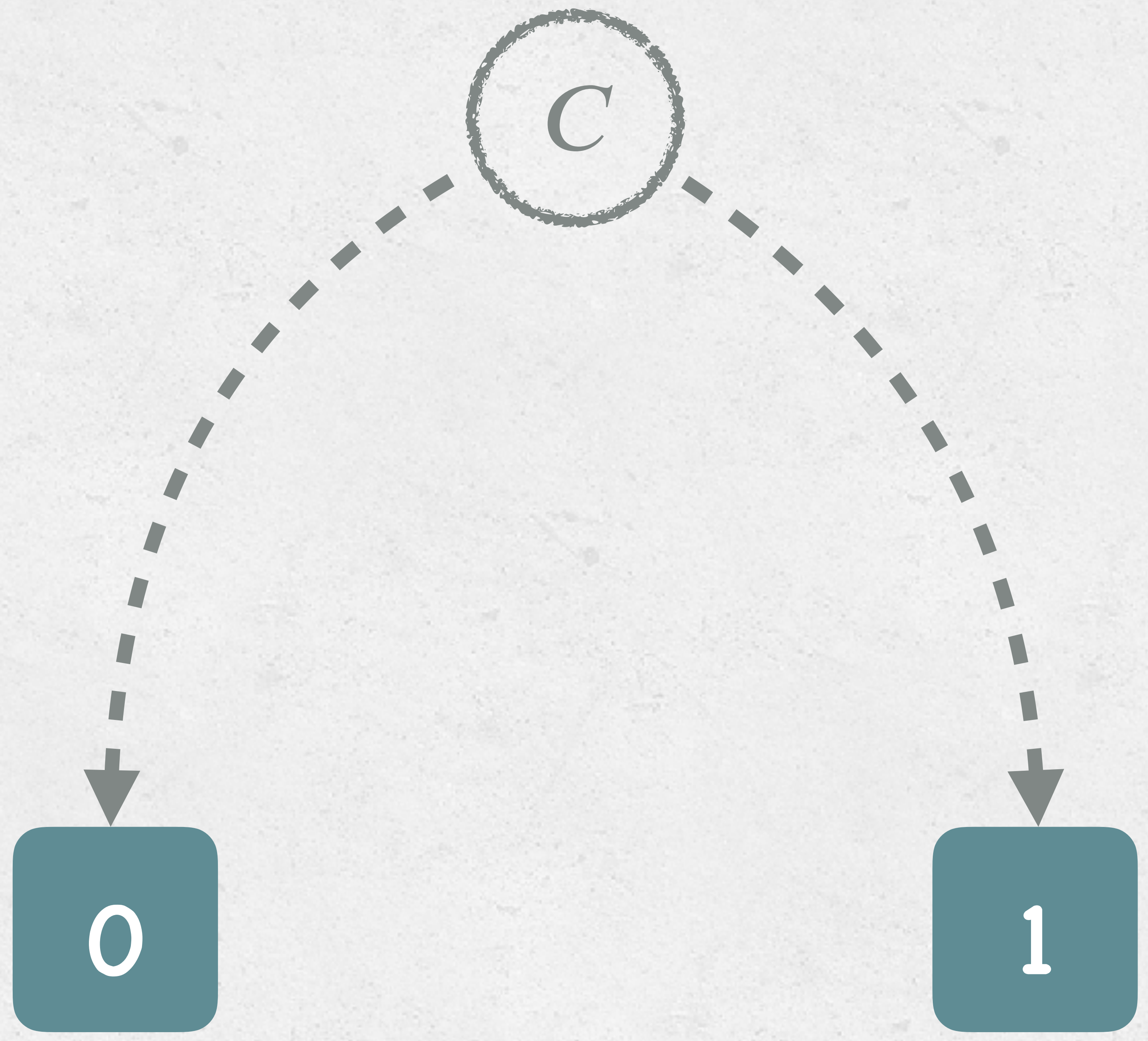


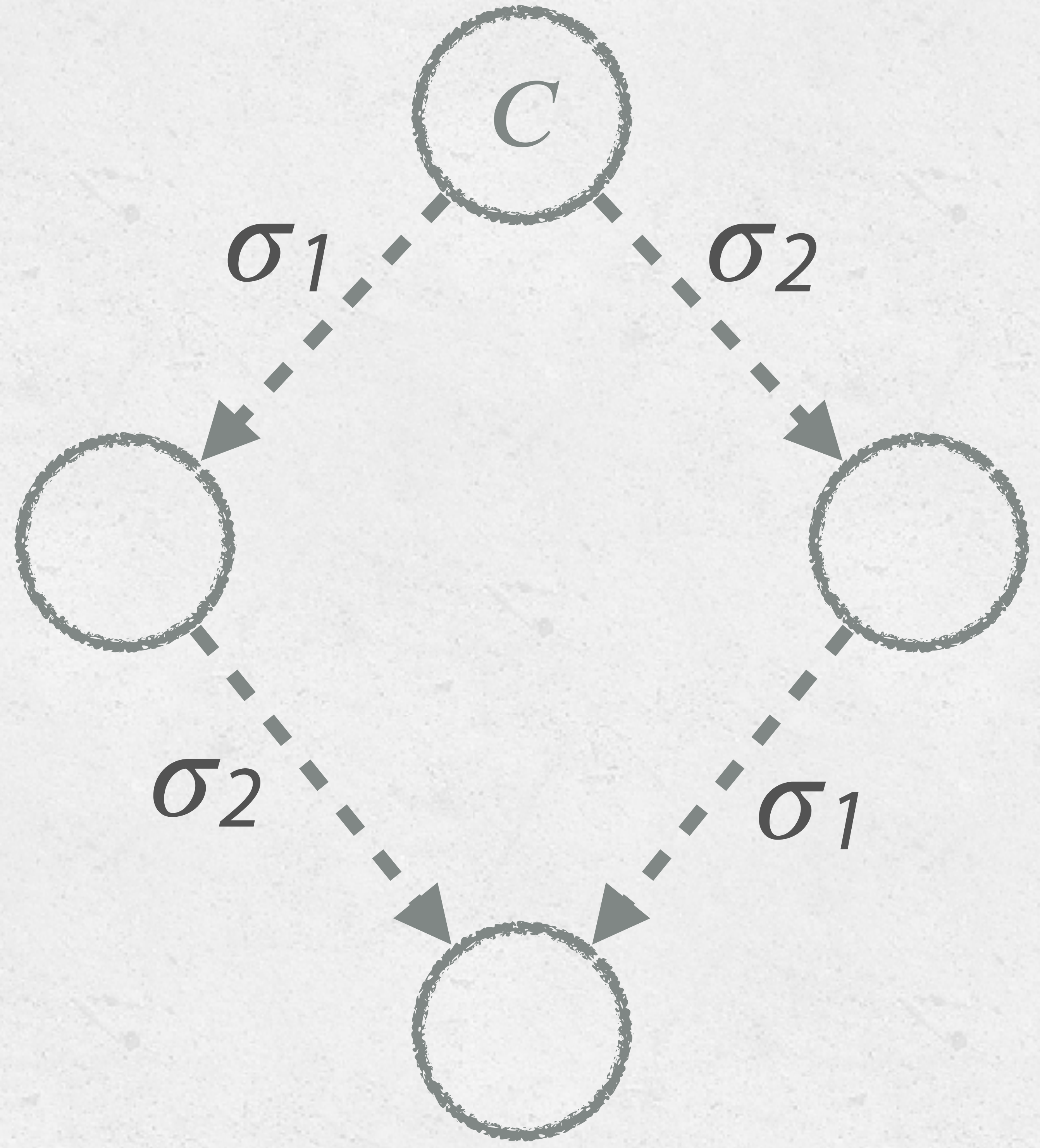
Observation: bivalent configurations are not themselves decided.

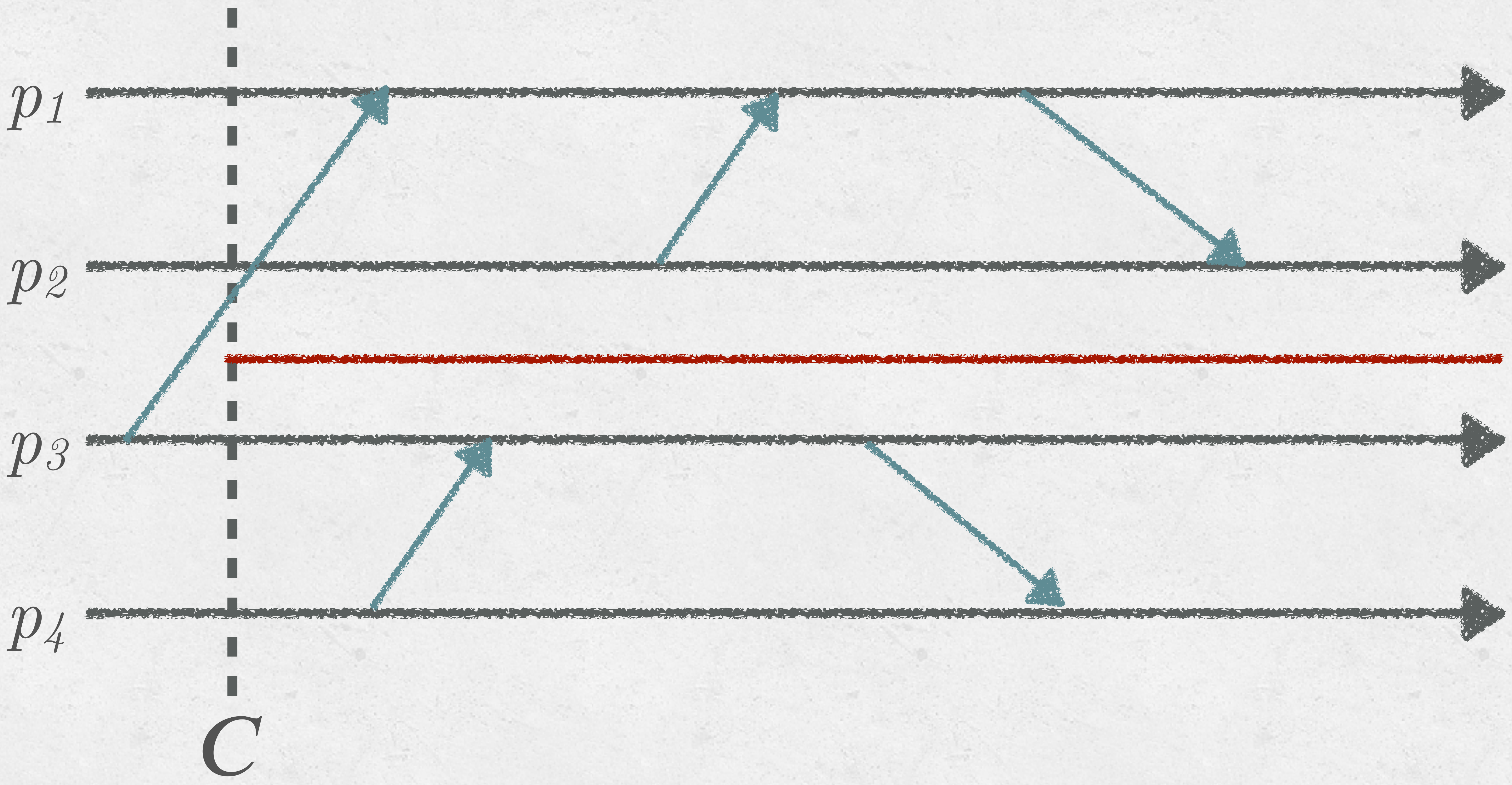
Observation: 1-valent and bivalent configurations are not reachable from 0-valent configurations.

0-valent and bivalent configurations are not reachable from 1-valent configurations.

Commutative Events

Lemma 1: *If two sequences of events, σ_1 and σ_2, are taken by* ***disjoint*** *sets of processes from configuration C, then $\sigma_1(\sigma_2(C)) = \sigma_2(\sigma_1(C))$.*





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These two configurations are indistinguishable to the rest of the processes.

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Delaying Events

Lemma 3 (The Delay Lemma): *For every bivalent configuration, C, and every event applicable to C, e, there exists a sequence of applicable events σ such that $C' = e(\sigma(C))$ is bivalent.*

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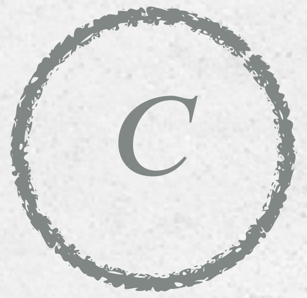

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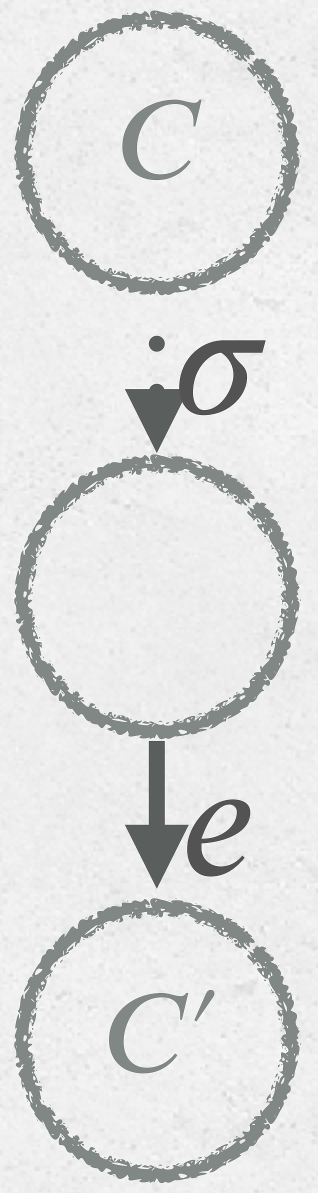

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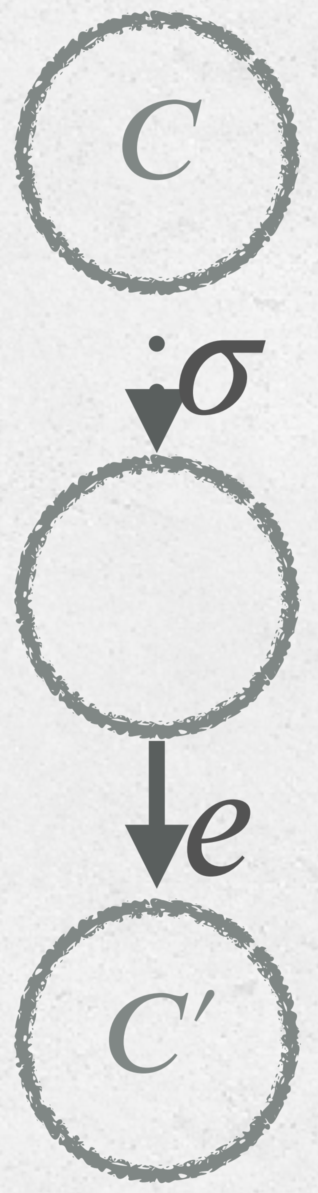

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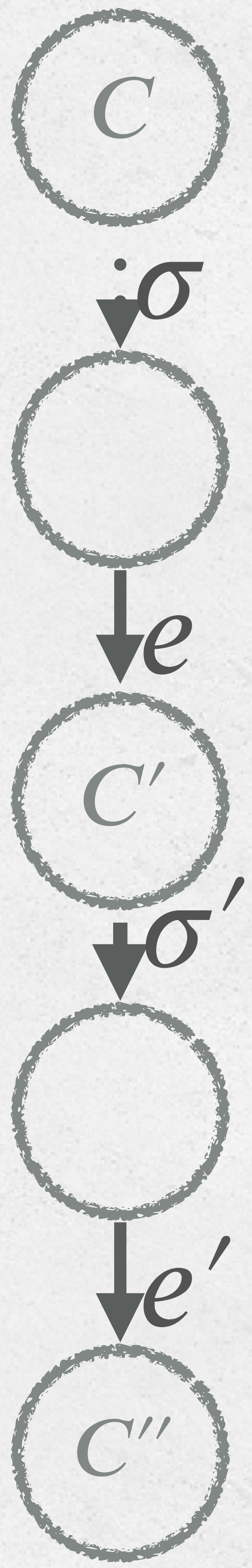

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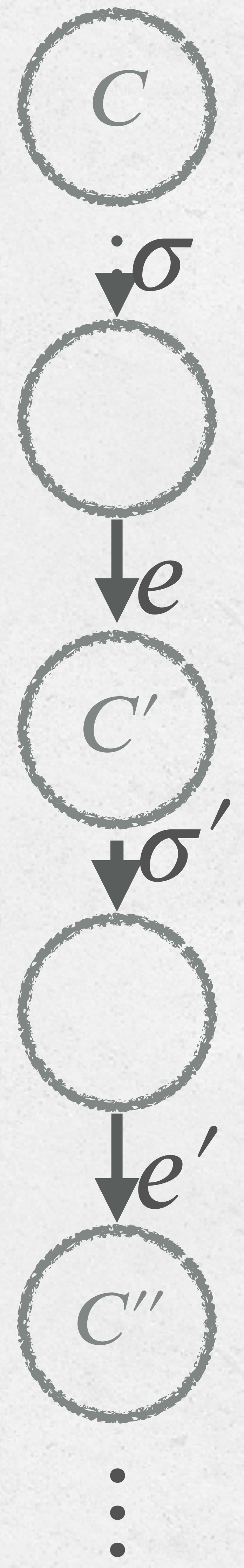

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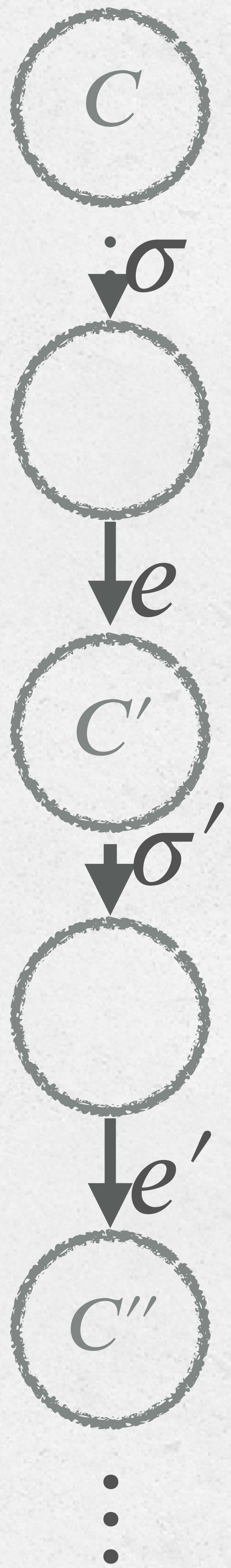

Every process takes infinitely many steps (i.e., no process is faulty). Every message sent is eventually delivered. This is an admissible execution.

We take infinitely many steps, and no process decides! The protocol fails to meet the termination property of the spec.

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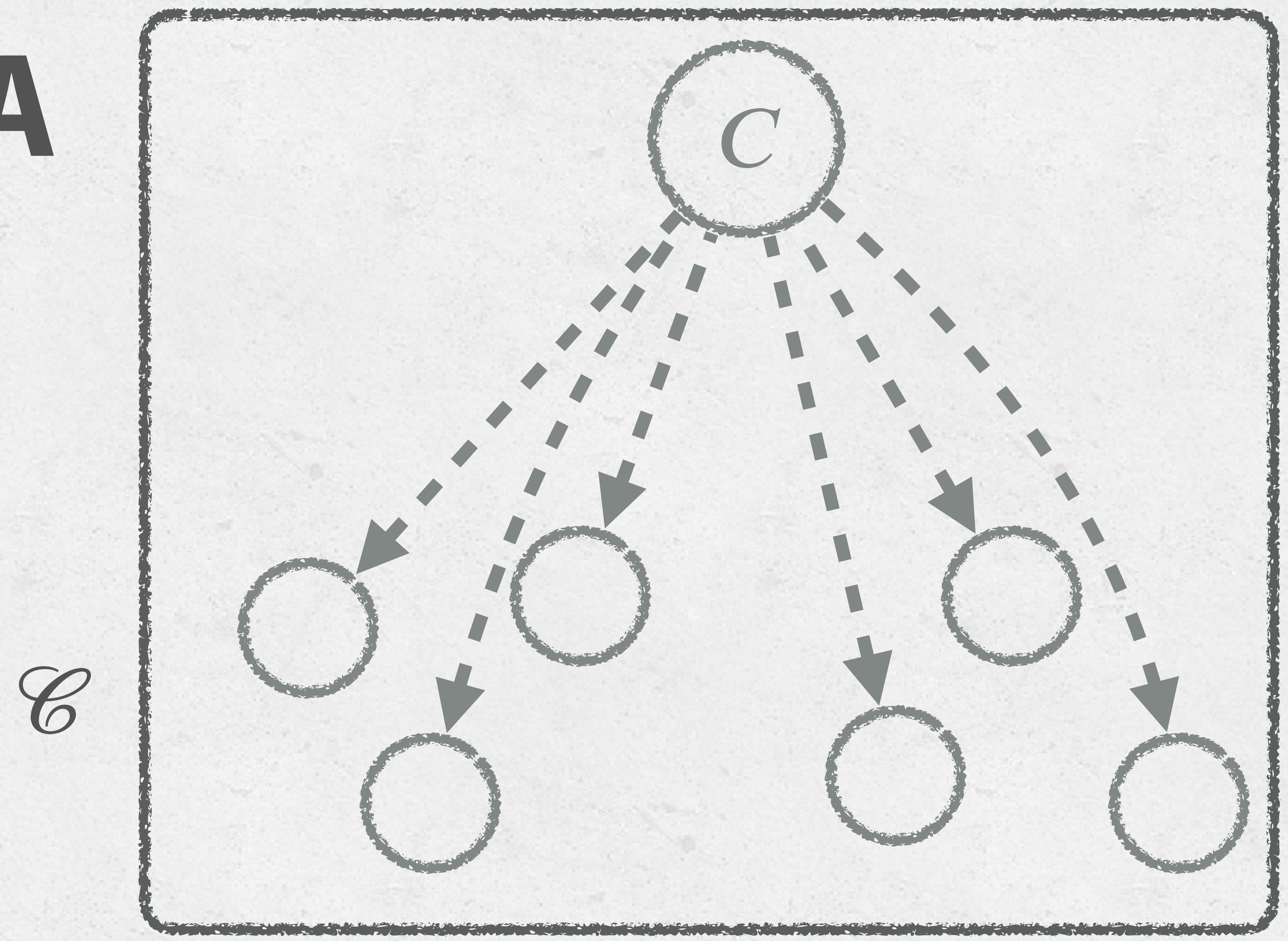

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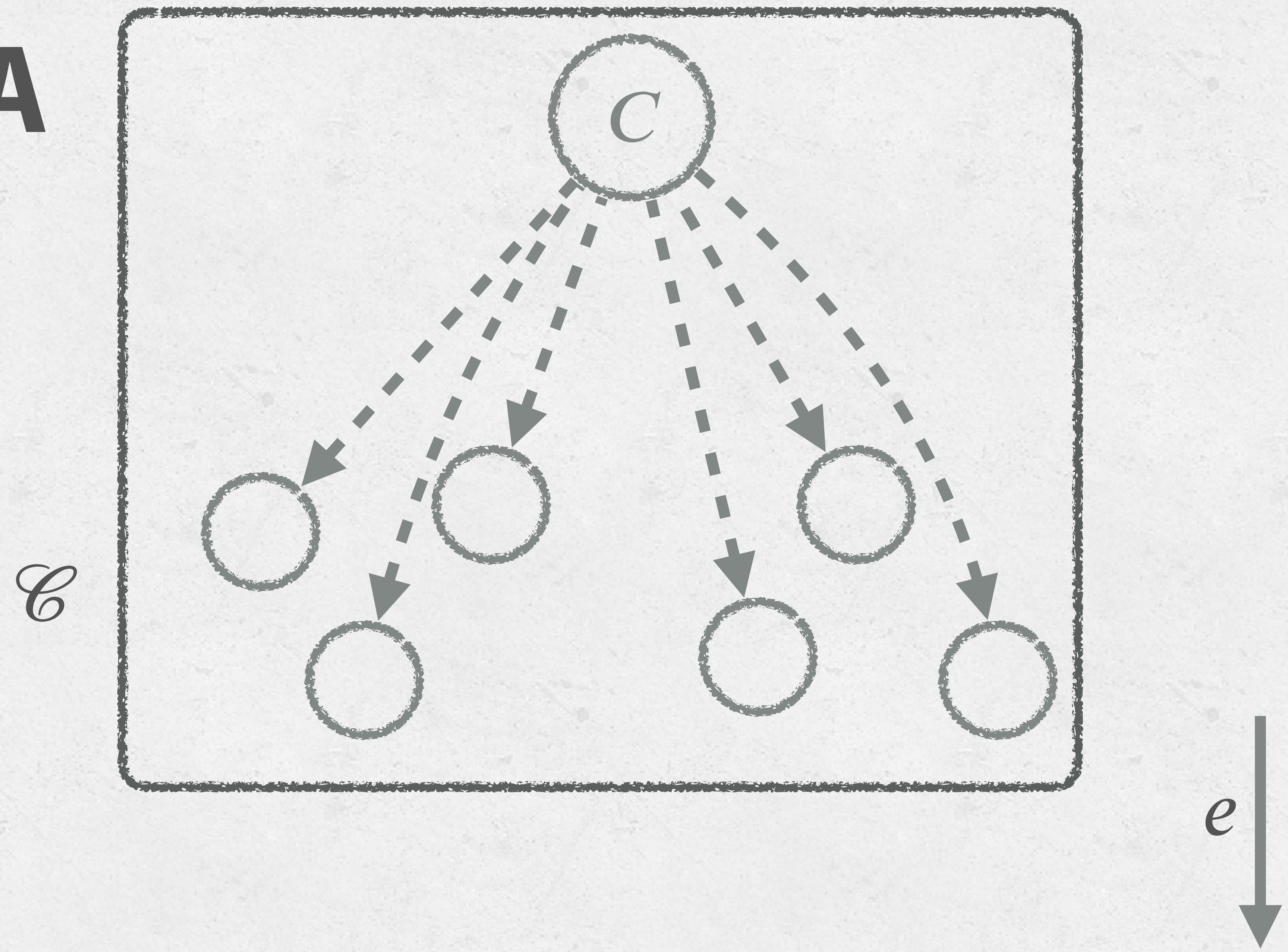

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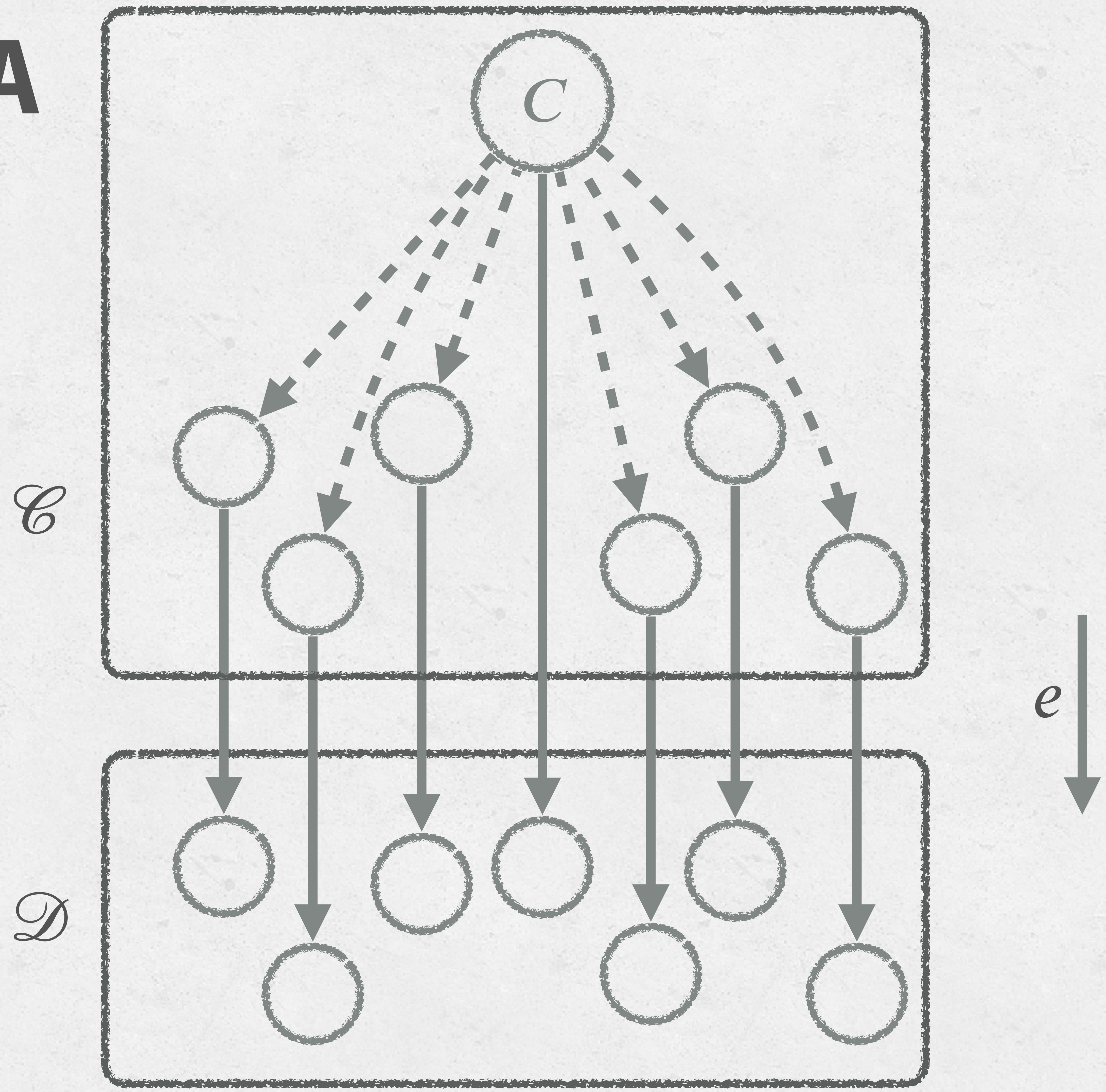

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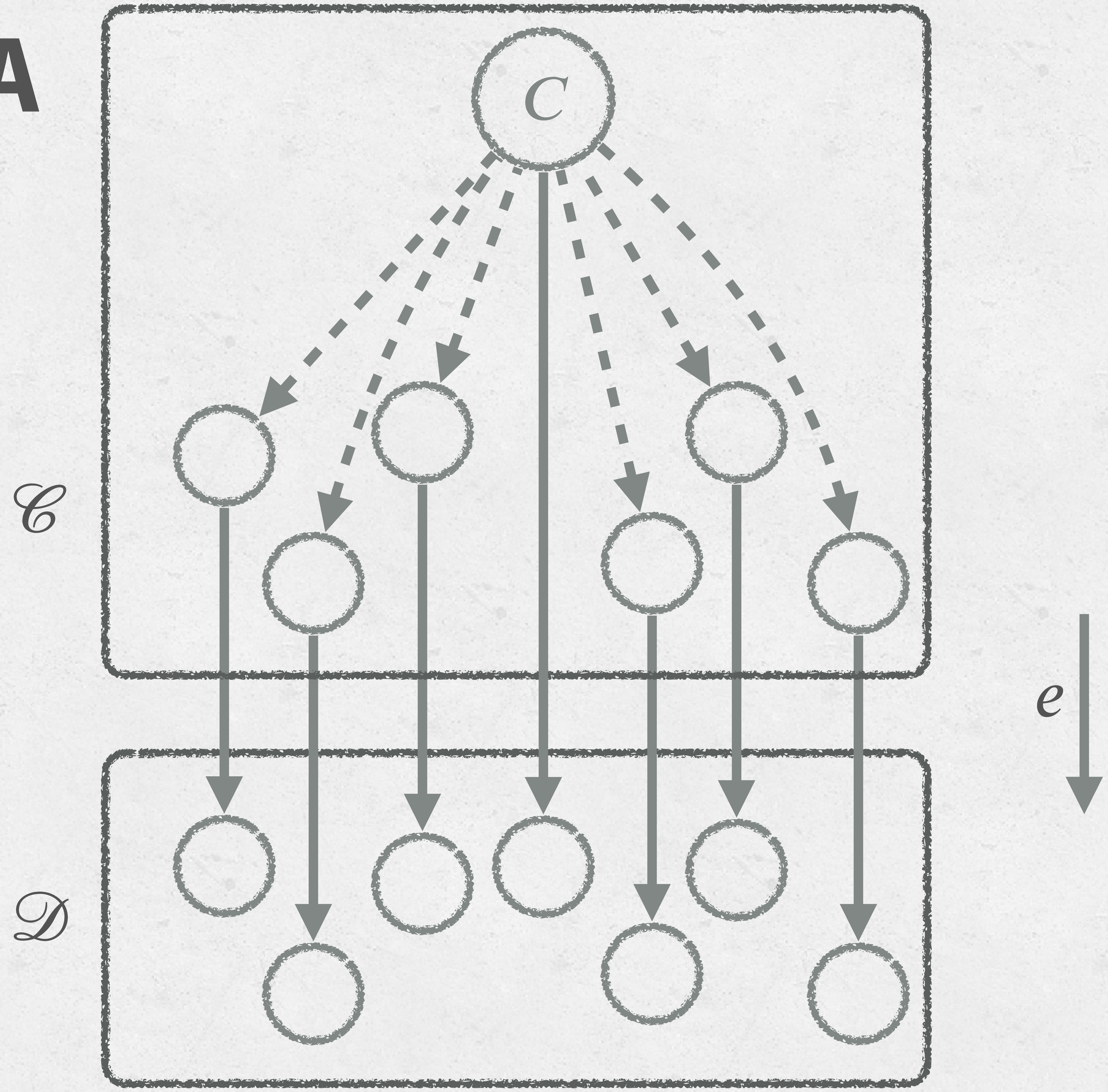

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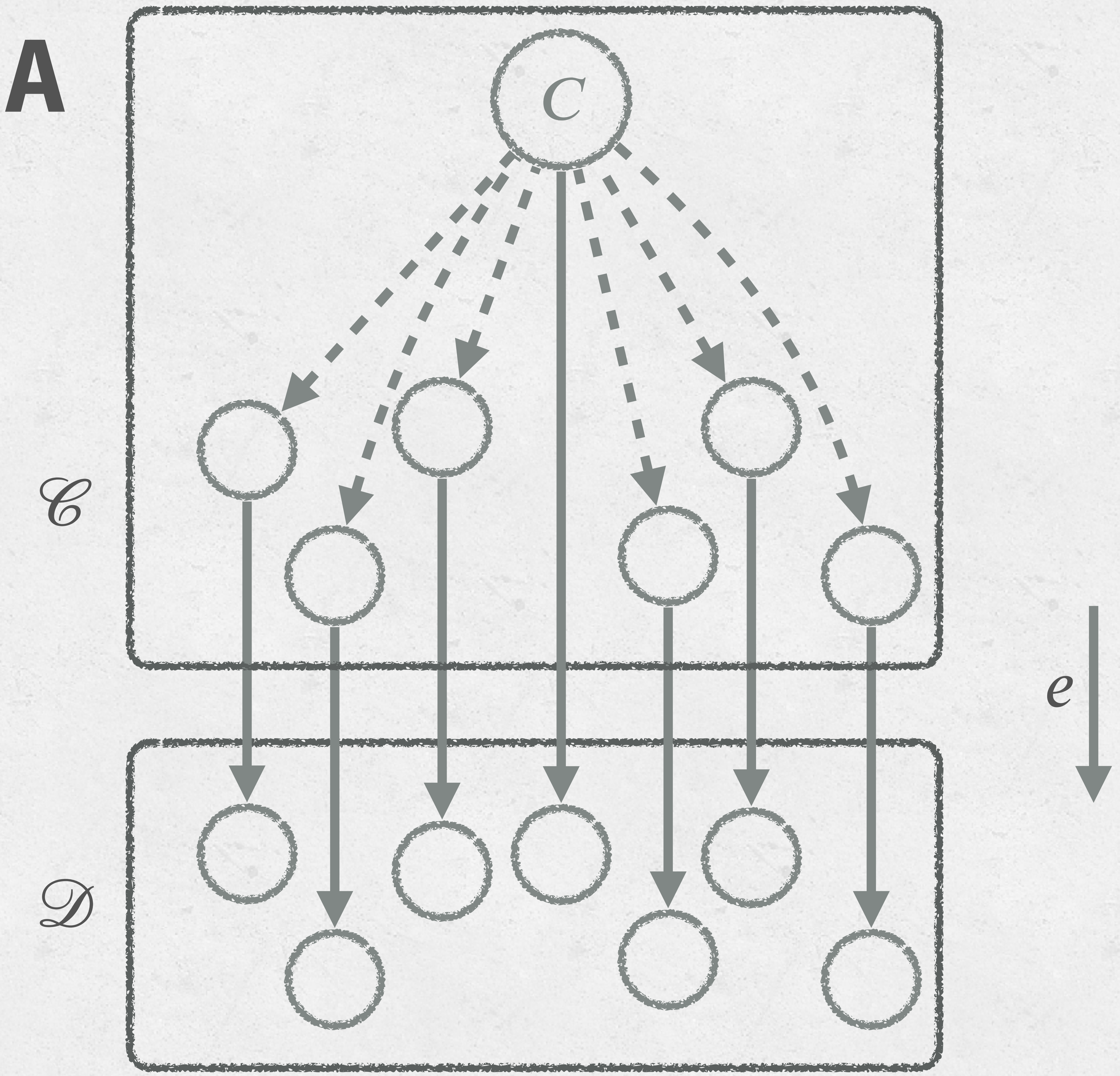

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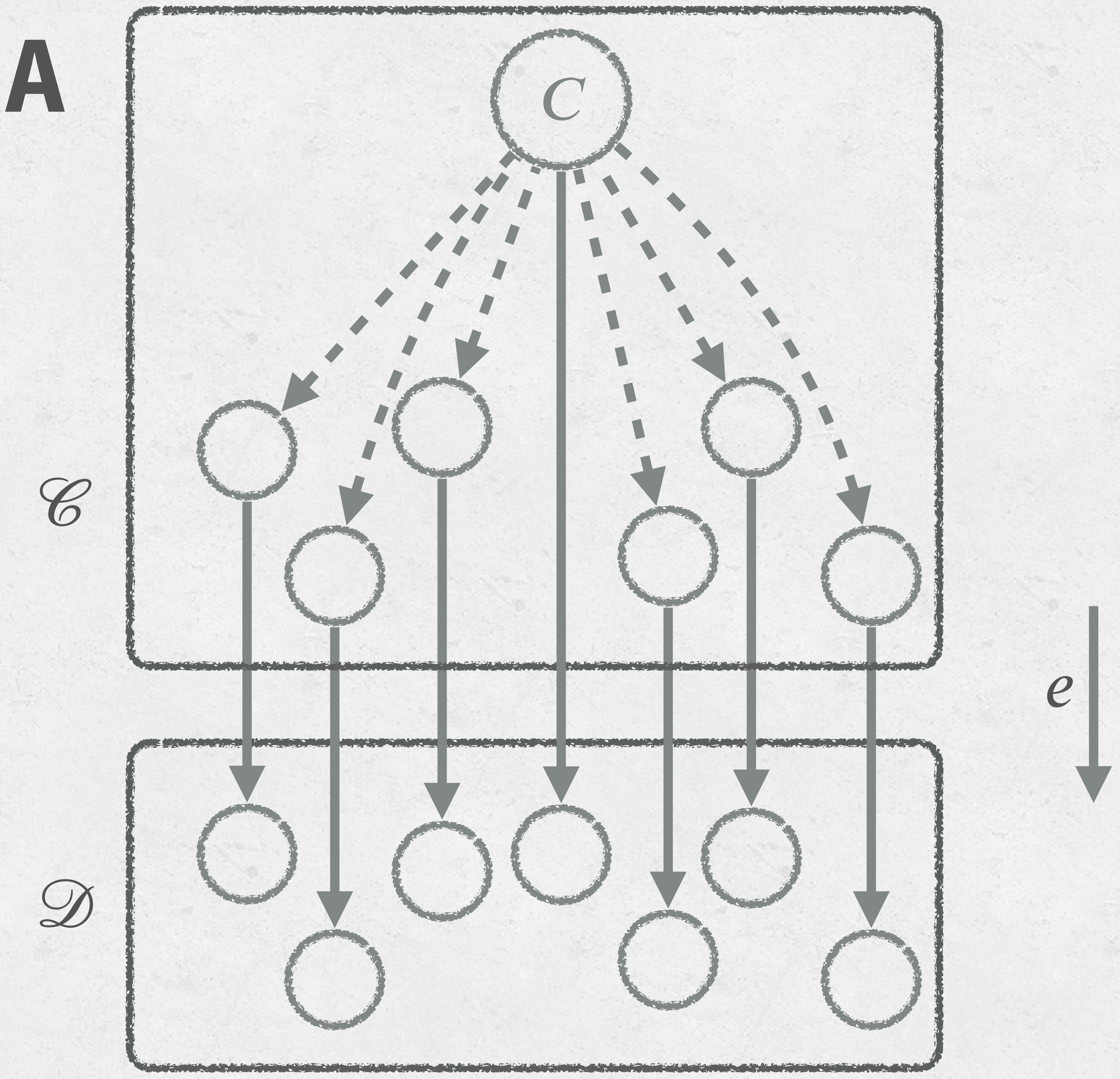

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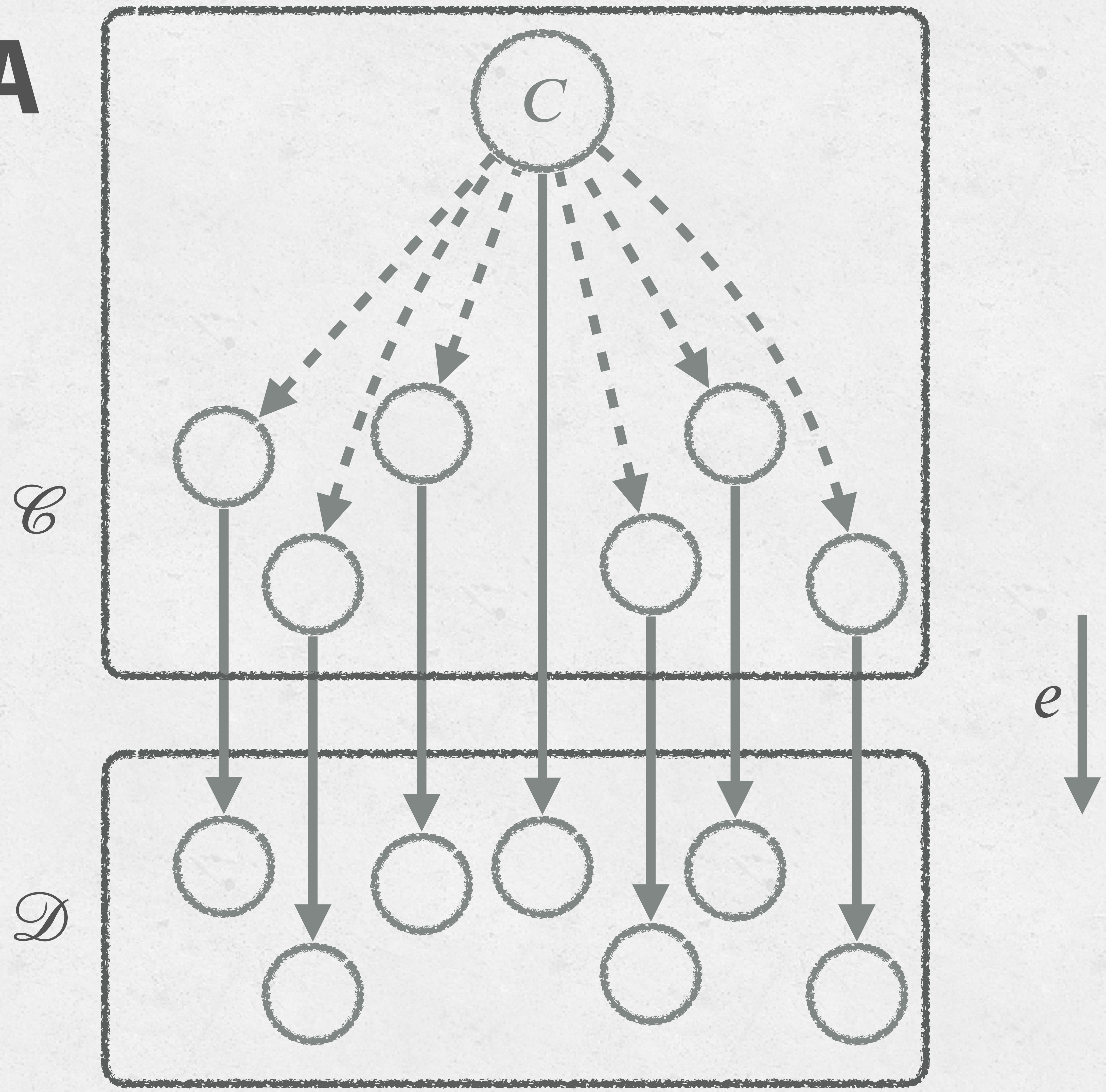

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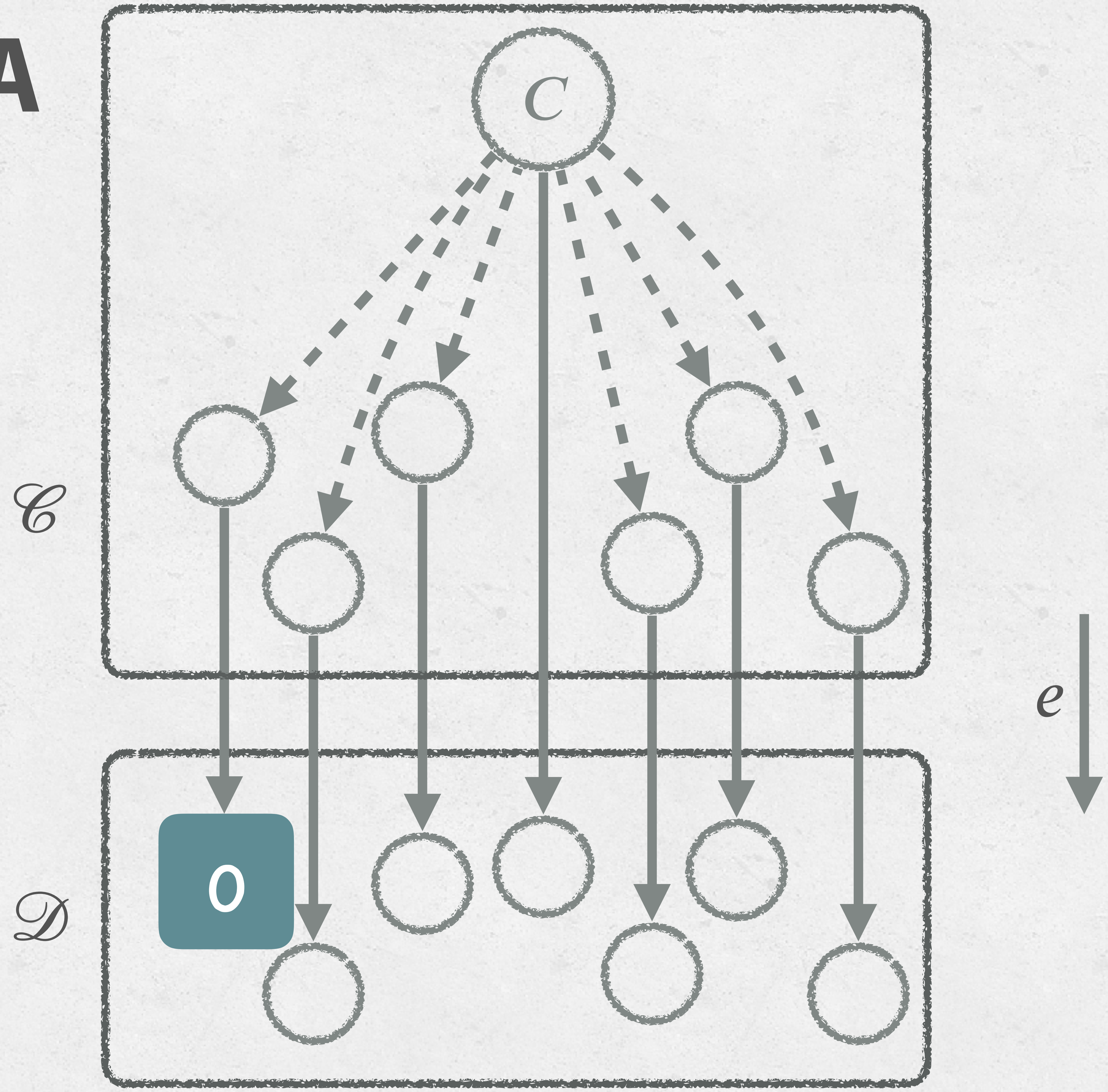

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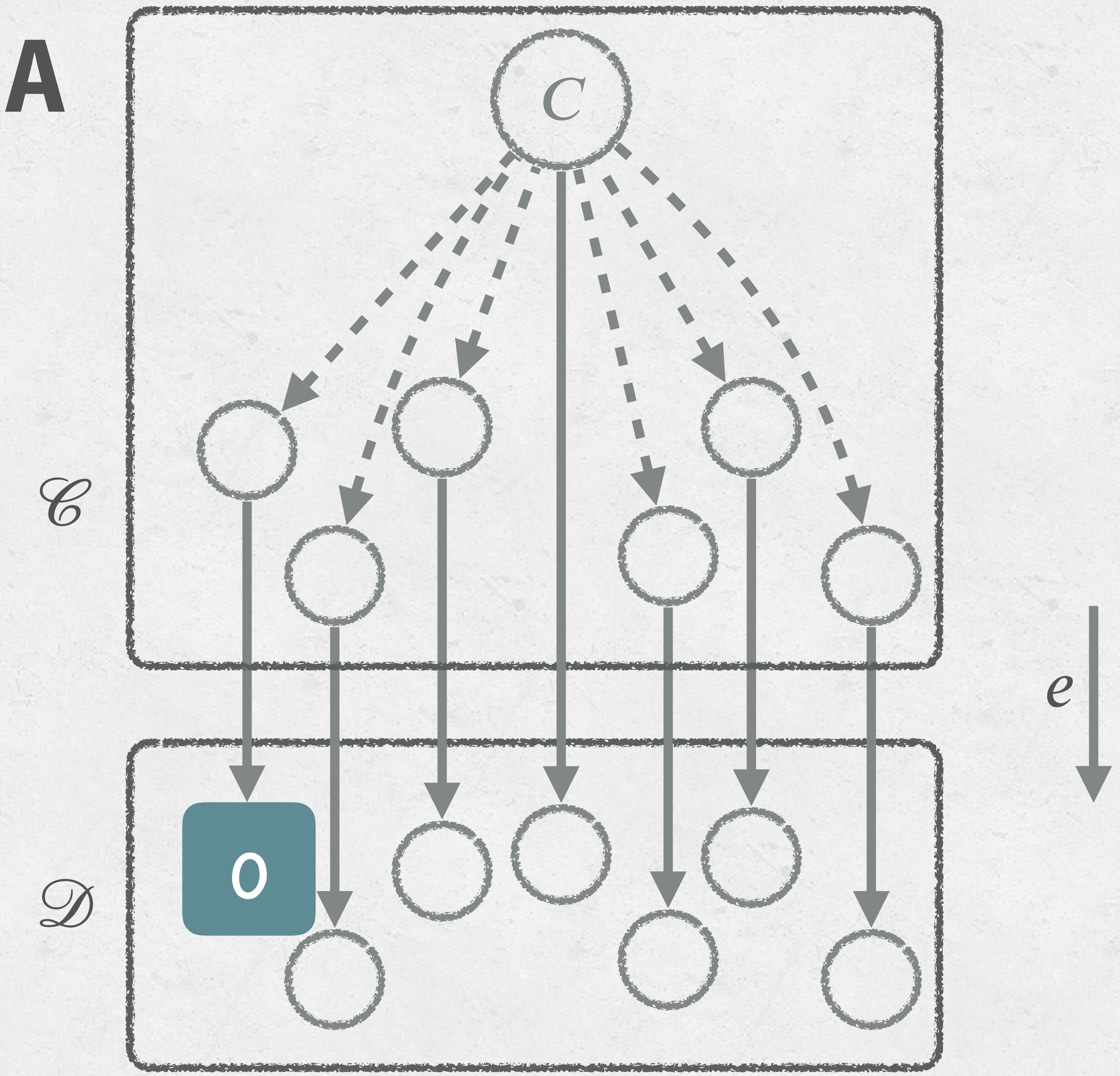

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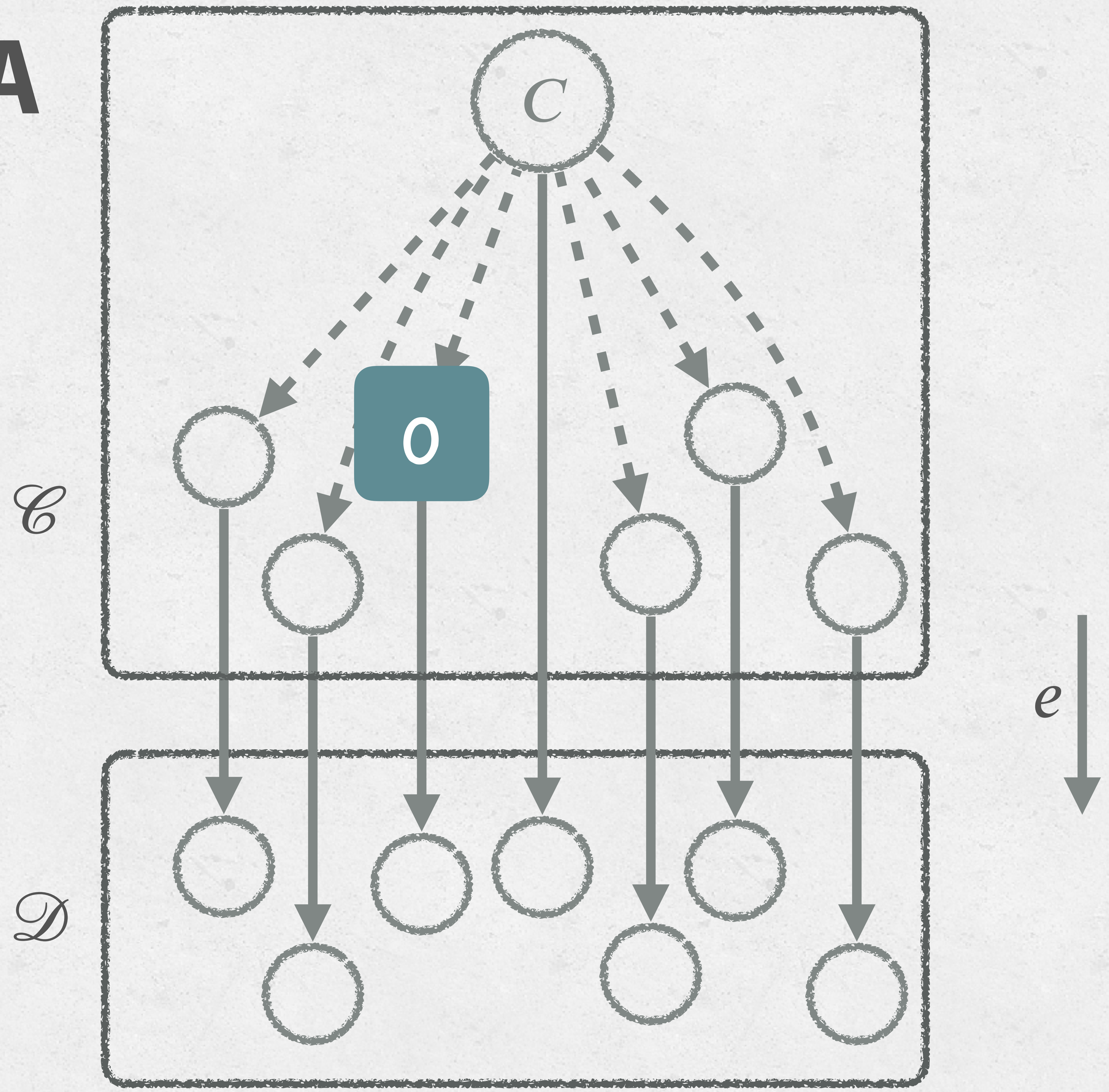

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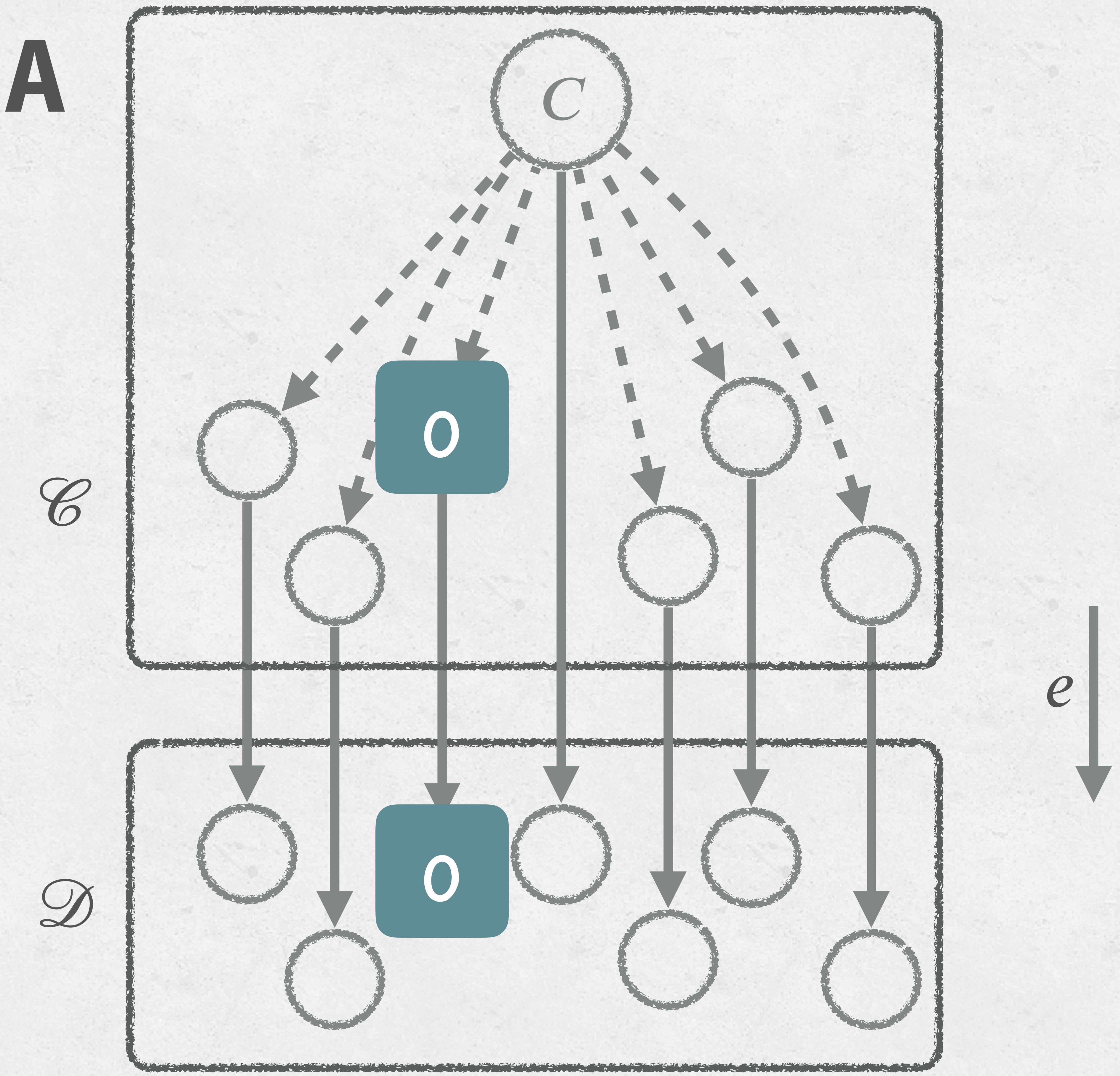

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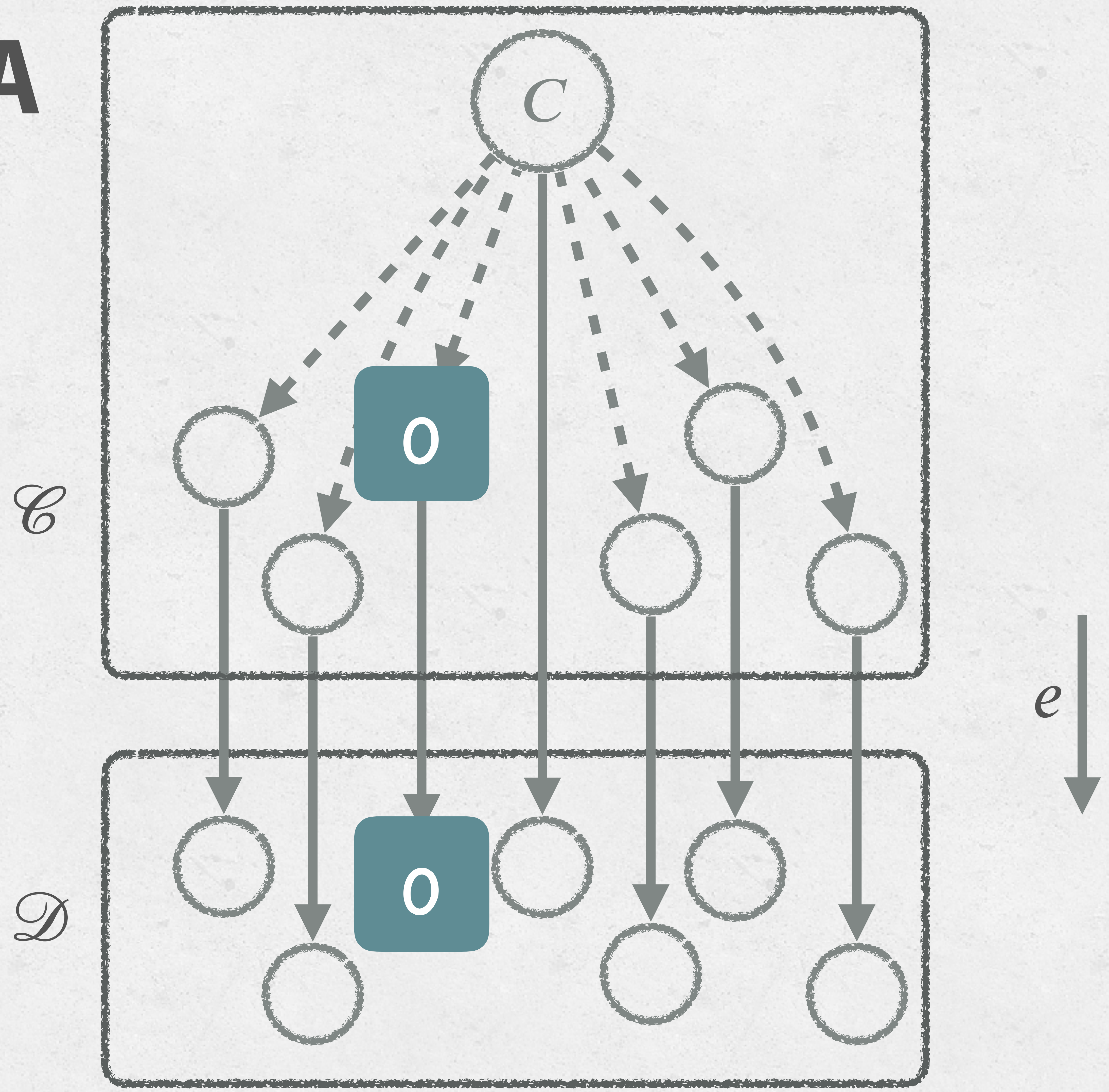

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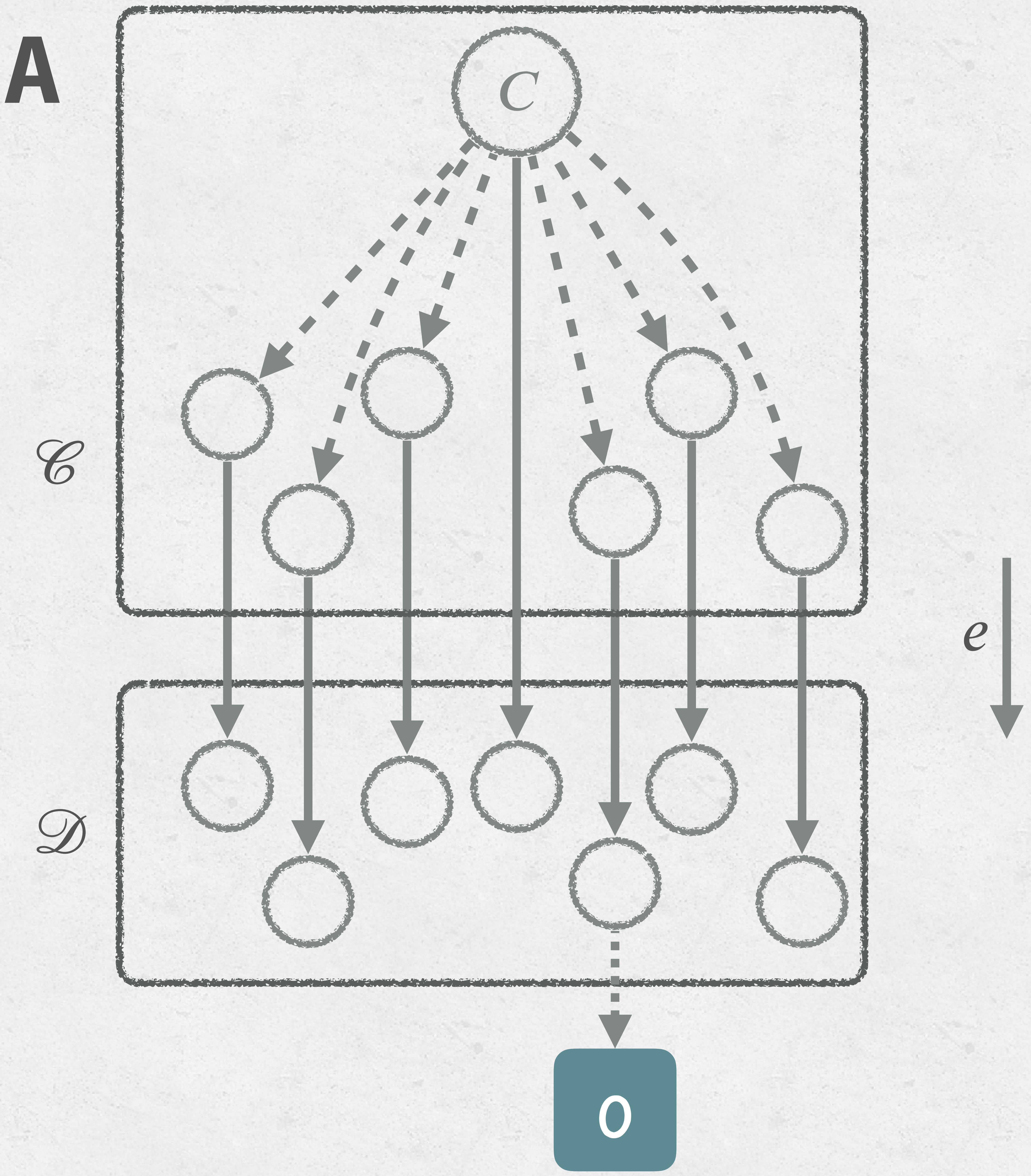

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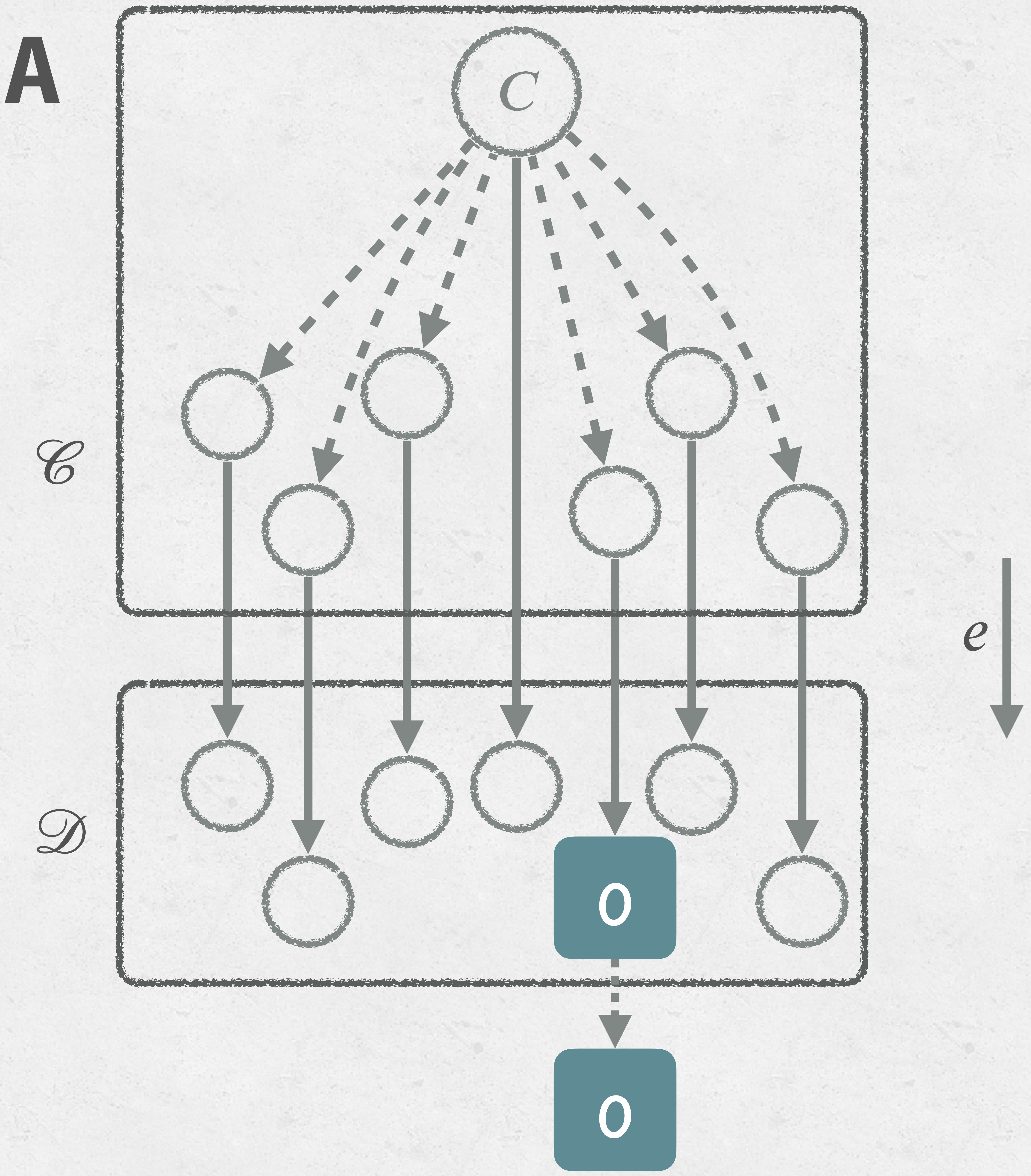

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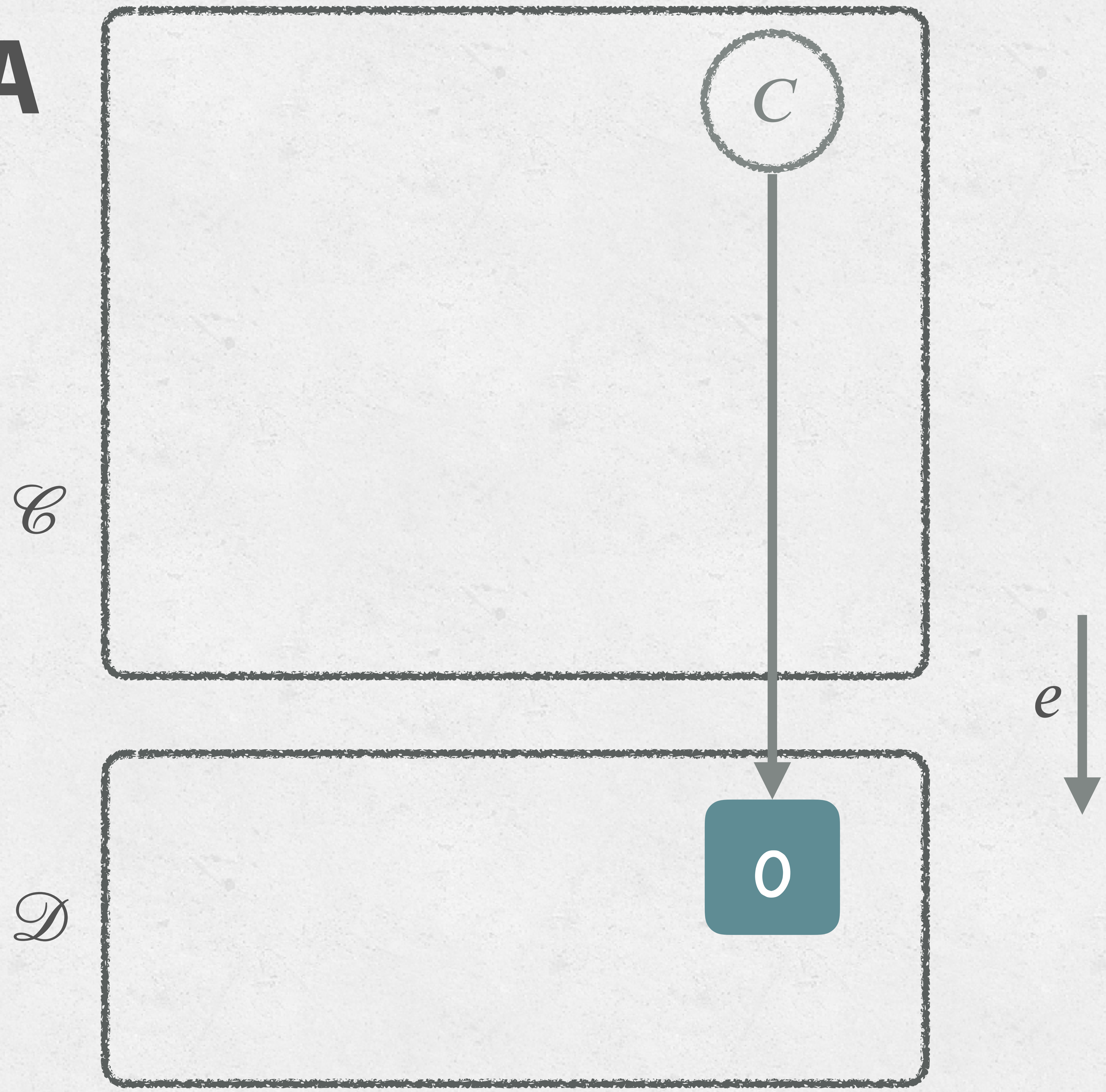

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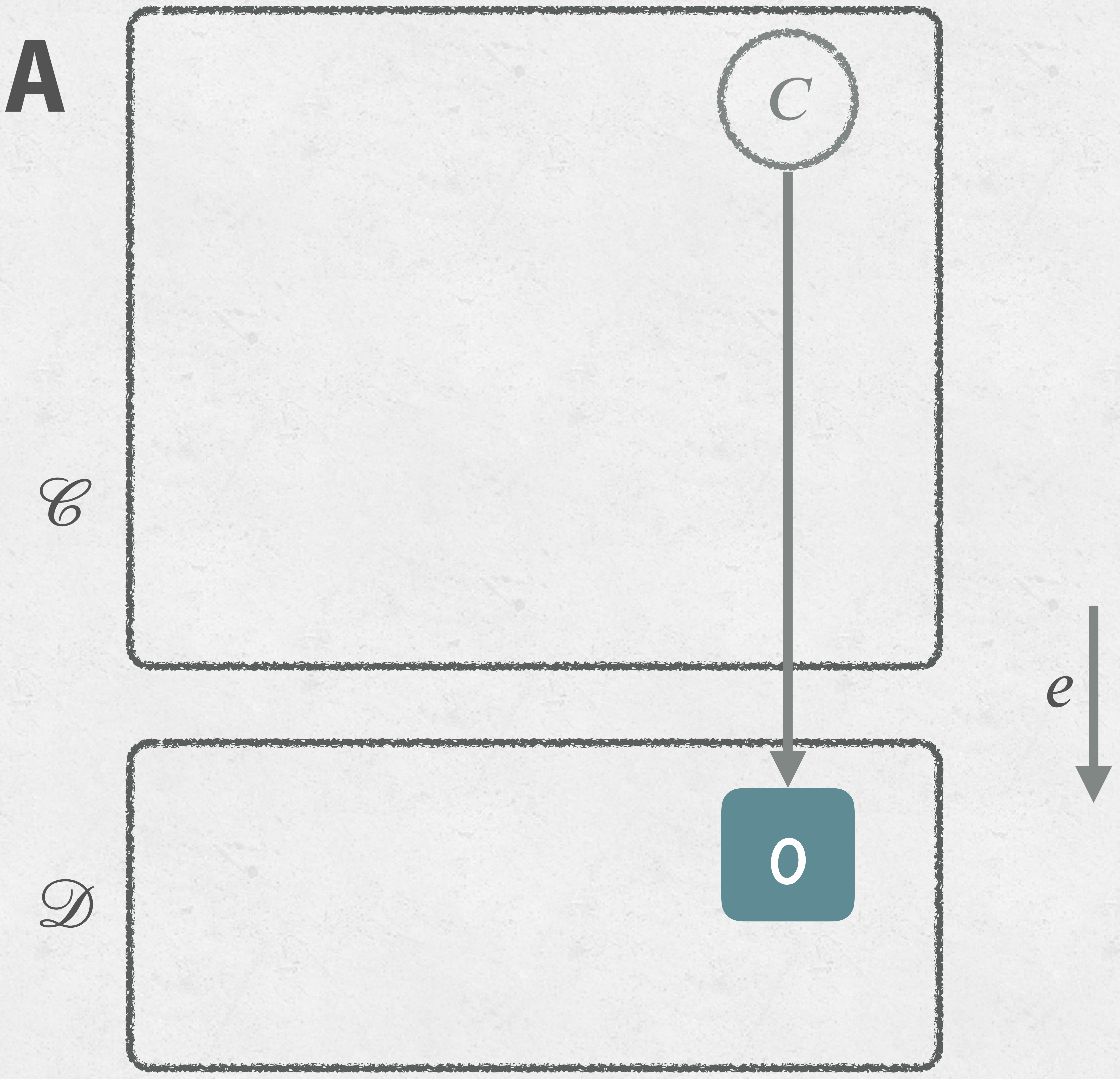

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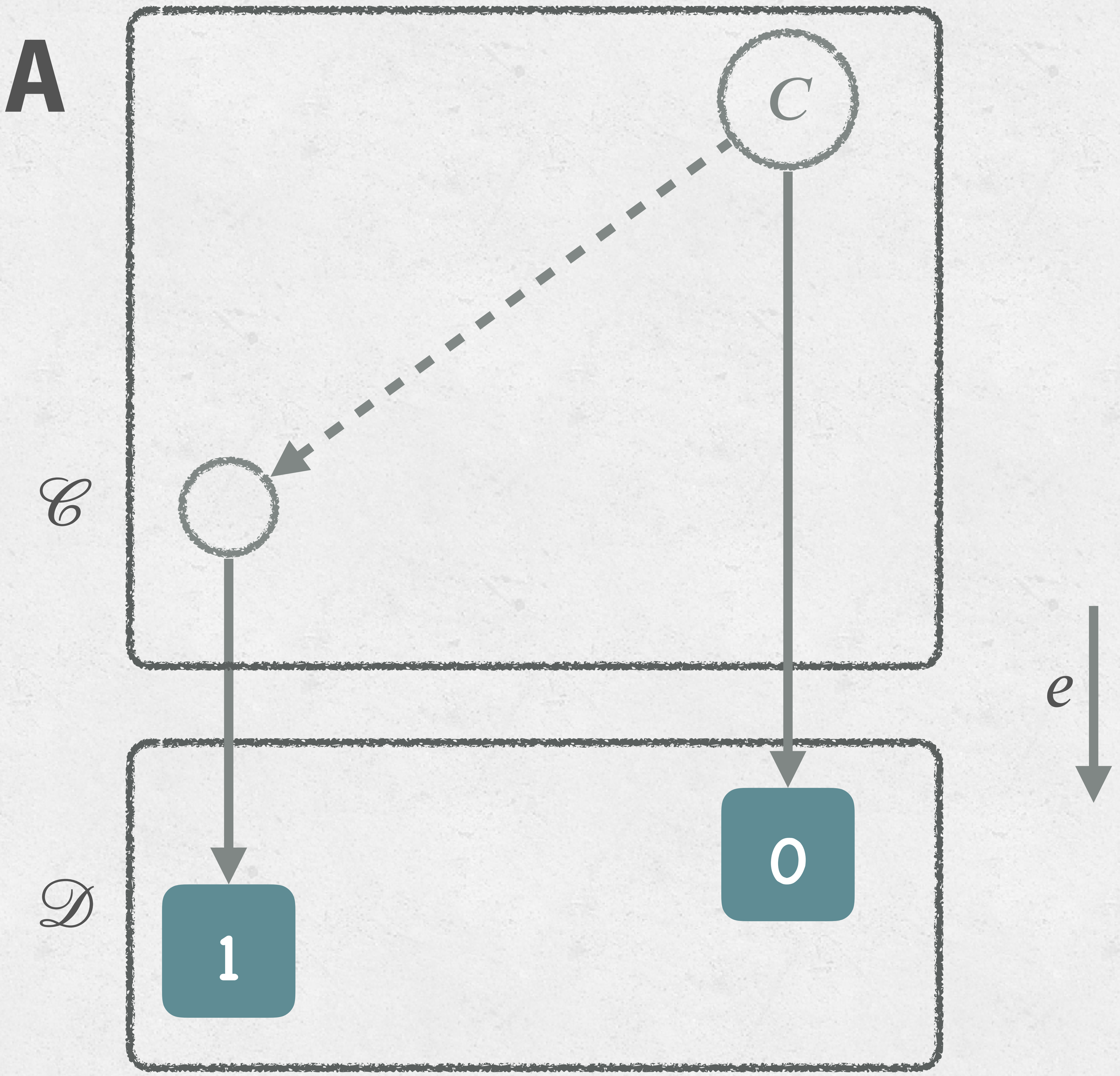

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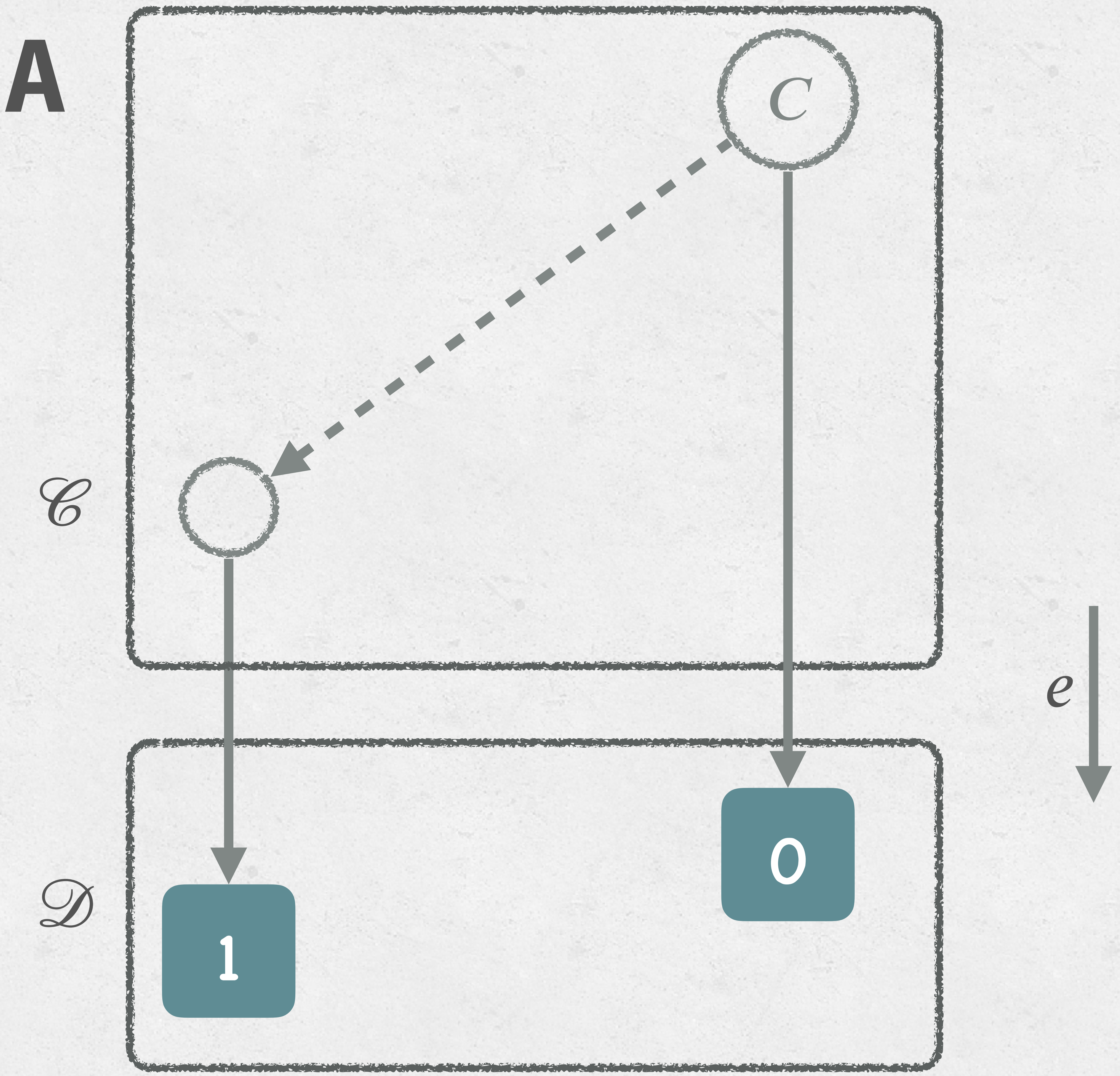

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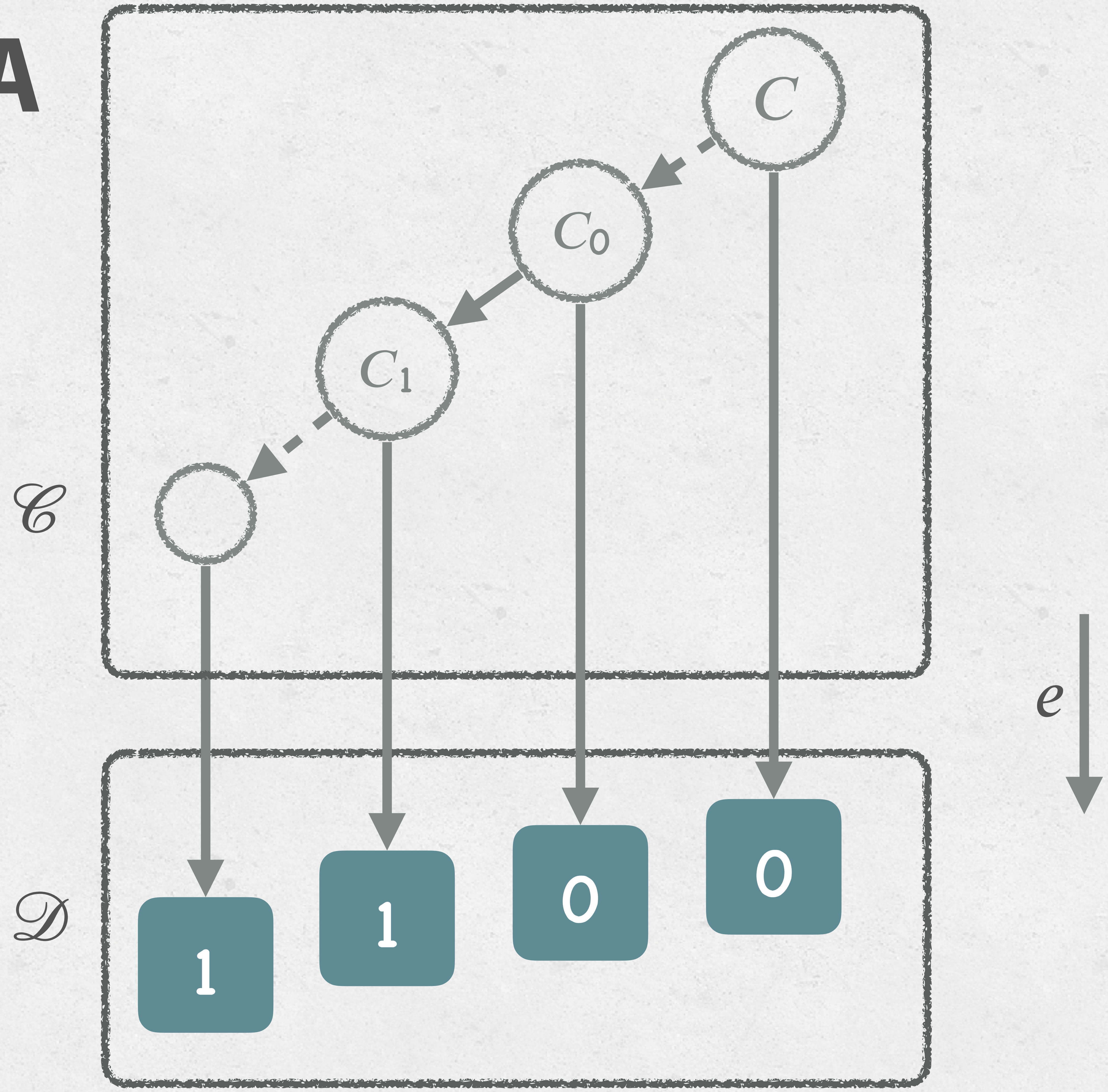

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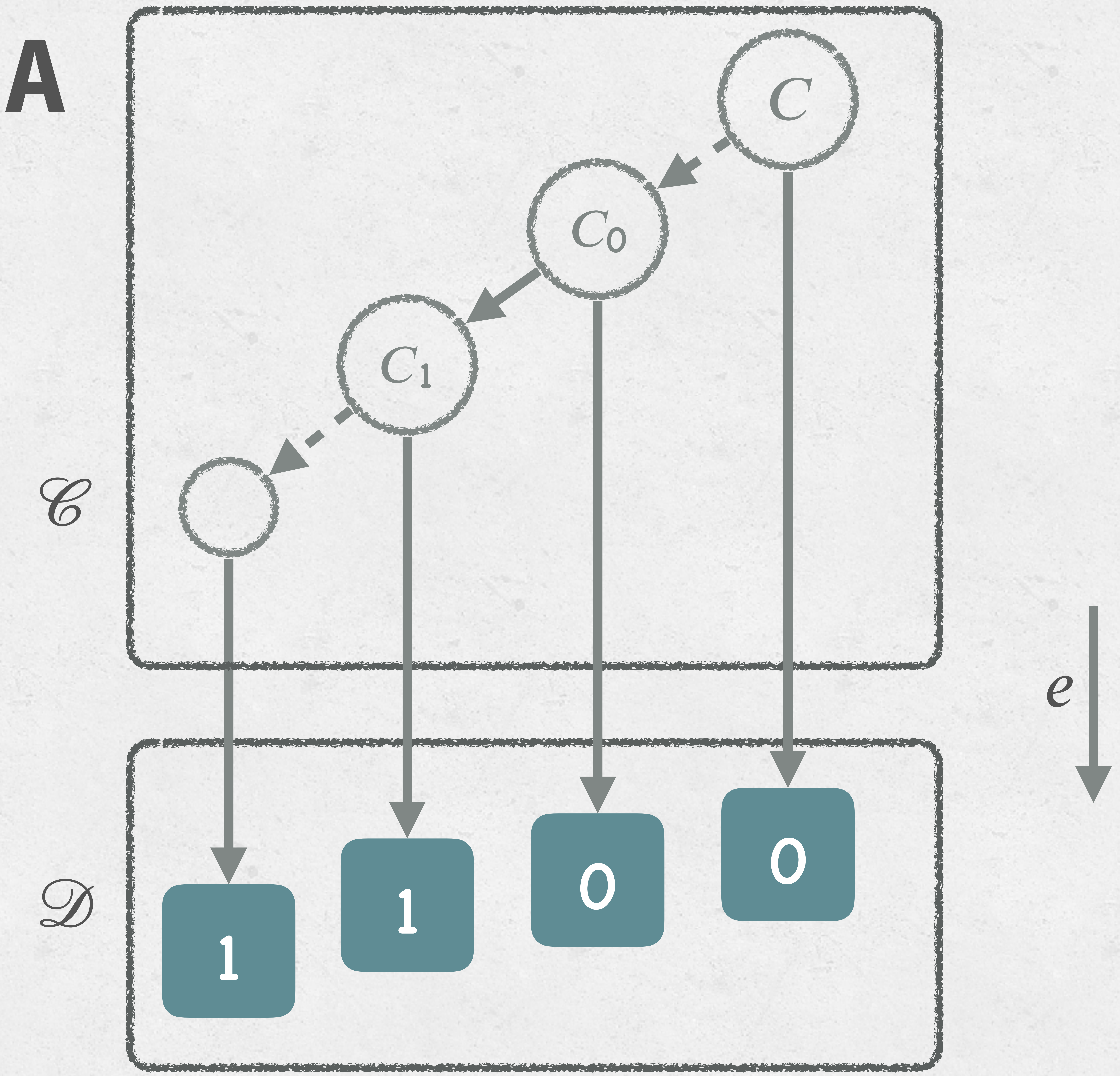

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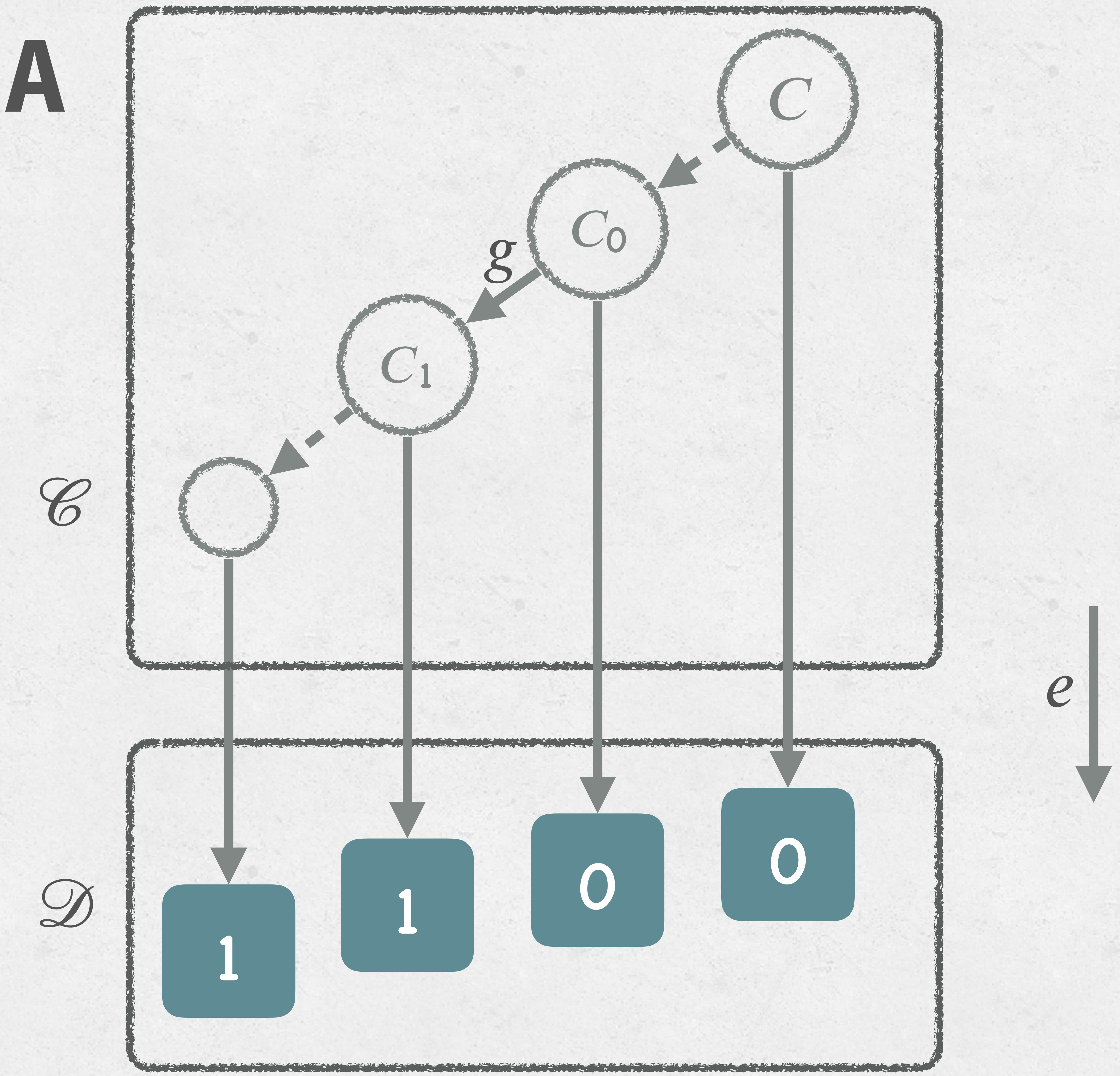

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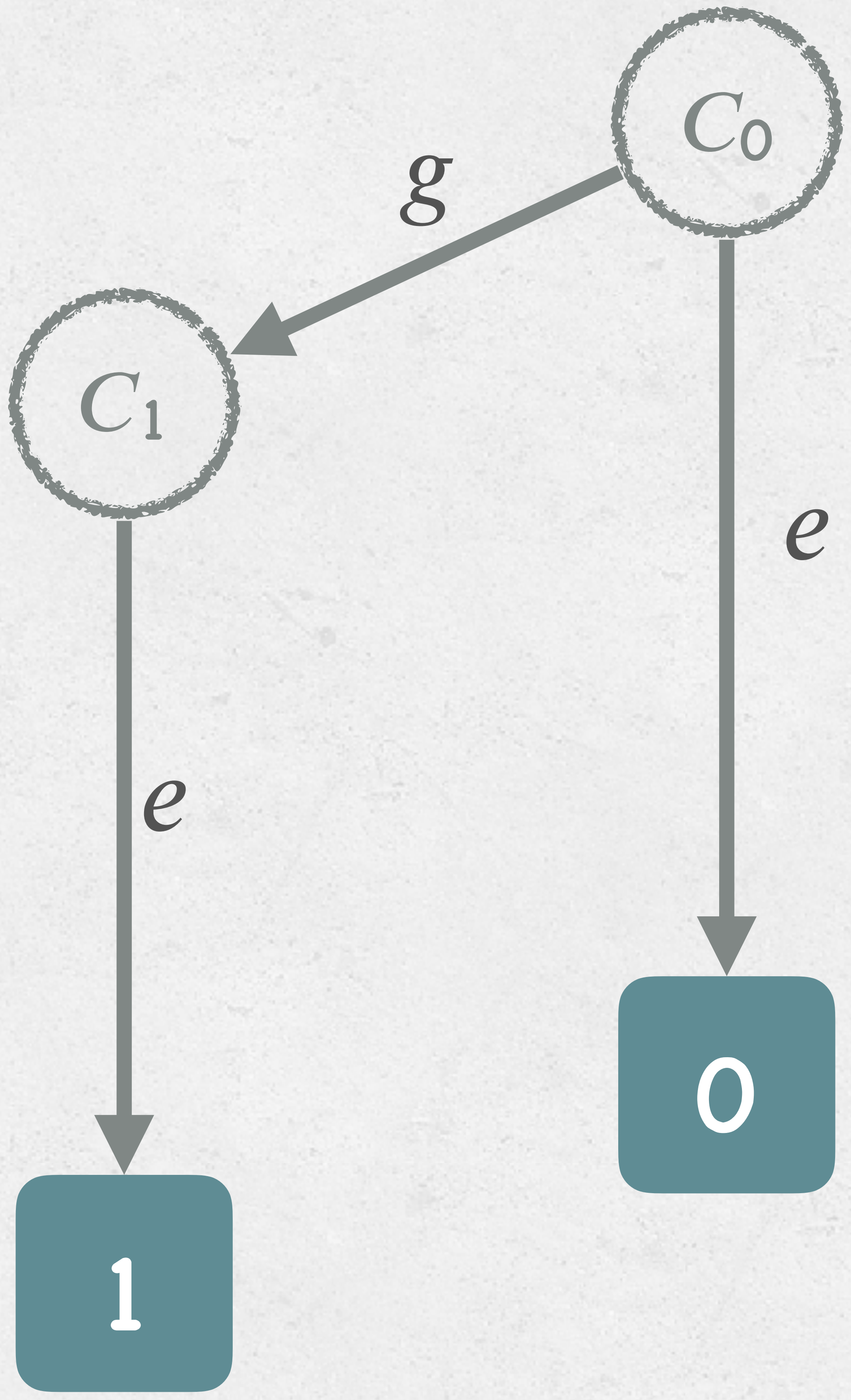

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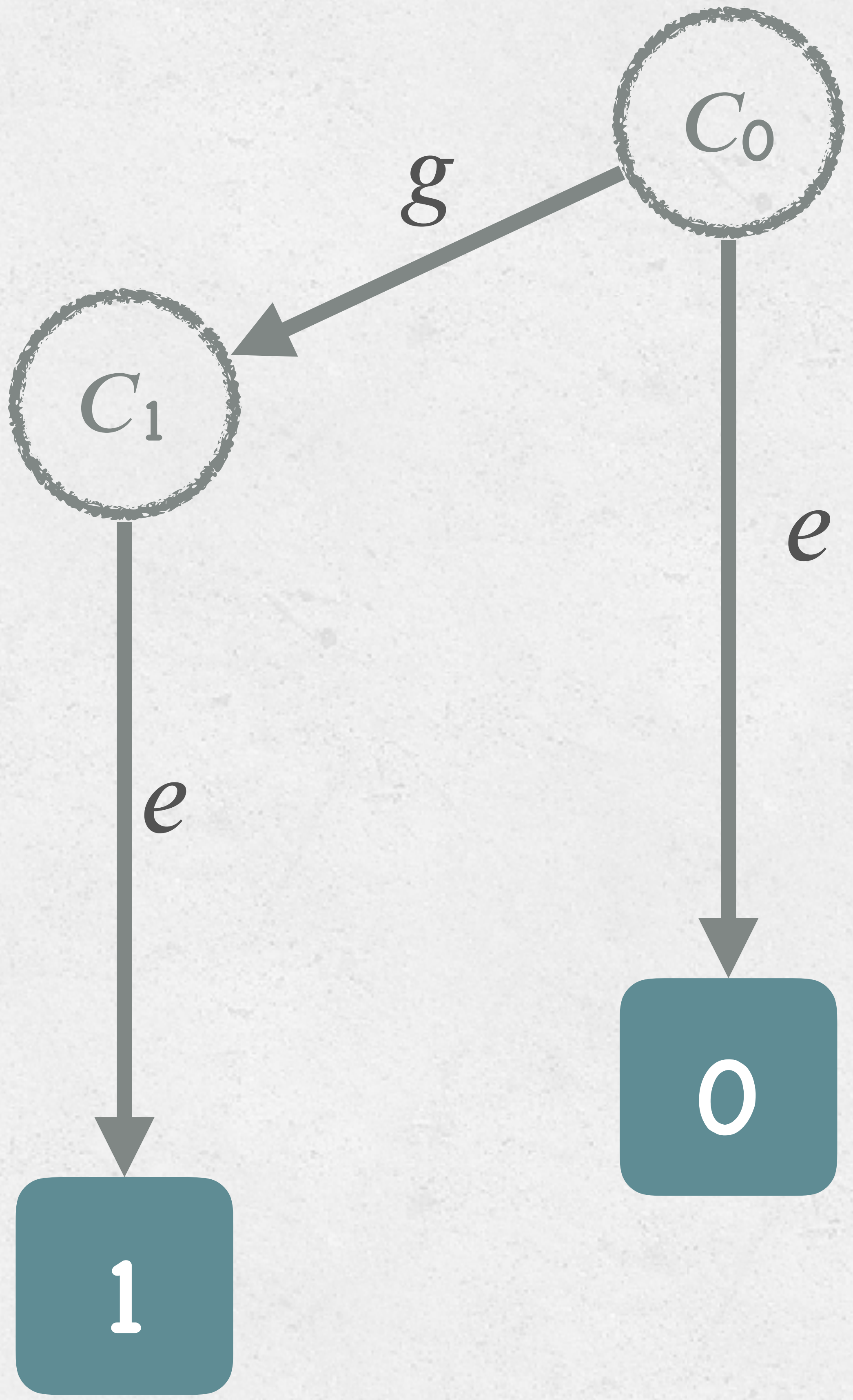

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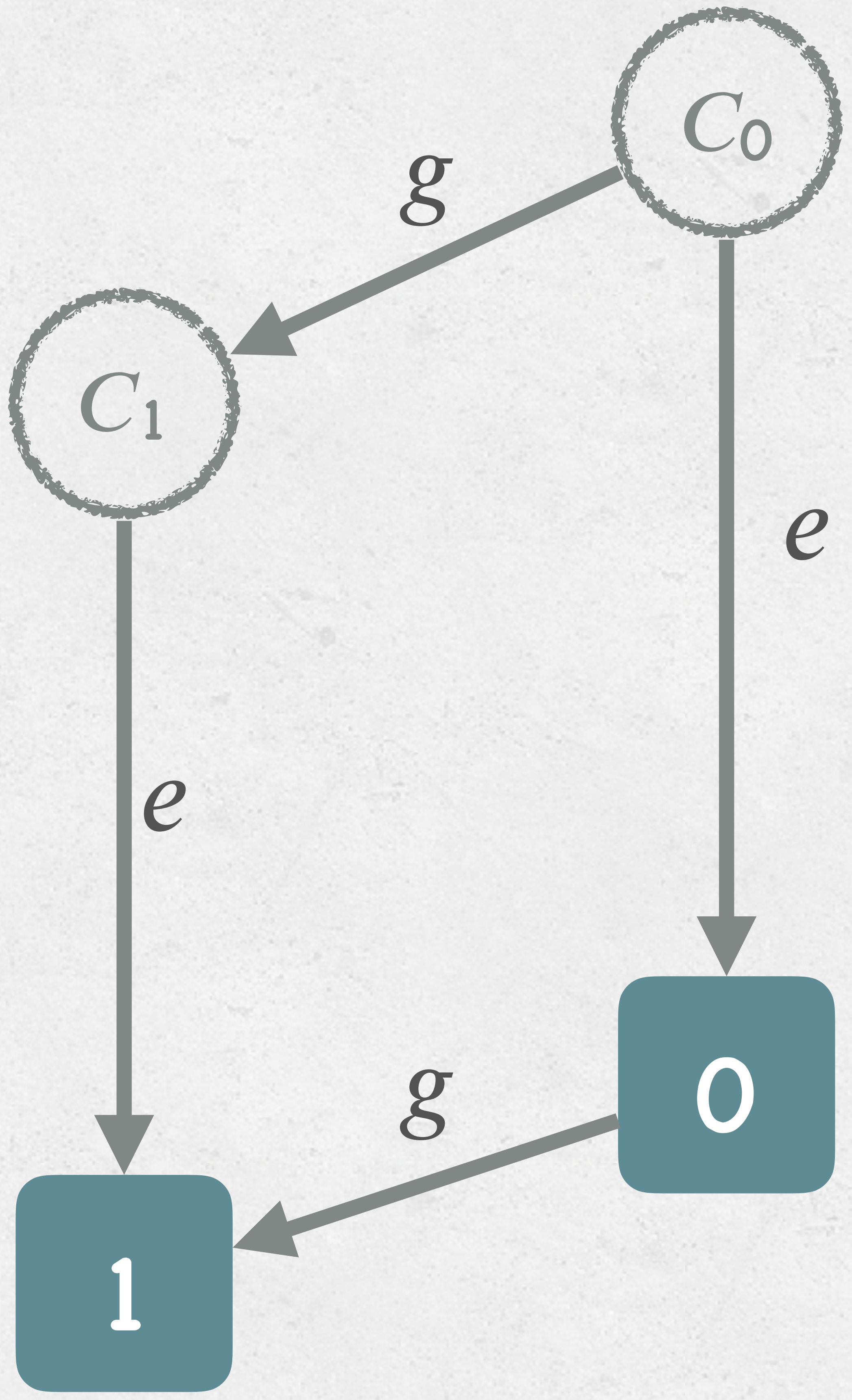

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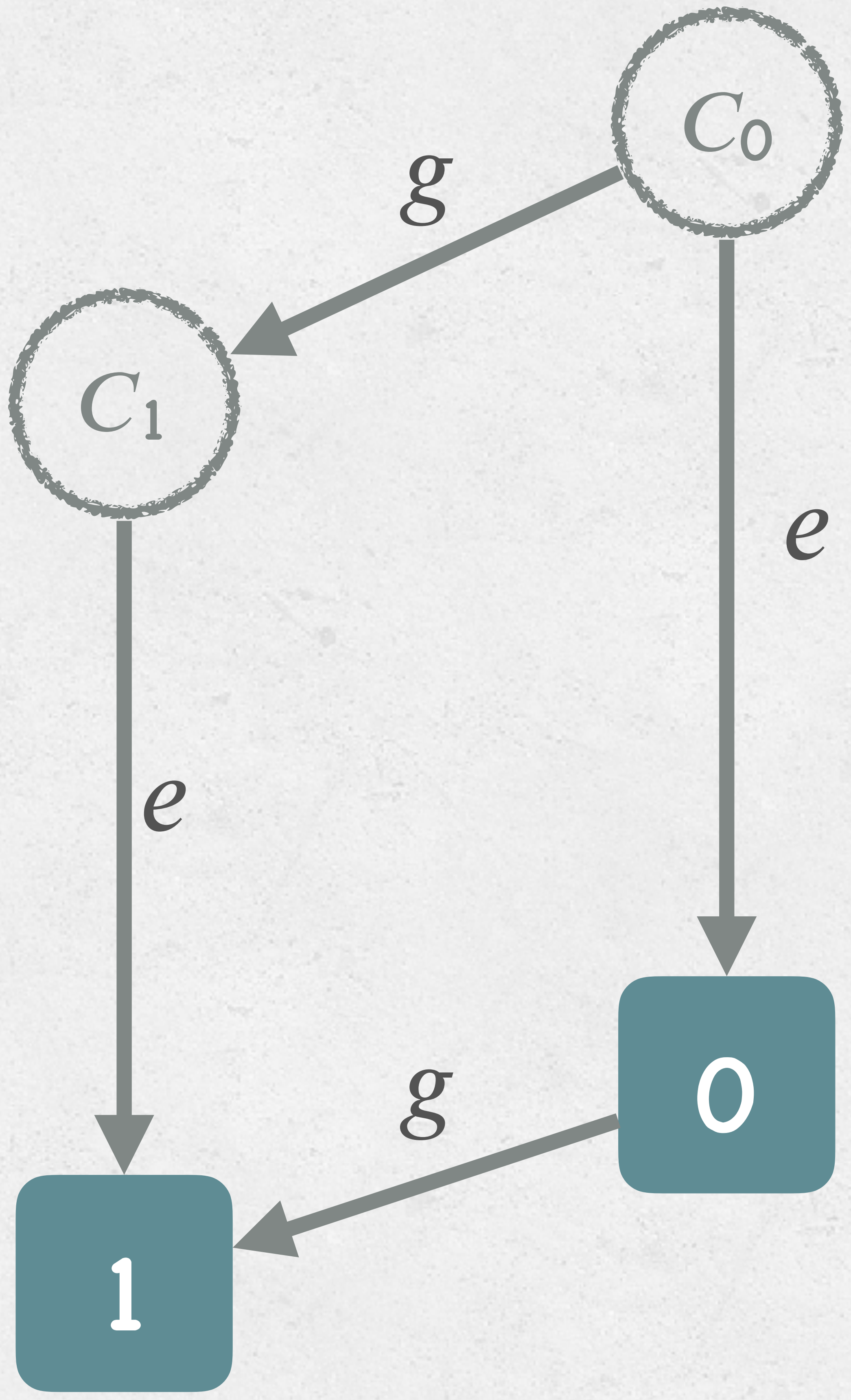

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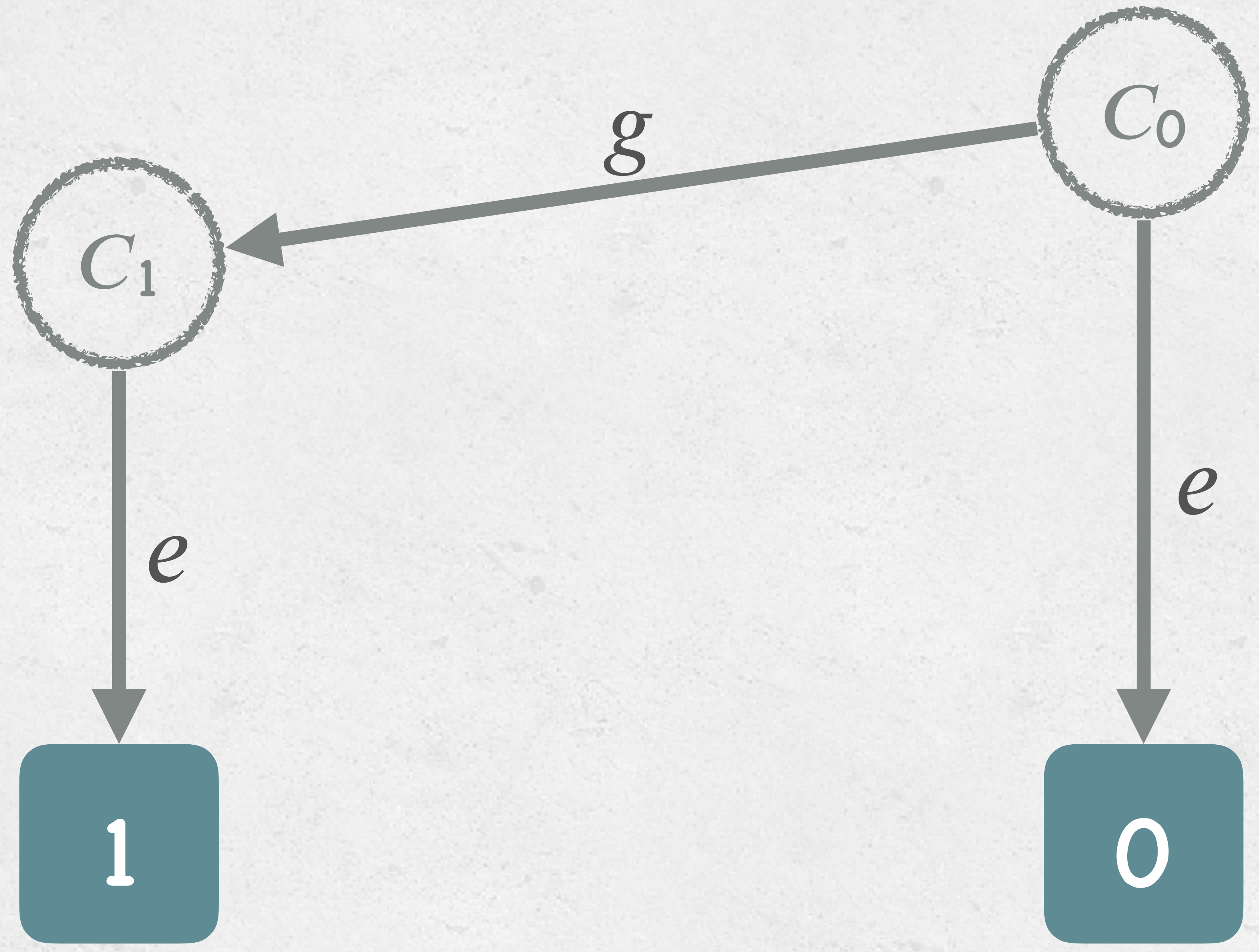
Since the protocol is correct and tolerates one failure, it must be able to reach a decided configuration, A, without p taking steps.



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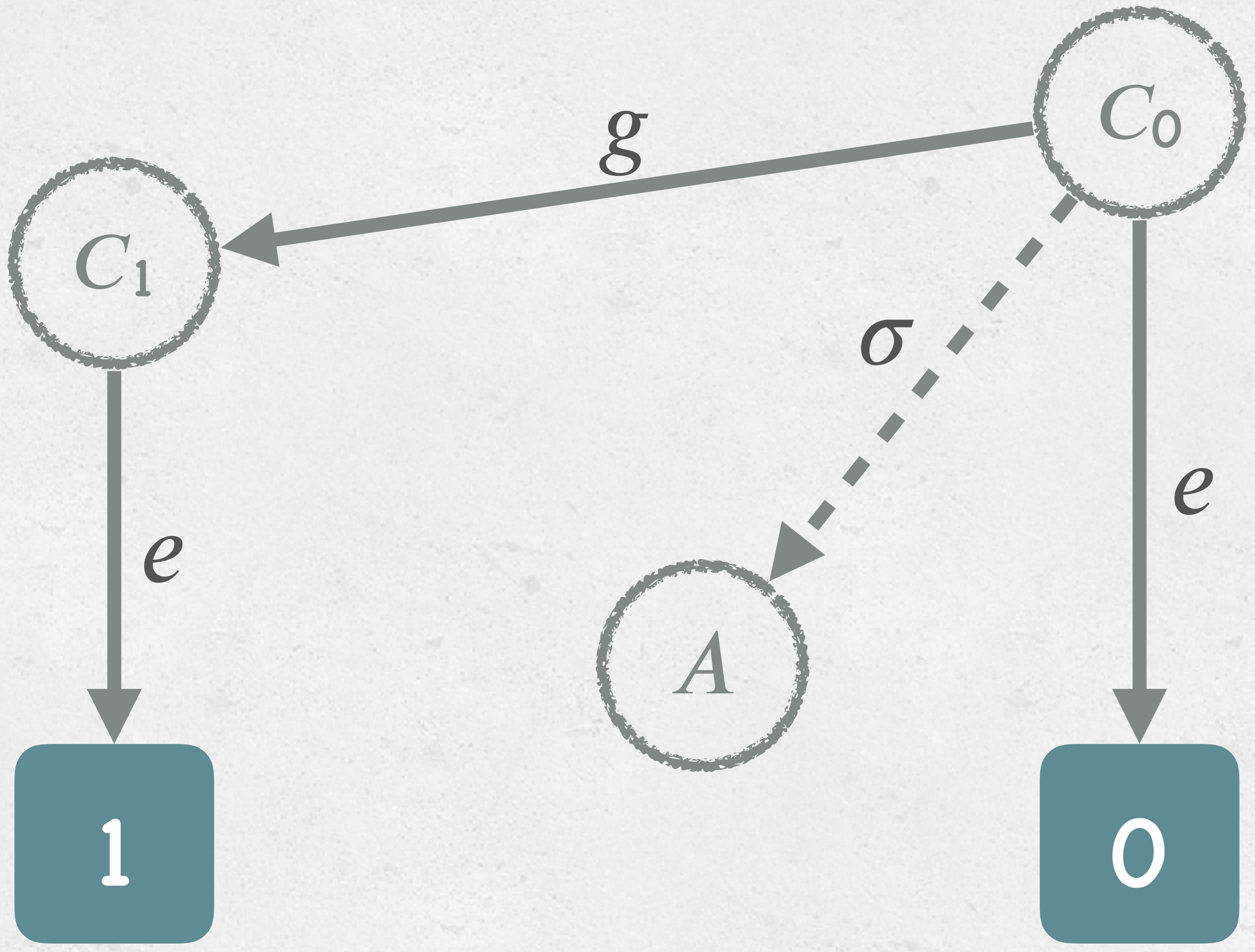

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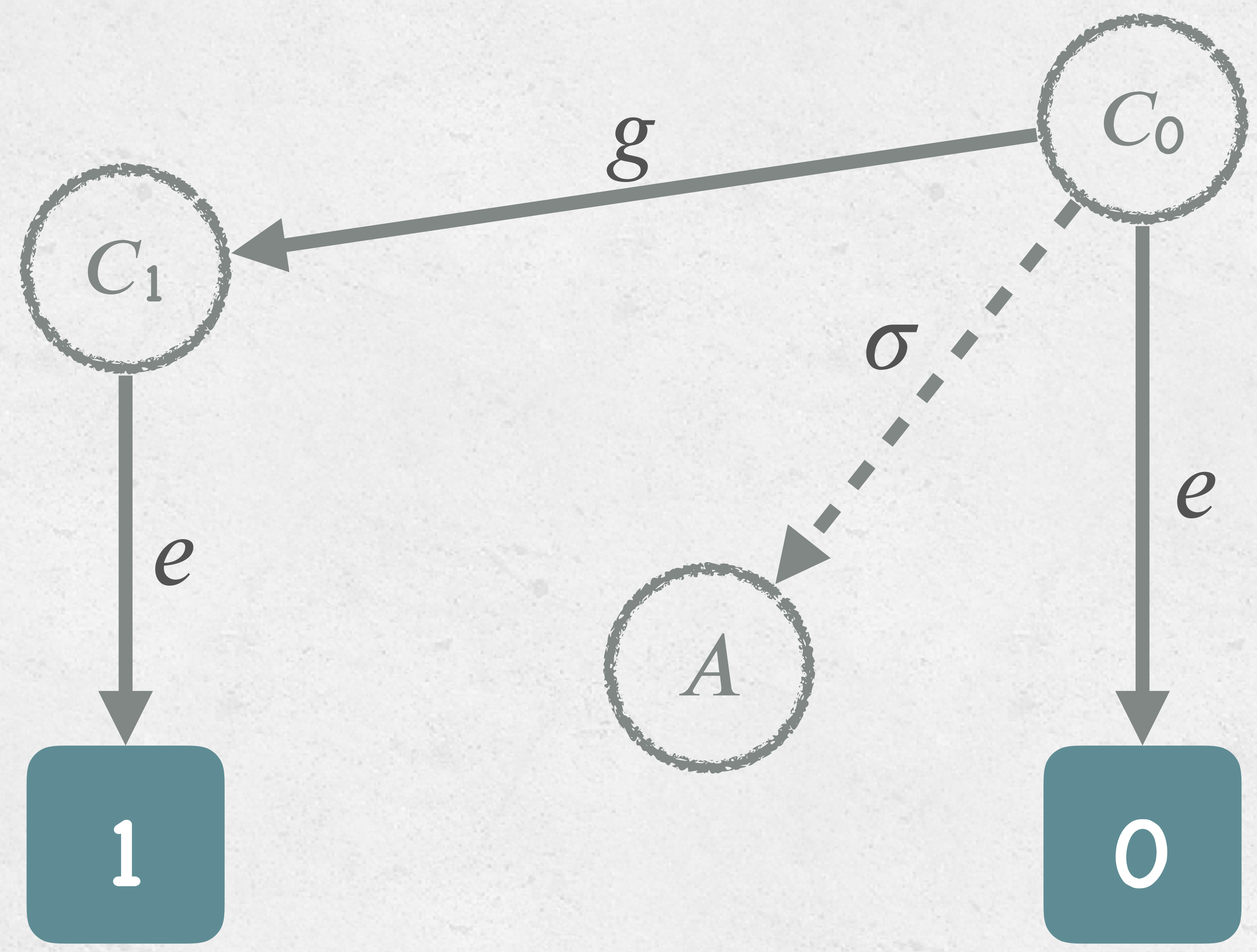

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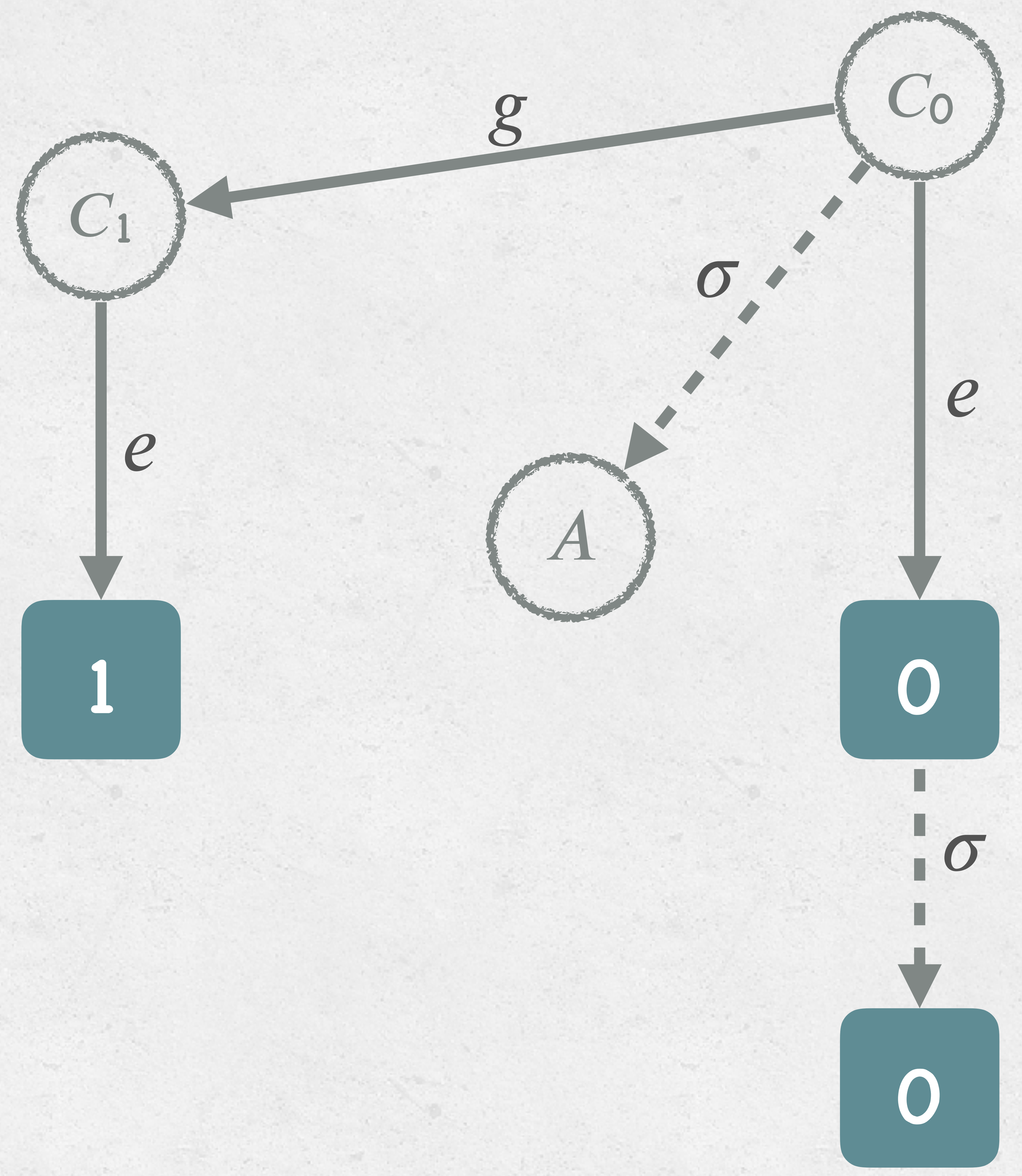

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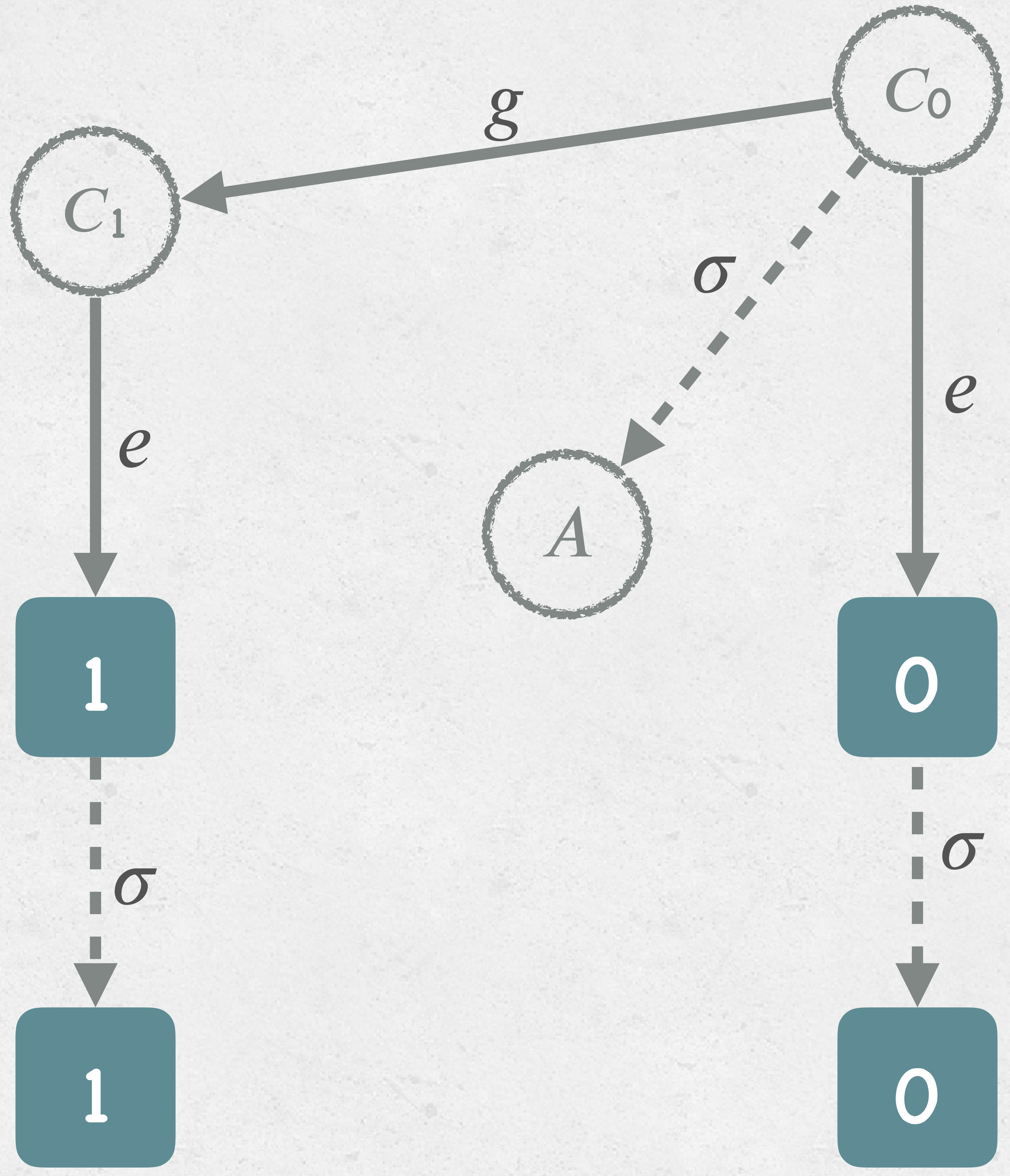

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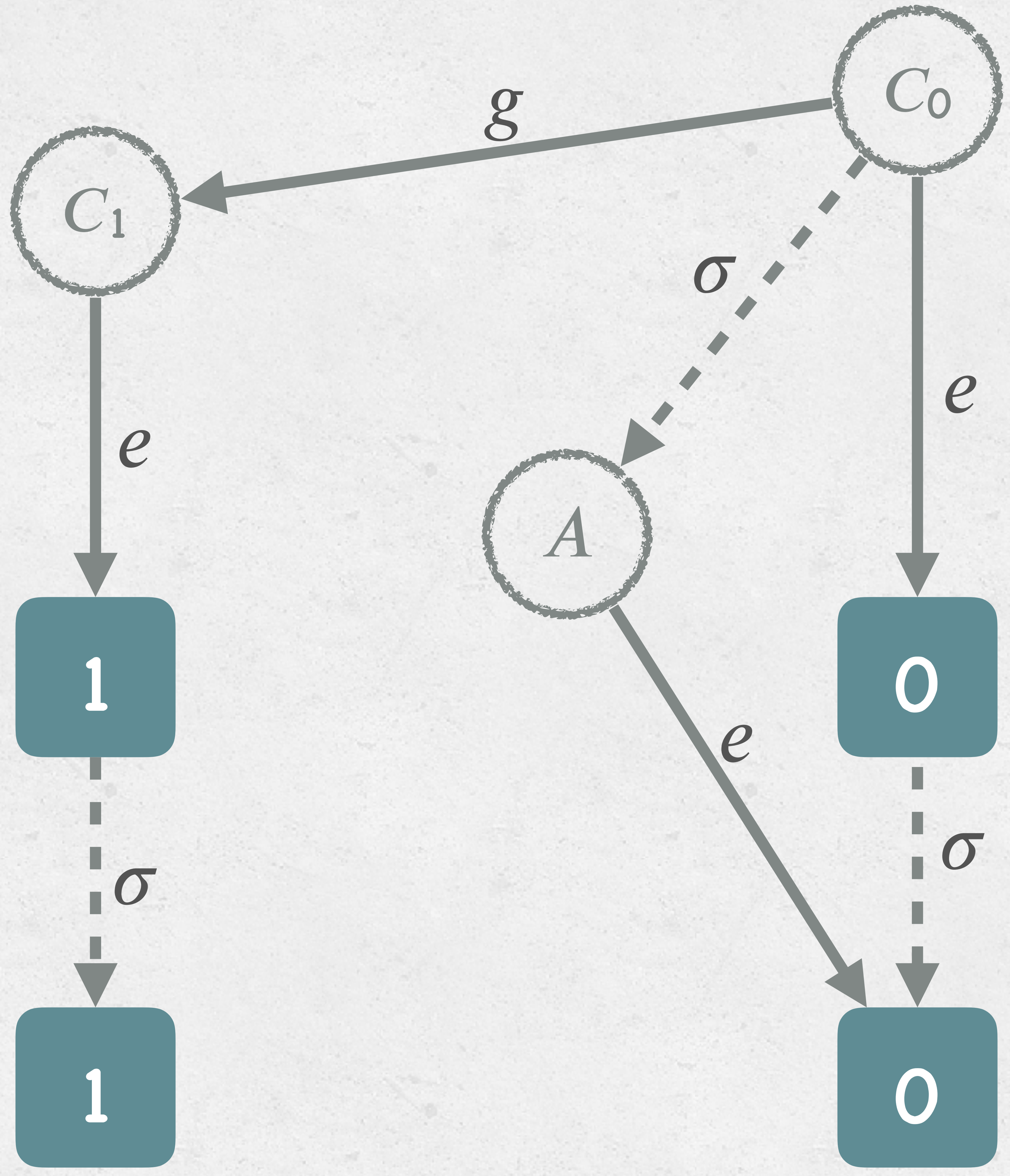

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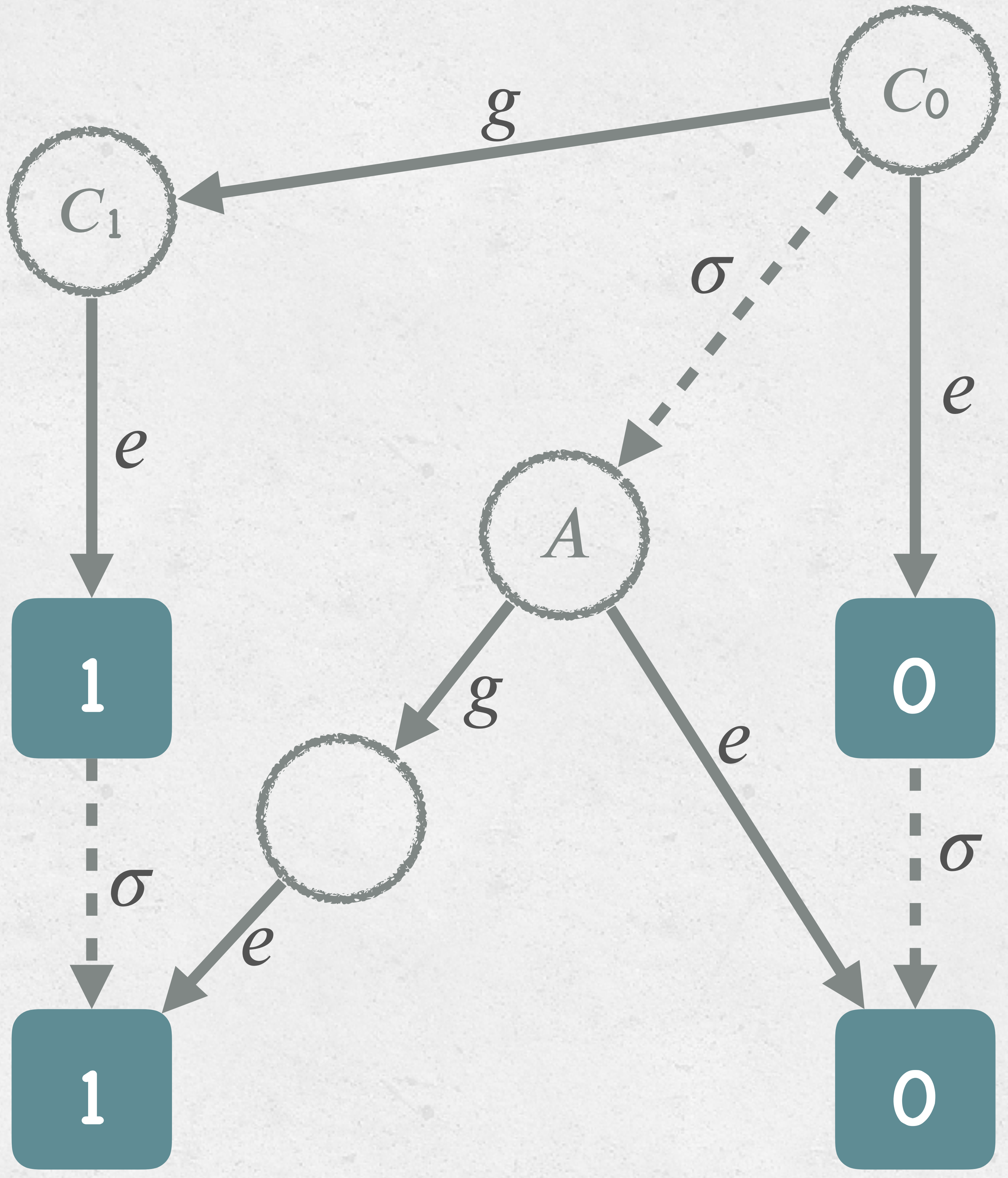

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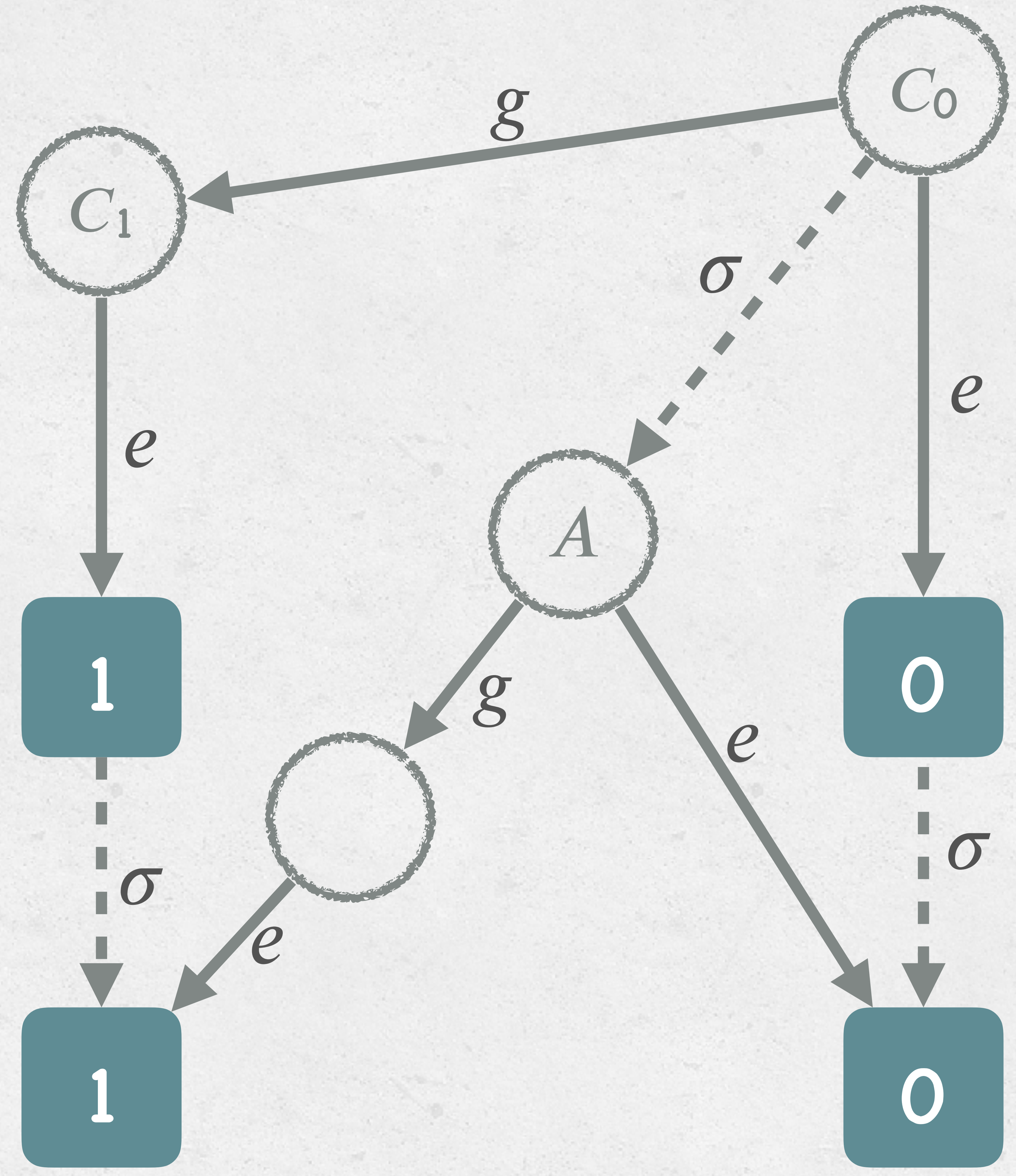

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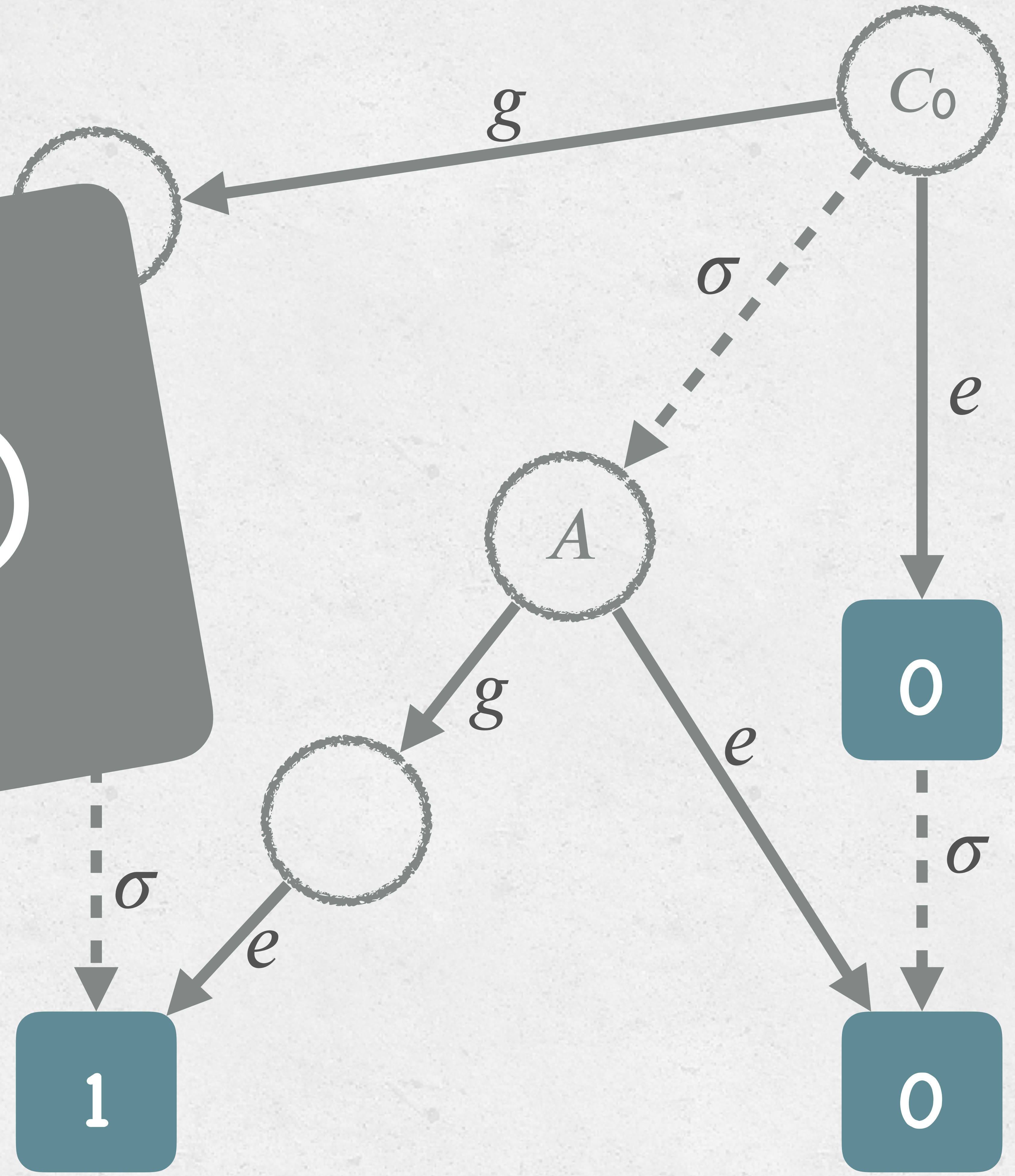
QED



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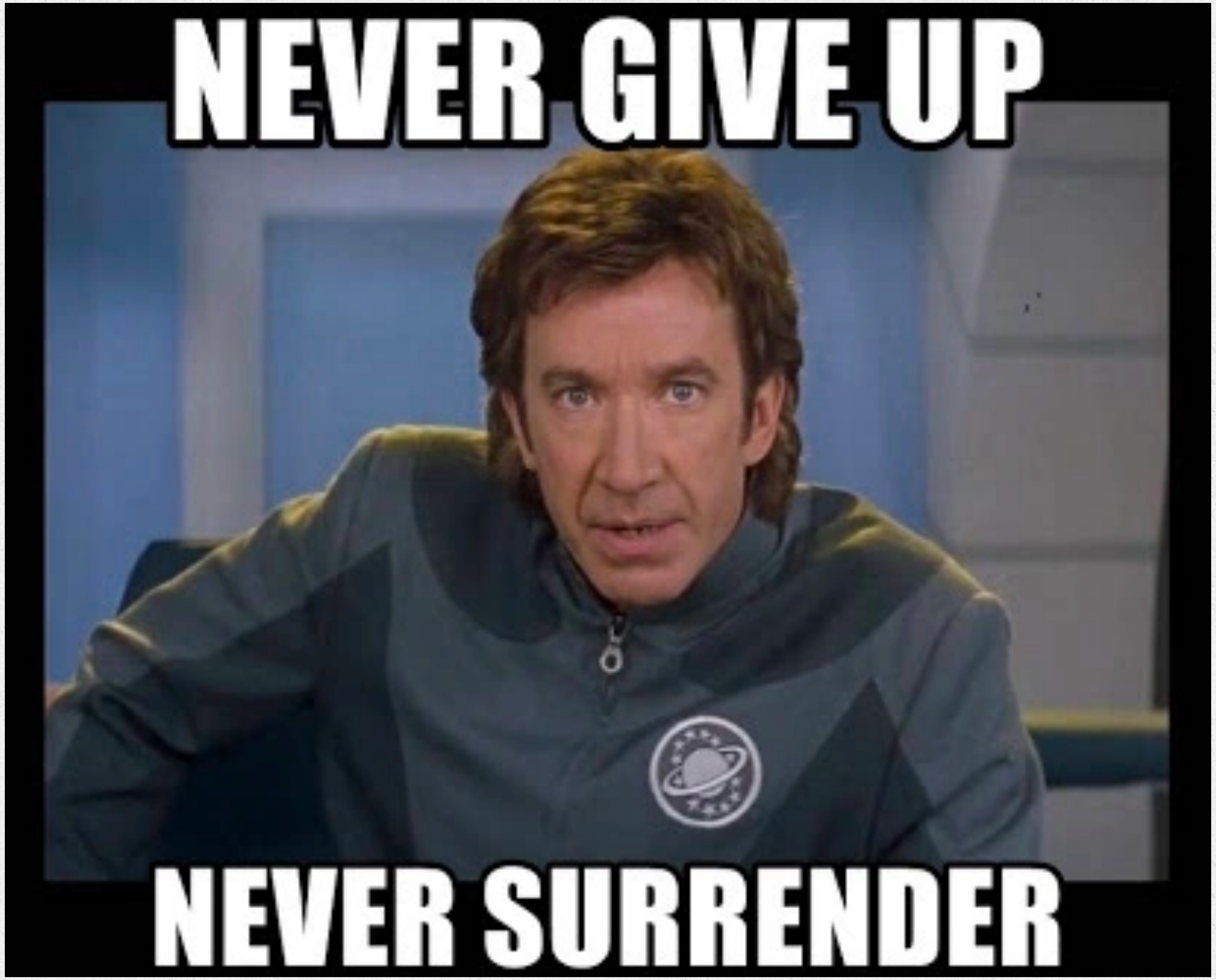

Is It Over? Do We Give Up Now?

Options:

- Only guarantee termination during periods of synchrony (**Paxos**); implies that no configuration is ever dead
- Use randomization to guarantee termination with probability 1 (**Ben-Or**)
- Strengthen the assumptions (consensus is solvable in a synchronous system)
- Constrain/weaken the problem

Some Related Problems

- ***k*-set Agreement:** allows up to k different decision values
- **Generalized Lattice Agreement:** processes decide on sets of values, all decision sets are comparable by $\subseteq$
- **Shared read/write register:** processes can read and write to a register

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Also solvable! And useful!

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