Text Categorization

CSE 454

Categorization

- Given:
  - A description of an instance, $x \in X$, where X is the *instance language* or *instance space*.
  - A fixed set of categories: $C = \{c_1, c_2, \ldots c_n\}$
- Determine:
  - The category of x: $c(x) \in C$, where $c(x)$ is a categorization function whose domain is X and whose range is C.

Sample Category Learning Problem

- Instance language: <size, color, shape>
  - size ∈ {small, medium, large}
  - color ∈ {red, blue, green}
  - shape ∈ {square, circle, triangle}
- C = {positive, negative}
- D:

Example	Size	Color	Shape	Category
1	small	red	circle	positive
2	large	red	circle	positive
3	small	red	triangle	negative
4	large	blue	circle	negative

Another Example: County *vs.* Country?

Example: County *vs.* Country?

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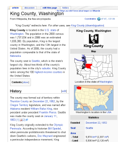

Text Categorization

- Assigning documents to a fixed set of categories, *e.g.*
- Web pages
  - Yahoo-like classification
- Newsgroup Messages
  - Recommending
  - Spam filtering
- News articles
  - Personalized newspaper
- Email messages
  - Routing
  - Prioritizing
  - Folderizing
  - spam filtering

Learning for Text Categorization

- Hard to construct text categorization functions.
- Learning Algorithms:
  - **Bayesian (naïve)**
  - Neural network
  - Relevance Feedback (Rocchio)
  - Rule based (C4.5, Ripper, Slipper)
  - Nearest Neighbor (case based)
  - Support Vector Machines (SVM)

Applications of ML

- Credit card fraud
- Product placement / consumer behavior
- Recommender systems
- Speech recognition

Most mature & successful area of AI

Learning for Categorization

- A ***training example*** is an instance $x \in X$, paired with its correct category $c(x)$: $<x, c(x)>$ for an unknown categorization function, c.
- Given a set of training examples, D.

{<  , county>, <  , country>,…

- Find a hypothesized categorization function, $h(x)$, such that: $\forall < x, c(x) > \in D : h(x) = c(x)$

Consistency

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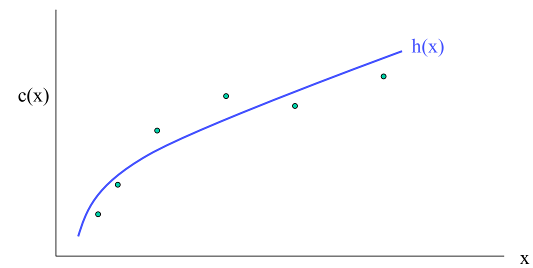
Function Approximation

May not be any perfect fit
Classification ~ discrete functions



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General Learning Issues

- Many hypotheses are usually consistent with the training data.
- Bias
  - Any criteria other than consistency with the training data that is used to select a hypothesis.
- Classification accuracy
  - % of instances classified correctly
  - (Measured on independent test data.)
- Training time
  - Efficiency of training algorithm
- Testing time
  - Efficiency of subsequent classification

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Generalization

- Hypotheses must generalize to correctly classify instances not in the training data.
- Simply memorizing training examples is a consistent hypothesis that does not generalize.
- *Occam's razor*:
  - Finding a *simple* hypothesis helps ensure generalization.

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Why is Learning Possible?

Experience alone never justifies any conclusion about any unseen instance.

Learning occurs when

PREJUDICE meets DATA!

Learning a “Frobnitz”

Bias

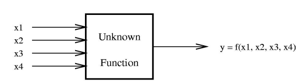
- The nice word for prejudice is “bias”.
- What kind of hypotheses will you consider?
  - What is allowable ***range*** of functions you use when approximating?
- What kind of hypotheses do you prefer?

Some Typical Biases

- Occam’s razor
  - ***“It is needless to do more when less will suffice”***
  - ***– William of Occam,***
    - ***died 1349 of the Black plague***
- MDL – Minimum description length
- Concepts can be approximated by
- ... conjunctions of predicates
  - ... by linear functions
  - ... by short decision trees

Frobnitz?

A Learning Problem



Example	x_1	x_2	x_3	x_4	y
1	0	0	1	0	0
2	0	1	0	0	0
3	0	0	1	1	1
4	1	0	0	1	1
5	0	1	1	0	0
6	1	1	0	0	0
7	0	1	0	1	0

Hypothesis Spaces

- **Complete Ignorance.** There are $2^{16} = 65536$ possible boolean functions over four input features. We can't figure out which one is correct until we've seen every possible input-output pair. After 7 examples, we still have 2^9 possibilities.

x_1	x_2	x_3	x_4	y
0	0	0	0	?
0	0	0	1	?
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	?
1	0	0	0	?
1	0	0	1	1
1	0	1	0	?
1	0	1	1	?
1	1	0	0	0
1	1	0	1	?
1	1	1	0	?
1	1	1	1	?

Terminology

- **Training example.** An example of the form $\langle \mathbf{x}, f(\mathbf{x}) \rangle$.
- **Target function (target concept).** The true function f.
- **Hypothesis**. A proposed function h believed to be similar to f.
- **Concept**. A boolean function. Examples for which $f(\mathbf{x}) = 1$ are called **positive examples** or **positive instances** of the concept. Examples for which $f(\mathbf{x}) = 0$ are called **negative examples** or **negative instances.**
- **Classifier**. A discrete-valued function. The possible values $f(\mathbf{x}) \in \{1, \ldots, K\}$ are called the **classes** or **class labels**.
- **Hypothesis Space**. The space of all hypotheses that can, in principle, be output by a learning algorithm.
- **Version Space**. The space of all hypotheses in the hypothesis space that have not yet been ruled out by a training example.

Two Strategies for ML

- Restriction bias: use prior knowledge to specify a restricted hypothesis space.
  - Naïve Bayes Classifier
- Preference bias: use a broad hypothesis space, but impose an ordering on the hypotheses.
  - Decision trees.

Bayesian Methods

- Learning and classification methods based on probability theory.
  - Bayes theorem plays a critical role in probabilistic learning and classification.
  - Uses *prior* probability of each category given no information about an item.
- Categorization produces a ***posterior*** probability distribution over the possible categories given a description of an item.

Axioms of Probability Theory

- All probabilities between 0 and 1

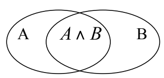
  $0 \leq P(A) \leq 1$

- Probability of truth and falsity

  P(true) = 1   P(false) = 0.

- The probability of disjunction is:

  $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$



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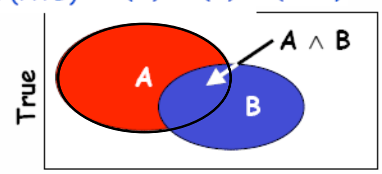
Probability: Simple & Logical

- The definitions imply that certain logically related events must have related probabilities

E.g. $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$



de Finetti (1931): an agent who bets according to probabilities that violate these axioms can be forced to bet so as to lose money regardless of outcome.

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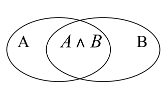
Conditional Probability

- P(A | B) is the probability of A given B
- Assumes:
  - B is all and only information known.
- Defined by:

$$P(A \mid B) = \frac{P(A \wedge B)}{P(B)}$$



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Independence

- A and B are *independent* iff:

  $P(A \mid B) = P(A)$

  $P(B \mid A) = P(B)$

  These two constraints are logically equivalent

- Therefore, if A and B are independent:

$$P(A \mid B) = \frac{P(A \wedge B)}{P(B)} = P(A)$$

$$P(A \wedge B) = P(A)P(B)$$

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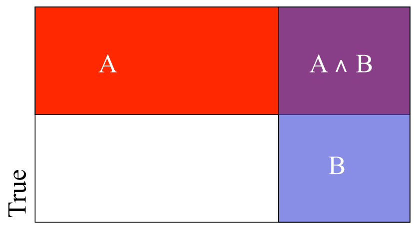
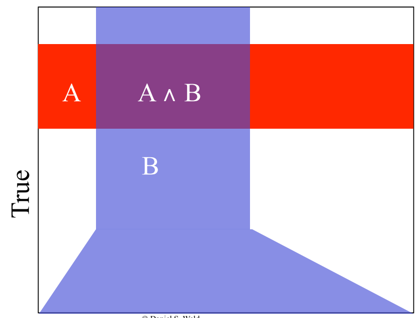
Independence

$P(A \wedge B) = P(A)P(B)$



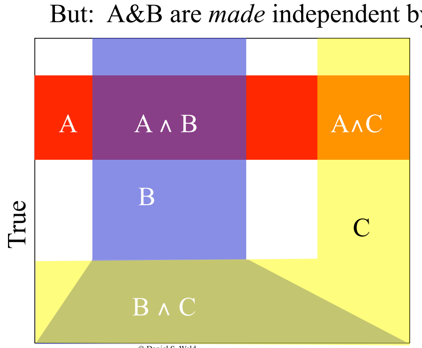
Conditional Independence

A&B *not* independent, since P(A|B) < P(A)



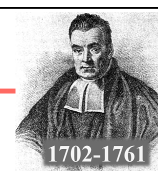
Conditional Independence

But: A&B are *made* independent by ¬C



$P(A|B,\neg C) = P(A|\neg C)$

Bayes Theorem



$$P(H\,|\,E) = \frac{P(E\,|\,H)P(H)}{P(E)}$$

Simple proof from definition of conditional probability:

$$P(H\,|\,E) = \frac{P(H \wedge E)}{P(E)}$$ (Def. cond. prob.)

$$P(E\,|\,H) = \frac{P(H \wedge E)}{P(H)}$$ (Def. cond. prob.)

$$P(H \wedge E) = P(E\,|\,H)P(H)$$ (Mult both sides of 2 by P(H).)

QED: $$P(H\,|\,E) = \frac{P(E\,|\,H)P(H)}{P(E)}$$ (Substitute 3 in 1.)

Bayesian Categorization

- Let set of categories be $\{c_1, c_2, \ldots c_n\}$
- Let E be description of an instance.
- Determine category of E by determining for each c_i

$$P(c_i \mid E) = \frac{P(c_i)P(E \mid c_i)}{P(E)}$$

- P(E) can be determined since categories are complete and disjoint.

$$\sum_{i=1}^{n} P(c_i \mid E) = \sum_{i=1}^{n} \frac{P(c_i)P(E \mid c_i)}{P(E)} = 1$$

$$P(E) = \sum_{i=1}^{n} P(c_i)P(E \mid c_i)$$

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Bayesian Categorization (cont.)

- Need to know:
  - Priors: P(c_i)
  - Conditionals: P(E | c_i)

Huh???

- P(c_i) are easily estimated from data.
  - If n_i of the examples in D are in c_i, then P(c_i) = n_i / |D|
- Assume instance is a conjunction of binary features:

$$E = e_1 \wedge e_2 \wedge \cdots \wedge e_m$$

- Too many possible instances (exponential in m) to estimate all P(E | c_i)

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Naïve Bayesian Motivation

- Problem: Too many possible instances (exponential in m) to estimate all P(E | c_i)
- If we assume features of an instance are independent given the category (c_i) (*conditionally independent*).

$$P(E \mid c_i) = P(e_1 \wedge e_2 \wedge \cdots \wedge e_m \mid c_i) = \prod_{j=1}^{m} P(e_j \mid c_i)$$

- Therefore, we then only need to know P(e_j | c_i) for each feature and category.

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Naïve Bayes Example

- C = {allergy, cold, well}
- e_1 = sneeze; e_2 = cough; e_3 = fever
- E = {sneeze, cough, ¬fever}

Prob	Well	Cold	Allergy
P(c_i)	0.9	0.05	0.05
P(sneeze|c_i)	0.1	0.9	0.9
P(cough|c_i)	0.1	0.8	0.7
P(fever|c_i)	0.01	0.7	0.4

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Naïve Bayes Example (cont.)

Probability	Well	Cold	Allergy
$P(c_i)$	0.9	0.05	0.05
$P(\text{sneeze} \mid c_i)$	0.1	0.9	0.9
$P(\text{cough} \mid c_i)$	0.1	0.8	0.7
$P(\text{fever} \mid c_i)$	0.01	0.7	0.4

E={sneeze, cough, ¬fever}

P(well | E) = (0.9)(0.1)(0.1)(0.99)/P(E)=0.0089/P(E)
P(cold | E) = (0.05)(0.9)(0.8)(0.3)/P(E)=0.01/P(E)
P(allergy | E) = (0.05)(0.9)(0.7)(0.6)/P(E)=0.019/P(E)

Most probable category: allergy
P(E) = 0.089 + 0.01 + 0.019 = 0.0379
P(well | E) = 0.23
P(cold | E) = 0.26
P(allergy | E) = 0.50

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Estimating Probabilities

- Normally, probabilities are estimated based on observed frequencies in the training data.
- If D contains n_i examples in category c_i, and n_{ij} of these n_i examples contains feature e_j, then:

$$P(e_j \mid c_i) = \frac{n_{ij}}{n_i}$$

- However, estimating such probabilities from small training sets is error-prone.
- If due only to chance, a rare feature, e_k, is always false in the training data, $\forall c_i : P(e_k \mid c_i) = 0$.
- If e_k then occurs in a test example, E, the result is that $\forall c_i$: $P(E \mid c_i) = 0$ and $\forall c_i$: $P(c_i \mid E) = 0$

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Smoothing

- To account for estimation from small samples, probability estimates are adjusted or *smoothed*.
- Laplace smoothing using an m-estimate assumes that each feature is given a prior probability, p, that is assumed to have been previously observed in a "virtual" sample of size m.

$$P(e_j \mid c_i) = \frac{n_{ij} + mp}{n_i + m} \quad = (n_{ij} + 1) / (n_i + 2)$$

- For binary features, p is simply assumed to be 0.5.

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Naïve Bayes for Text

- Modeled as generating a bag of words for a document in a given category by repeatedly sampling with replacement from a vocabulary $V = \{w_1, w_2, \ldots w_m\}$ based on the probabilities $P(w_j \mid c_i)$.
- Smooth probability estimates with Laplace m-estimates assuming a uniform distribution over all words ($p = 1/|V|$) and $m = |V|$
  - Equivalent to a virtual sample of seeing each word in each category exactly once.

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Text Naïve Bayes Algorithm (Train)

Let V be the vocabulary of all words in the documents in D
For each category $c_i \in C$
Let D_i be the subset of documents in D in category c_i
$P(c_i) = |D_i| / |D|$
Let T_i be the concatenation of all the documents in D_i
Let n_i be the total number of word occurrences in T_i
For each word $w_j \in V$
Let n_{ij} be the number of occurrences of w_j in T_i
Let $P(w_i \mid c_i) = (n_{ij} + 1) / (n_i + |V|)$

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Text Naïve Bayes Algorithm (Test)

Given a test document X
Let n be the number of word occurrences in X
Return the category:

$$\underset{c_i \in C}{\text{argmax}}\, P(c_i) \prod_{i=1}^{n} P(a_i \mid c_i)$$

where a_i is the word occurring the ith position in X

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Naïve Bayes Time Complexity

- Training Time: $O(|D|L_d + |C||V|)$ where L_d is the average length of a document in D.
  - Assumes V and all D_i, n_i, and n_{ij} pre-computed in $O(|D| L_d)$ time during one pass through all of the data.
  - Generally just $O(|D|L_d)$ since usually $|C||V| < |D|L_d$
- Test Time: $O(|C| L_t)$ where L_t is the average length of a test document.
- Very efficient overall, linearly proportional to the time needed to just read in all the data.

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Easy to Implement

- But…
- If you do… it probably won't work…

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Probabilities: Important Detail!

- $P(\text{spam} \mid E_1 \dots E_n) = \prod_i P(\text{spam} \mid E_i)$

Any more potential problems here?

- We are multiplying lots of small numbers
  Danger of underflow!
  - $0.5^{57} = 7\text{ E -18}$
- Solution? Use logs and add!
  - $p_1 * p_2 = e^{\log(p1)+\log(p2)}$
  - Always keep in log form

Underflow Prevention

- Multiplying lots of probabilities, which are between 0 and 1 by definition, can result in floating-point underflow.
- Since $\log(xy) = \log(x) + \log(y)$, it is better to perform all computations by summing logs of probabilities rather than multiplying probabilities.
- Class with highest final un-normalized log probability score is still the most probable.

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Naïve Bayes Posterior Probabilities

- Classification results of naïve Bayes
  - I.e. the class with maximum posterior probability…
  - Usually fairly accurate (?!?!?)
- However, due to the inadequacy of the conditional independence assumption…
  - Actual posterior-probability estimates *not* accurate.
  - Output probabilities generally very close to 0 or 1.

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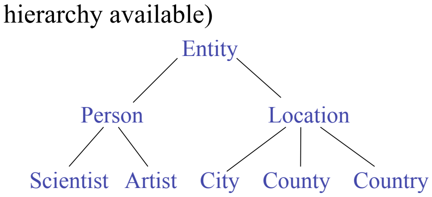
Multi-Class Categorization

- Pick the category with max probability
- Create many 1 vs other classifiers
- Use a hierarchical approach (wherever hierarchy available)



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