| Computer Vision | Prof. Steve Seitz |
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| CSE 490CV, EE400B | Winter 2002 |

## Midterm

Handed out: Friday, Feb 1
Due: Friday Feb 8, at the beginning of class

Directions: Please provide answers to the questions in the space provided. This take-home midterm is open book and open notes, but do not discuss it or collaborate with other students.

## Problem 1. (10 pts)

Suppose image $\mathrm{I}(\mathrm{x}, \mathrm{y})$ is a linear gradient:

$$
I(x, y)=a x+b y+c
$$

and H is the following filter mask:

| 0 | 1 | 0 |
| :---: | :---: | :---: |
| 1 | -4 | 1 |
| 0 | 1 | 0 |

A. Give the equation for $\mathrm{G}(\mathrm{x}, \mathrm{y})$, where G is the cross-correlation of H and F . Show your work.
B. What kind of filter is this, and why would it produce this result?

Problem 2. (10 pts)
A. Write pseudo-code for a procedure to detect a parabola in an image. The parabola will be composed of edge pixels, but may be incomplete, as shown below (the parabola is shown in bold). The image may also contain edges that aren't on the parabola (the lines in the image below). The equation for a parabola is $y=a x^{2}+b x+c$
Your method should return the parabola that contains the most edge pixels in the image. Be sure and explicitly specify any formulas that are needed (instead of just saying "compute the solutions of ..."). Assume that the edge pixels have already been identified so that $\mathrm{I}(\mathrm{x}, \mathrm{y})=1$ if $(\mathrm{x}, \mathrm{y})$ is an edge, 0 otherwise.


Hint: this may be solved using a variant of a technique that we discussed in lecture!
B. What is the running time of your algorithm? Be sure and define any symbols that you use.

## Problem 3. (10 pts)

Suppose that you are given two images H and I and asked to compute the optical flow between them, given the knowledge that all points in $H$ flow in the same direction, specified by a vector [a b $]^{\mathrm{T}}$. Different points may move different amounts in this direction.
A. Give the optical flow constraint equation for this case. Show your work.
B. Give the expression for the estimate of optical flow by solving the above equation.
C. Is there an aperture problem in this case? Explain why or why not. Also describe any situation in which the optical flow is ambiguous (in terms of directions of edges in the image).

## Problem 4. (10 pts)

Define a $3 \times 3$ directional smoothing filter H that blurs horizontal edges, but not vertical edges. Its behavior should match the two filtering results shown below. Assume that the images are padded with an extra one-pixel wide border using the reflection technique described in lecture.


| 0 | 0 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 |

Before filtering

| 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

Before filtering

$\longrightarrow$| 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: |
| .25 | .25 | .25 | .25 | .25 |
| .75 | .75 | .75 | .75 | .75 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

After filtering

## Problem 5. (10 pts)

The reverse of a blurring operation is a sharpening operation that enhances edges of an image. In this problem you will derive a sharpening filter. Note that $H \otimes F$ is the cross-correlation of filter H with image F .
A. For the 1D curve F , suppose $F-H \otimes F$ is as shown below left. Sketch the curves: $F-H \otimes F$ and $2 F-H \otimes F$ below left. Explain how the operation $2 F-H \otimes F$ enhances the edge.




$2 F-H \otimes F$


E
B. Given the 2 D smoothing filter H , as defined above, determine the $3 \times 3$ filter E that performs enhancement in 2 D . E is defined by the following relation:

$$
E \otimes F=2 F-H \otimes F
$$

## Problem 6. (10 pts)

In this problem, you will use K-means to cluster optical flow vectors. Assume that each flow vector is represented as $(\mathrm{u}, \mathrm{v})$ and the distance between $(\mathrm{u}, \mathrm{v})$ and $(\mathrm{p}, \mathrm{q})$ is defined to be $\sqrt{(u-p)^{2}+(v-q)^{2}}$

A. Suppose the initial means are

$\longleftarrow$ and $\longrightarrow$


In the leftmost picture above, circle the flow vectors assigned to the second of these two initial means.
B. Suppose the initial means are $\square$ and $\downarrow$ (assume that these two vectors have the same length). In the center picture above, circle the flow vectors assigned to the second of these two initial means.
C. In the space below, draw the updated mean vectors, given the clusters in part B (hint-add the vectors in each cluster by connecting the arrows, then divide the length of the resulting vector by the number of vectors in each cluster).
D. In the rightmost picture, circle the flow vectors assigned the second arrow from part C.

