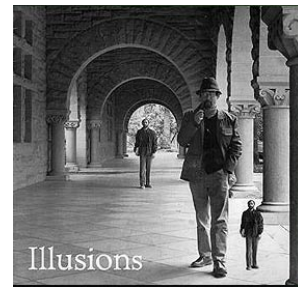


Announcements

- Midterm out today
- Project 1 demos

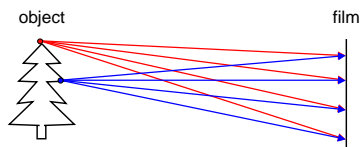
Projection



Today's Readings

- Nalwa 2.1 (handout)

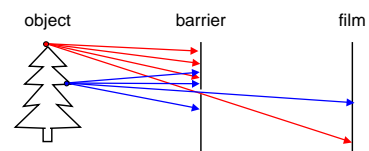
Image formation



Let's design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

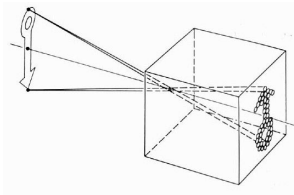
Pinhole camera



Add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**
- How does this transform the image?
 - it gets inverted

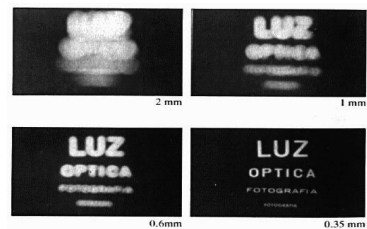
Camera Obscura



The first camera

- Known to Aristotle
- How does the aperture size affect the image?

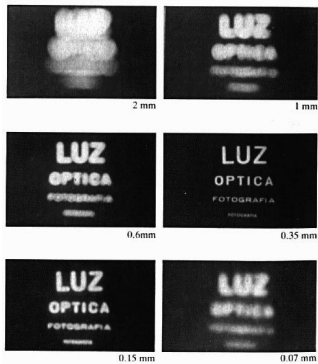
Shrinking the aperture



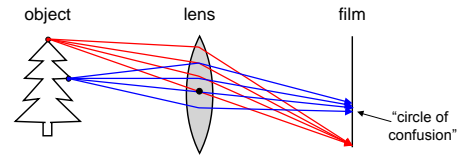
Why not make the aperture as small as possible?

- Less light gets through
- *Diffraction* effects...

Shrinking the aperture



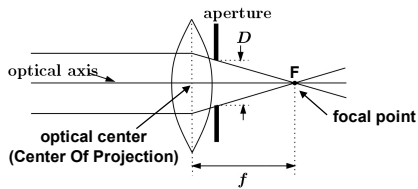
Adding a lens



A lens focuses light onto the film

- There is a specific distance at which objects are "in focus"
 - other points project to a "circle of confusion" in the image
- Changing the shape of the lens changes this distance

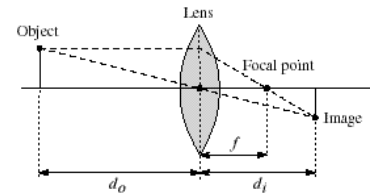
Lenses



A lens focuses parallel rays onto a single focal point

- focal point at a distance f beyond the plane of the lens
 - f is a function of the shape and index of refraction of the lens
- Aperture of diameter D restricts the range of rays
 - aperture may be on either side of the lens
- Lenses are typically spherical (easier to produce)

Lenses

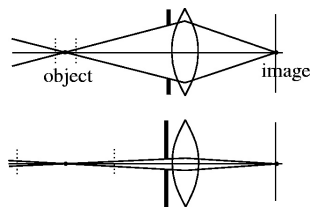


Thin lens equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

- Any object point satisfying this equation is in focus
- What is the shape of the focus region?
- How can we change the focus region?

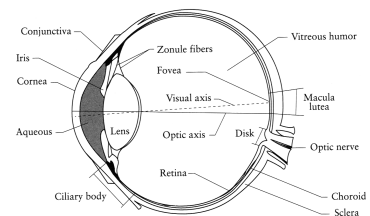
Depth of field



Changing the aperture size affects depth of field

- A smaller aperture increases the range in which the object is approximately in focus

The eye



The human eye is a camera

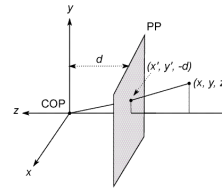
- **Iris** - colored annulus with radial muscles
- **Pupil** - the hole (aperture) whose size is controlled by the iris
- What's the "film"?
 - photoreceptor cells (rods and cones) in the **retina**

Digital camera

A digital camera replaces film with a sensor array

- Each cell in the array is a **Charge Coupled Device**
 - light-sensitive diode that converts photons to electrons
 - <http://www.howstuffworks.com/digital-camera2.htm>

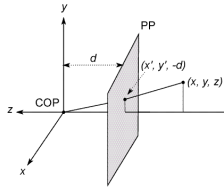
Modeling projection



The coordinate system

- We will use the pin-hole model as an approximation
- Put the optical center (**Center Of Projection**) at the origin
- Put the image plane (**Projection Plane**) *in front of* the COP
 - Why? This way the image is right-side-up
- The camera looks down the *negative* z axis
 - we need this if we want right-handed-coordinates

Modeling projection



Projection equations

- Compute intersection with PP of ray from (x,y,z) to COP
- Derived using similar triangles (on board)

$$(x, y, z) \rightarrow \left(-d\frac{x}{z}, -d\frac{y}{z}, -d\right)$$

- We get the projection by throwing out the last coordinate:

$$(x, y, z) \rightarrow \left(-d\frac{x}{z}, -d\frac{y}{z}\right)$$

Homogeneous coordinates

Is this a linear transformation?

- no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \\ 1 \end{bmatrix} \Rightarrow \left(-d\frac{x}{z}, -d\frac{y}{z}\right)$$

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**
- Can also formulate as a 4x4 (today's reading does this)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ -z/d \end{bmatrix} \Rightarrow \left(-d\frac{x}{z}, -d\frac{y}{z}\right)$$

divide by fourth coordinate

Perspective Projection

How does multiplying the projection matrix change the transformation?

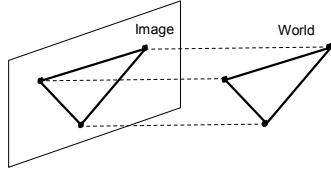
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \\ 1 \end{bmatrix} \Rightarrow \left(-d\frac{x}{z}, -d\frac{y}{z}\right)$$

$$\begin{bmatrix} -d & 0 & 0 & 0 \\ 0 & -d & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} -dx \\ -dy \\ z \\ 1 \end{bmatrix} \Rightarrow \left(-d\frac{x}{z}, -d\frac{y}{z}\right)$$

Orthographic projection

Special case of perspective projection

- Distance from the COP to the PP is infinite



- Also called "parallel projection"
- What's the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Other types of projection

Scaled orthographic

- Also called "weak perspective"

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/d \end{bmatrix} \Rightarrow (dx, dy)$$

Affine projection

- Also called "paraperspective"

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f , principle point (x'_c, y'_c) , pixel size (s_x, s_y)
- blue parameters are called "extrinsics," red are "intrinsics"

Projection equation

$$\mathbf{X} = \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi} \mathbf{X}$$

- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

$$\mathbf{\Pi} = \begin{bmatrix} -f s_x & 0 & x'_c & 1 & 0 & 0 \\ 0 & -f s_y & y'_c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_{3 \times 3} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{f}_{3 \times 3} & \mathbf{T}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$$

intrinsics projection rotation translation identity matrix

- The definitions of these parameters are **not** completely standardized
 - especially intrinsics—varies from one book to another

Summary

Things to take away from this lecture

- Properties of a pinhole camera
 - effects of aperture size
- Properties of lens-based cameras
 - focal point, optical center, aperture
 - thin lens equation
 - depth of field
 - circle of confusion
- Modeling projection
 - homogeneous coordinates
 - projection matrix and its elements
 - orthographic, weak perspective, affine models
- Camera parameters
 - intrinsics, extrinsics