## Announcements

- Project 1
- Grading session this afternoon
- Artifacts due Friday (voting TBA)
- Project 2 out (online)
- Signup for panorama kits ASAP (weekend slots go quickly...)
- help session at end of class



## How to do it?

Basic Procedure

- Take a sequence of images from the same position - Rotate the camera about its optical center
- Compute transformation between second image and first
- Shift the second image to overlap with the first
- Blend the two together to create a mosaic
- If there are more images, repeat


## Aligning images



How to account for warping?

- Translations are not enough to align the images
- Photoshop demo


## Image reprojection



The mosaic has a natural interpretation in 3D

- The images are reprojected onto a common plane
- The mosaic is formed on this plane

Image reprojection
Basic question

- How to relate two images from the same camera center? - how to map a pixel from PP1 to PP2


## Answer

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2

Image reprojection

Observation

- Rather than thinking of this as a 3D reprojection, think of it as a 2D image warp from one image to another

Homographies
Perspective projection of a plane

- Lots of names for this:
- homography, texture-map, colineation, planar projective map
- Modeled as a 2D warp using homogeneous coordinates


To apply a homography $\mathbf{H}$

- Compute $\mathbf{p}^{\prime}=\mathbf{H p} \quad$ (regular matrix multiply)
- Convert p' from homogeneous to image coordinates
- divide by w (third) coordinate

Image warping with homographies


## Panoramas

What if you want a $360^{\circ}$ field of view?


Spherical projection


- Convert to spherical coordinates

$$
(\sin \theta \cos \phi, \sin \phi, \cos \theta \cos \phi)=(\hat{x}, \hat{y}, \hat{z})
$$

- Convert to spherical image coordinates

$$
(\tilde{x}, \tilde{y})=(s \theta, s \phi)+\left(\tilde{x}_{c}, \tilde{y}_{c}\right)
$$

- $s$ defines size of the final image
" often convenient to set $s=$ camera focal length

unwrapped sphere



## Spherical reprojection



## Spherical reprojection




Spherical image stitching


What if you don't know the camera rotation?

- Solve for the camera rotations
- Note that a pan (rotation) of the camera is a translation of the sphere!
- Use feature matching to solve for translations of spherical-warped images



## RANSAC

## Assembling the panorama

Same basic approach works for any transformation

- Translation, rotation, homographies, etc.
- Very useful tool


## General version

- Randomly choose a set of $K$ correspondences - Typically K is the minimum size that lets you fit a model
- Fit a model (e.g., homography) to those correspondences
- Count the number of inliers that "approximately" fit the model - Need a threshold on the error
- Repeat as many times as you can
- Choose the model that has the largest set of inliers
- Refine the model by doing a least squares fit using ALL of the inliers


## Problem: Drift



Error accumulation

- small errors accumulate over time


## Problem: Drift



## Solution

- add another copy of first image at the end
- this gives a constraint: $y_{n}=y_{1}$
- there are a bunch of ways to solve this problem
- add displacement of $\left(y_{1}-y_{n}\right) /(n-1)$ to each image after the first - compute a global warp: $y^{\prime}=y+a x$
- run a big optimization problem, incorporating this constraint ") best solution, but more complicated » known as "bundle adjustment"



## Project 2 (out today)





## Alpha Blending



Encoding blend weights: $I(x, y)=(\alpha R, \alpha G, \alpha B, \alpha)$
color at $\mathrm{p}=\left(\alpha_{1} R_{1}, \alpha_{1} G_{1}, \alpha_{1} B_{1}\right)+\left(\alpha_{2} R_{2}, \alpha_{2} G_{2}, \alpha_{2} B_{2}\right)+\left(\alpha_{3} R_{3}, \alpha_{3} G_{3}, \alpha_{3} B_{3}\right)$
Implement this in two steps:

1. accumulate: add up the ( $\alpha$ premultiplied) $\mathrm{RGB} \alpha$ values at each pixel
2. normalize: divide each pixel's accumulated RGB by its $\alpha$ value

Q: what if $\alpha=0$ ?


## Forward warping



Send each pixel $f(x, y)$ to its corresponding location

$$
\left(x^{\prime}, y^{\prime}\right)=h(x, y) \text { in the second image }
$$

Q: what if pixel lands "between" two pixels?

## Forward warping



Send each pixel $f(x, y)$ to its corresponding location $\left(x^{\prime}, y^{\prime}\right)=h(x, y)$ in the second image

Q: what if pixel lands "between" two pixels?
A: distribute color among neighboring pixels ( $x^{\prime}, y^{\prime}$ )

- Known as "splatting"

Inverse warping


Get each pixel $g\left(x^{\prime}, y^{\prime}\right)$ from its corresponding location $(x, y)=h^{-1}\left(x^{\prime}, y^{\prime}\right)$ in the first image

Q: what if pixel comes from "between" two pixels?

Inverse warping


Get each pixel $g\left(x^{\prime}, y^{\prime}\right)$ from its corresponding location
$(x, y)=h^{-1}\left(x^{\prime}, y^{\prime}\right)$ in the first image
Q: what if pixel comes from "between" two pixels?
A: resample color value

- We discussed resampling techniques before - nearest neighbor, bilinear, Gaussian, bicubic

Forward vs. inverse warping
Q: which is better?
A: usually inverse-eliminates holes

- however, it requires an invertible warp function-not always possible..

