## Computer Vision

# CSE 455 <br> Image Coordinates and Resizing 

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What is an image?

## Eyes: projection onto retina



## Model: pinhole camera



## Model: pinhole camera



## At each point we record incident light



## An image is a matrix of light



## Values in matrix = how much light

| $\begin{array}{llll}  & \\ 0 & 1 & \text { Columns } \\ 0 & 6 \end{array}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 100 |  |  | 102 | 121 |  |  |  |  |  |  |  |  |
|  | ${ }_{100}$ | 102 | 187 | 102 | 132 | 14613 | 136150 | 15 | ${ }_{182}^{12}$ |  |  | 104105 |  |
| 2 | ${ }^{100}$ |  | 107 | 102 | 122 | 14613 | 36 158 |  |  |  | 15120 | ${ }^{04} 105$ |  |
| Rows 3 | ${ }^{100}$ | 102 | 187 | 102 | 132 | ${ }_{14} 13$ | ${ }^{36} 1150$ | 56 | ${ }^{182}$ | 12215 | 15104 | 04 105 |  |
| 4 | ${ }^{100}$ | 102 | 107 | 102 | 13 | 13 | 1361180 | A | ${ }^{122}$ | 12 | 15104 | 15 |  |
| 5 | ${ }_{100}$ | 102 | 107 | 102 | 13 |  | ${ }_{60} 188$ | 86 128 | ${ }^{8} 122$ | 12115 | 15104 | O4 105 | $1{ }^{13}$ |
| 6 | ${ }_{100}$ | 102 | 107 | 102 | 12 | $\infty$ | 2050 |  |  |  | 202 |  |  |
|  | 100 |  | 107 |  |  |  |  |  |  |  | 5 | ${ }^{\circ}$ |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | ${ }_{10}$ | 12 | 10 | 12 | 132 | 6 |  | ${ }_{56} 128$ | ${ }^{128}$ | 12115 | ${ }^{15} 104$ | 104 105 |  |
|  | ${ }_{10}$ | 12 | 107 | 102 | 132 | 8 |  | ${ }_{56} 148$ | 18812 | 1215 | ${ }_{15} 104$ | ${ }^{104} 105$ |  |
|  |  |  |  | 122 | 132 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | 146 |  |  |  |  |  | 04 105 |  |

## Values in matrix = how much light

- Higher = more light
- Lower = less light
- Bounded
- No light = 0
- Sensor/device limit = max
- Typical ranges:
- [0-255], fit into byte
- [0-1], floating point
- Called pixels



## Addressing pixels

- Ways to index:
- (r,c)

- We use ( $x, y$ )
- So does your homework!
- Arbitrary
- Only thing that matters is consistency


## Color image: 3d tensor in colorspace



## RGB information in separate "channels"

Remember: we can match "real" colors using a mix of primaries.

Each channel encodes one primary. Adding the light produced from each primary mimics the original color.


## Addressing pixels

- We use ( $x, y, c$ )
- $(1,2,0)$ :
- column 1, row 2, channel 0
- Be consistent
- But do what we do for homeworks :-)
- Also for size:
- $1920 \times 1080 \times 3$ image:
- 1920 px wide
- 1080 pxtall
- 3 channels



## How do we store them?



## Storage: row major vs column major



Column Major


## Storage: row major vs column major

HW


WH


## Typically use row-major or HW



In 3d we have more choices!


## HWC: channels interleaved



## CHW: channels separated



## CHW Pop quiz

We'll use CHW, it's what a lot of other libraries use.
In an array for a $1920 \times 1080 \times 3$ image what entry would contain the pixel $(15,192,2)$ ?

Formula:
$x+y^{*} W+z^{*} W^{*} H$



## CHW Pop quiz

In an array for a $1920 \times 1080 \times 3$ image what entry would contain the pixel $(15,192,2)$ ?

In general for ( $x, y, z$ ) of image ( $\mathrm{W}, \mathrm{H}, \mathrm{C}$ )
$x+y^{*} W+z^{*} W^{*} H$
$15+192 * 1920+2 * 1920 * 1080=4,515,855$
Remember, everything is 0 indexed
Where does $(0,0,0)$ go?
Position $0+0+0=0$


## In your homework



## Image interpolation and resizing

## An image is kinda like a function

An image is a mapping from indices to pixel value:

- Im: |x|x|->R

We may want to pass in non-integers:

- Im': RxRxI->R



## A note on coordinates in images


integer pixels

## A note on coordinates in images



We can think of their values as being at the centers.

## A note on coordinates in images



Now we can move to a real coordinate system.

## A note on coordinates in images



On the image

## A note on coordinates in images

So, the value of the pixel ( $x, y$ ) is now centered at ( $x, y$ ).


## A note on coordinates in images

But there are other real-valued points.

0123 .


## A note on coordinates in images

This point is:


## Just be careful

This point is:


## Interpolation

- How do we find out the VALUE of a noninteger point, when the image only comes with integer points, ie $(25,45,3)$.
- For our assignment:

1. Nearest-Neighbor Interpolation
2. Bilinear Interpolation

## Nearest neighbor: what it sounds like

$f(x, y, z)=\operatorname{lm}($ round $(x)$, round $(y), z)$

- Looks blocky
- Common pitfall: Integer division rounds down in C
- Note: z is still int


Triangle interpolation: for less structured image (alternate approach)

Sometimes you have a regular grid, sometimes you don't.

When you don't, you can look for triangles!


## Triangle interpolation: for less structured image

Sometimes you have a regular grid, sometimes you don't.

When you don't look for triangles!


Triangle interpolation: for less structured image

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## Triangle interpolation: for less structured image

Sometimes you have a regular grid, sometimes you don't.

When you don't look for triangles!


Triangle interpolation: for less structured image
Weighted sum using triangles:
$\mathrm{Q}=\mathrm{V} 1 * \mathrm{~A} 1+\mathrm{V} 2 * \mathrm{~A} 2+\mathrm{V} 3 * \mathrm{~A} 3$
WHY?
V1 is the furthest from q and A 1 gives the smallest area.

V2 is next furthest from 1 and A2 gives the next smallest area...

Should normalize this based on
 total area, but we won't use this.

Bilinear interpolation: for grids, pretty good; easier than triangles

This time find the closest pixels in a box


## Bilinear interpolation: for grids, pretty good

This time find the closest pixels in a box

Bilinear interpolation: for grids, pretty good
This time find the closest pixels in a box

Bilinear interpolation: for grids, pretty good
This time find the closest pixels in a box

Same plan, weighted sum based on area of opposite rectangle
$\mathrm{Q}=\mathrm{V} 1 * \mathrm{~A} 1+\mathrm{V} 2 * \mathrm{~A} 2+\mathrm{V} 3^{*} \mathrm{~A} 3+\mathrm{V} 4^{*} \mathrm{~A} 4$


Bilinear interpolation: for grids, pretty good

$$
\begin{aligned}
& \mathrm{A} 1=\mathrm{d} 2 * \mathrm{~d} 4 \\
& \mathrm{~A} 2=\mathrm{d} 1^{*} \mathrm{~d} 4 \\
& \mathrm{~A} 3=\mathrm{d} 2^{*} \mathrm{~d} 3 \\
& \mathrm{~A} 4=\mathrm{d} 1^{*} \mathrm{~d} 3 \\
& \mathrm{q}=\mathrm{V} 1^{*} \mathrm{~A} 1+\mathrm{V} 2^{*} \mathrm{~A} 2+\mathrm{V} 3^{*} \mathrm{~A} 3+\mathrm{V} 4^{*} \mathrm{~A} 4
\end{aligned}
$$



Bilinear interpolation: for grids, pretty good

- Smoother than NN
- More complex
- 4 lookups
- Some math
- Often the right tradeoff of speed vs final result


Bicubic sampling: more complex, maybe better?

- A cubic interpolation of 4 cubic interpolations
- Smoother than bilinear, no "star"
- 16 nearest neighbors
- Fit 3rd order poly:


Bilinear


Bicubic

- $f(x)=a+b x+c x^{\wedge} 2+d x^{\wedge} 3$
- Interpolate along axis
- Fit another poly to interpolated values

Bicubic vs bilinear


Bicubic vs bilinear


Resize algorithm:

- For each pixel in new image:

1. Map to old im coordinates
2. Interpolate value
3. Set new value in image


## What about shrinking?

- NN and Bilinear only look at small area
- Lots of artifacting
- Staircase pattern on diagonal lines
- We'll fix this next class with filters!



## So what is this interpolation useful for?

## Image resizing!

Say we want to increase the size of an image...

This is a beautiful image of a sunset... it's just very small...


## Image resizing!

Say we want to increase the size of an image...

This is a beautiful image of a sunset... it's just very small...

Say we want to increase size $4 \times 4$ -
> 7x7


Resize $4 x 4$-> 7x7
$\begin{array}{llll}0 & 1 & 2 & 3\end{array}$

- Create our new image
0
- 

$\sim$
$m$

|  |  | 1 | 1 | 1 | - |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  | 1 |  |  |
| - |  |  |  |  |  |  |
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## Resize $4 \times 4$-> 7x7

- Create our new image
- Match up coordinates



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- System of equations
- $\quad a X+b=Y$
- $\quad a^{*}-.5+b=-.5$
- $\quad a * 6.5+b=3.5$



## Resize $4 \times 4$-> 7x7

- Create our new image
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- $\quad a X+b=Y$
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- $\quad a * 6.5+b=3.5$
- $a^{*} 7=4$



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- $a^{*} 7=4$
- $\quad a=4 / 7$



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$$
-a^{*}-.5+b=-.5
$$



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- $\quad a * 6.5+b=3.5$
- $a=4 / 7$

$$
\begin{aligned}
& \quad a^{*} .5+b=-.5 \\
& -\quad 4 / 7^{*}-1 / 2+b=-1 / 2
\end{aligned}
$$



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- System of equations
- $\quad a X+b=Y$
- $\quad a^{*}-.5+b=-.5$
- $\quad a * 6.5+b=3.5$
- $a=4 / 7$

$$
\begin{array}{ll} 
& a^{*}-.5+b=-.5 \\
- & 4 / 7^{*}-1 / 2+b=-1 / 2 \\
- & -4 / 14+b=-7 / 14
\end{array}
$$



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- System of equations
- $\quad a X+b=Y$
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& -\quad 4 / 7^{*}-1 / 2+b=-1 / 2 \\
& -\quad-4 / 14+b=-7 / 14 \\
& -\quad b=-3 / 14
\end{aligned}
$$



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- System of equations
- $\quad a X+b=Y$
- $\quad a^{*}-.5+b=-.5$
- $\quad a * 6.5+b=3.5$
- $a=4 / 7$
- $b=-3 / 14$
- So, we can start with any coordinate $X$ of the big (new) image and use a and b to get Y on the smaller (old) image.



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 X-3 / 14=Y$



## Resize $4 \times 4$-> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 X-3 / 14=Y$



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 X-3 / 14=Y$
- Iterate over new pts



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 \mathrm{X}-3 / 14=\mathrm{Y}$
- Iterate over new pts
- Map to old coords ( Y is old)



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- $4 / 7$ X - $3 / 14=$ Y
- Iterate over new pts
- Map to old coords ( Y is old)
- $(1,3)$



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- $4 / 7$ X $-3 / 14=$ Y
- Iterate over new pts
- Map to old coords
- $(1,3)$
- 4/7*1-3/14
- $4 / 7 * 3-3 / 14$



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 \mathrm{X}-3 / 14=\mathrm{Y}$
- Iterate over new pts
- Map to old coords
- $(1,3)$
- 4/7*1-3/14
- $4 / 7 * 3-3 / 14$
- $(5 / 14,21 / 14)$



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 \mathrm{X}-3 / 14=\mathrm{Y}$
- Iterate over new pts
- Map to old coords
- $(1,3)$-> $(5 / 14,21 / 14)$



## Resize $4 \times 4$-> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 \mathrm{X}-3 / 14=\mathrm{Y}$
- Iterate over new pts
- Map to old coords
- $(1,3)$-> $(5 / 14,21 / 14)$
- Interpolate old values



## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 \mathrm{X}-3 / 14=\mathrm{Y}$
- Iterate over new pts
- Map to old coords
- $(1,3)$-> $(5 / 14,21 / 14)$
- Interpolate old values



## Resize $4 \times 4$-> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 X-3 / 14=Y$
- Iterate over new pts
- Map to old coords
- $(1,3)$-> $(5 / 14,21 / 14)$
- Interpolate old values
- Size of opposite rects



## Resize 4x4 -> 7x7

- Create our new image


V = Yval*Yar+Bval*Var+R1val*R1ar+R2val*R2ar

- For each channel c, put the interpolated value from that channel in position ( $1,3, c$ ).


## Resize 4x4 -> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 \mathrm{X}-3 / 14=\mathrm{Y}$
- Iterate over new pts
- Map to old coords
- $(1,3)$-> $(5 / 14,21 / 14)$
- Interpolate old values



## Resize 4x4 -> 7x7

- Create our new image results
- Match up coordinates
- $4 / 7 \mathrm{X}-3 / 14=\mathrm{Y}$
- Iterate over new pts
- Map to old coords
- $(1,3)$-> $(5 / 14,21 / 14)$
- Interpolate old values



## Resize $4 \times 4$-> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 \mathrm{X}-3 / 14=\mathrm{Y}$
- Iterate over new pts
- Map to old coords
- $(1,3)$-> $(5 / 14,21 / 14)$
- Interpolate old values
- Fill in the rest
- On outer edges use padding!



## Resize $4 \times 4$-> 7x7

- Create our new image
- Match up coordinates
- $4 / 7 \mathrm{X}-3 / 14=\mathrm{Y}$
- Iterate over new pts
- Map to old coords
- $(1,3)$-> $(5 / 14,21 / 14)$
- Interpolate old values
- Final result $7 \times 7$


We did it!


Let's do something interesting already!!

## Want to make image smaller



## $448 x 448$-> 64x64



## $448 x 448$-> 64x64



## $448 x 448$-> 64x64



## $448 x 448$-> 64x64



## $448 x 448$-> 64x64



## $448 x 448$-> 64x64




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## $448 x 448$-> 64x64



## $448 x 448$-> 64x64



## $448 \times 448$-> 64x64



## $448 x 448$-> 64x64



## $448 x 448$-> 64x64



## $448 \times 448$-> 64x64



## IS THIS ALL THERE IS??



## THERE IS A BETTER WAY!



# Next Time: Filtering 

