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Final Study Guide

Curves, surfaces, particle systems

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The final exam will be on Monday, December 11, 2:30pm-4:30pm. It will cover all material up to and including subdivision surfaces. The final will last 120 minutes and will be closed book. These problems are intended to get you to think in some depth about the material that was not covered by the midterm or the homeworks.

Problem 1 A Bézier curve of degree n, which (for the purposes of this problem) we'll denote by $Q^n(u)$, can be defined in terms of the locations of its $n+1$ control points $\{V_0, \ldots, V_n\}$:

$$Q^n(u) \quad = \quad \sum_{i=0}^{n} V_i \begin{pmatrix} n \\ i \end{pmatrix} u^i \, (1-u)^{n-i}$$

a) Use de Casteljau's algorithm to find the (approximate) position of the Bézier curves $Q^3(u)$ and $Q^4(u)$ defined by the two control polygons below at $u = 1/3$:

True or false:

b) Every Bézier curve $Q^1(u)$ is a line segment (assuming no repeated control points).

c) Every Bézier curve $Q^2(u)$ lies in a plane.

d) Moving one control point on a Bézier curve generally changes the whole curve.

Problem 2 More complex curves can be designed by piecing together different Bézier curves to make mathematical "splines." Two popular splines are the B-spline and the Catmull-Rom spline. If $\{B_0, B_1, B_2, B_3\}$ and $\{C_0, C_1, C_2, C_3\}$ are cubic B-spline and Catmull-Rom spline control points, respectively, then the corresponding Bézier control points $\{V_0, V_1, V_2, V_3\}$ can be constructed by the following identity:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 1 & 4 & 1 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 1 \end{bmatrix}\begin{bmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 0 & 6 & 0 & 0 \\ -1 & 6 & 1 & 0 \\ 0 & 1 & 6 & -1 \\ 0 & 0 & 6 & 0 \end{bmatrix}\begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

<u>True or false:</u>

a) B-splines and Catmull-Rom splines both have C^2 continuity.

b) Neither B-splines nor Catmull-Rom splines interpolate their control points.

c) B-splines and Catmull-Rom splines both provide local control.

Problem 2 (continued)

d) The points $\{B_0, B_1, B_2, B_3\}$ below are control points for a cubic B-spline. Construct, as carefully as you can on the diagram below, the Bézier control points $\{V_0, V_1, V_2, V_3\}$ corresponding to the same curve. Do not extend the curve to the endpoints.

e) The points $\{C_0, C_1, C_2, C_3\}$ below are control points for a cubic Catmull-Rom spline. Construct, as carefully as you can on the diagram below, the Bézier control points $\{V_0, V_1, V_2, V_3\}$ corresponding to the same curve. Do not extend the curve to the endpoints.

Problem 3

In class, we described the process of creating subdivision curves by starting with a sequence of splitting and averaging steps, followed by an evaluation mask that sends points to their limit positions. Let's assume that we have the following averaging mask:

$$(r_{-1}, r_0, r_1) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$$

Starting with the control polygon below:

1. Insert the vertices that correspond to the splitting step and label each with an **S**.
2. Apply the averaging mask, indicate the new vertex positions, and label each with an **A**.
3. Indicate the resulting polygon with solid lines.

Problem 4

We described Bézier tensor product surfaces as follows. Given a matrix V of control points V_{ij}, $i, j = 0, \dots n$, we construct a surface $S(u, v)$ by treating the rows of V as control points for Bézier curves $C_0(u), \dots, C_n(u)$, and then treating $C_0(u), \dots, C_n(u)$ as control points for Bézier curves parameterized by v.

a) Suppose that $n = 3$, and consider the resulting surface $S(u, v)$, $0 \leq u \leq 1$, $0 \leq v \leq 1$. Give pseudocode for an adaptive subdivision algorithm for displaying $S(u, v)$. Your algorithm should output quadrilaterals in 3D.

Problem 4 (continued)

b) As part of your pseudocode, you needed to call a function FlatEnough() to decide when to quit subdividing. What is a reasonable definition of this function?

Problem 4 (continued)

c) One problem with using adaptive subdivision for displaying surfaces is that it is possible to inadvertently produce "cracks" between the quadrilaterals. Describe why such cracks occur and how you might fix your algorithm so that it no longer produced them.

d) Write an expression for the normal to a tensor product cubic Bézier surface at $S(u, v)$.

Problem 5

Consider a swept surface defined by a trajectory curve $\mathbf{T}(v) = [x, y, z]^T = [\cos(2\pi v), \sin(2\pi v), 0]^T$ and a profile curve $\mathbf{C}(u) = [x, y]^T = [u, 0]^T$ for $0 \leq u, v \leq 1$.

a) Draw $\mathbf{T}(v)$ and then calculate and draw $\mathbf{T}'(v)$ at some point on the curve.

b) Draw $\mathbf{C}(u)$.

c) (T/F) Consider a Frenet frame positioned on $\mathbf{T}(v)$ as discussed in class. The normal vector n points at the origin.

d) (T/F) Position the profile curve $\mathbf{C}(u)$ in the normal plane of the frame as discussed in class, and vary u and v over their ranges. The surface swept out by C(u) is a sphere.

e) (T/F) There are no inflection points on $\mathbf{T}(v)$ where the curvature $\mathbf{T}''(v)$ goes to 0.

Problem 6

Consider a particle system implemented using only a vector field defined by $\mathbf{v} = \mathbf{h}(x, y, z)$ to control the motion of the particles. Assume that the particles are given a random direction and speed at time 0.

a) (T/F) After one time step, the particles must all be moving in the same direction.

b) (T/F) As time goes on, each particle will reach a maximum speed, depending on its mass.

c) (T/F) A very simple simulation like this can be useful to help visualize the dynamics of a flow field defined by $\mathbf{h}(t)$.