Hierarchical Modeling

Reading

- Angel, *Interactive Computer Graphics*, sections 8.1 - 8.6

Optional

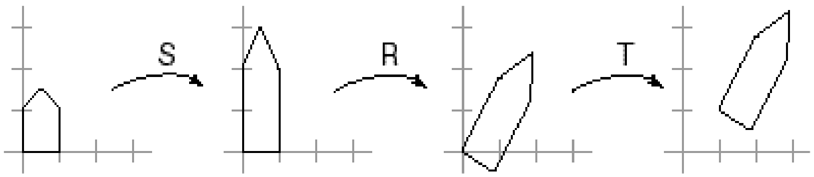
- Foley, *Computer Graphics, Chapter 5.*
- *OpenGL Programming Guide*, chapter 3

Symbols and instances

Most graphics APIs support a few geometric **primitives**:

- spheres
- cubes
- cylinders

These symbols are **instanced** using an **instance transformation**.



Q: What is the matrix for the instance transformation above?

Instancing in OpenGL

In OpenGL, instancing is created by modifying the **model-view** matrix:

```
glMatrixMode( GL_MODELVIEW );
glLoadIdentity();
glTranslatef( ... );
glRotatef( ... );
glScalef( ... );
house();
```

Do the transforms seem to be backwards? Why was OpenGL designed this way?

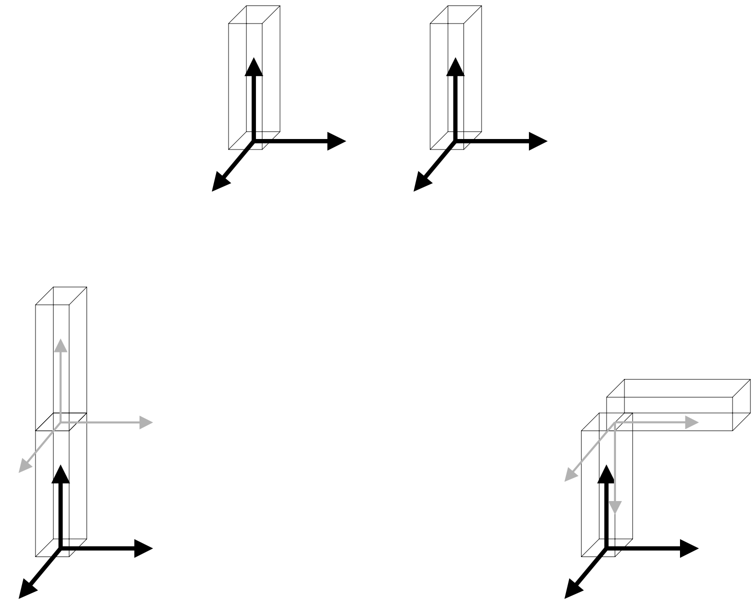
Instancing in real OpenGL

The advantage of right-multiplication is that it places the *earlier* transforms *closer* to the primitive.

```
glPushMatrix();
glTranslate( ... );
glRotate( ... );
house();
glPopMatrix();

glPushMatrix();
glTranslate( ... );
glRotate( ... );
house();
glPopMatrix();
```

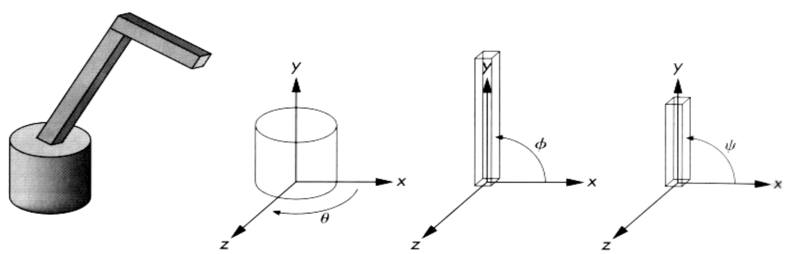
Connecting Primitives



3D Example: A robot arm

Consider this robot arm with 3 degrees of freedom:

- Base rotates about its vertical axis by θ
- Lower arm rotates in its *xy*-plane by ϕ
- Upper arm rotates in its *xy*-plane by ψ



Q: What matrix do we use to transform the base?

Q: What matrix for the lower arm?

Q: What matrix for the upper arm?

Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

```
Matrix M_model;
main()
{
    . . .
    robot_arm();
    . . .
}
robot_arm()
{
    M_model = R_y(theta);
    base();
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi);
    upper_arm();
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi)
                          *T(0,h2,0)*R_z(psi);
    lower_arm();
}
```

Do the matrix computations seem wasteful?

Robot arm implementation, better

Instead of recalculating the global matrix each time, we can just update it *in place*:

```
Matrix M_model;
main()
{
    . . .
    M_model = Identity();
    robot_arm();
    . . .
}
robot_arm()
{
    M_model *= R_y(theta);
    base();
    M_model *= T(0,h1,0)*R_z(phi);
    upper_arm();
    M_model *= T(0,h2,0)*R_z(psi);
    lower_arm();
}
```

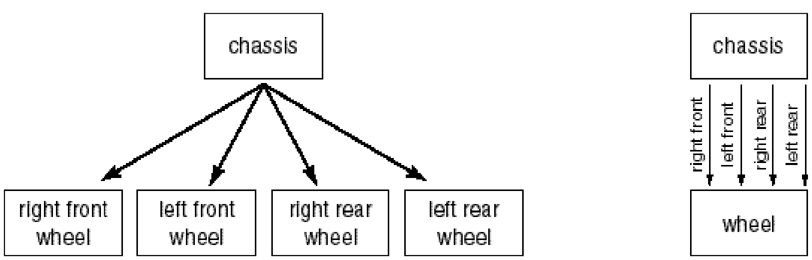
Robot arm implementation, OpenGL

OpenGL maintains a global state matrix called the **model-view matrix**.

```
main()
{
    . . .
    glMatrixMode( GL_MODELVIEW );
    glLoadIdentity();
    robot_arm(a, b, c);
    . . .
}
robot_arm(theta, phi, psi)
{
    glRotatef( theta, 0.0, 1.0, 0.0 );
    base();
    glTranslatef( 0.0, h1, 0.0 );
    glRotatef( phi, 0.0, 0.0, 1.0 );
    lower_arm();
    glTranslatef( 0.0, h2, 0.0 );
    glRotatef( psi, 0.0, 0.0, 1.0 );
    upper_arm();
}
```

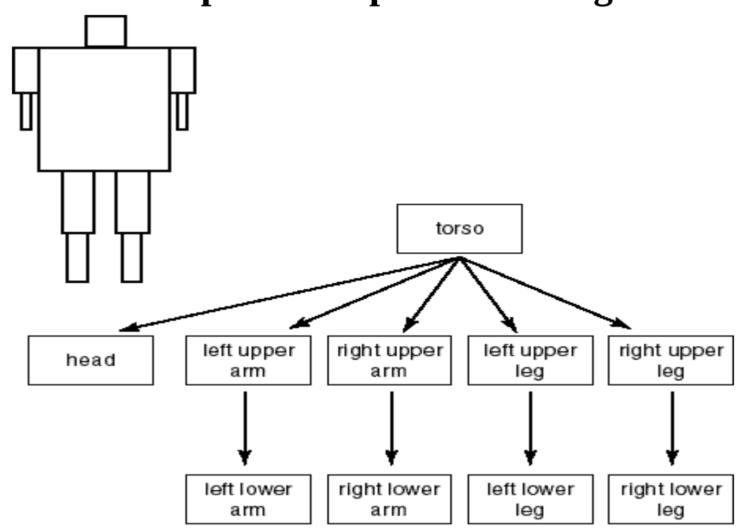
Hierarchical modeling

Hierarchical models can be composed of instances using trees or DAGs:



- edges contain geometric transformations
- nodes contain geometry (and possibly drawing attributes)

A complex example: human figure



Q: What's the most sensible way to traverse this tree?

Human figure implementation

The traversal can be implemented by saving the model-view matrix on a stack:

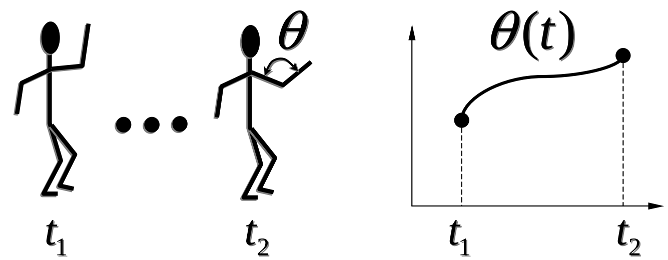
```
figure()
{
    glPushMatrix();
        glTranslate( ... );
        glRotate( ... );
        torso();
        glPushMatrix();
            glTranslate( ... );
            glRotate( ... );
            head();
        glPopMatrix();
        glPushMatrix();
            glTranslate( ... );
            glRotate( ... );
            left_upper_leg();
        glPopMatrix();
        . . .
    glPopMatrix();
}
```

Animation

The above examples are called **articulated models**:

- rigid parts
- connected by joints

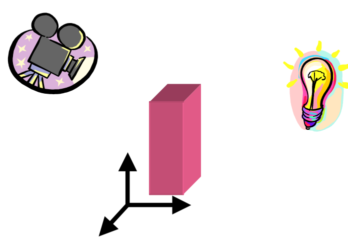
They can be animated by specifying the joint angles (or other display parameters) as functions of time.



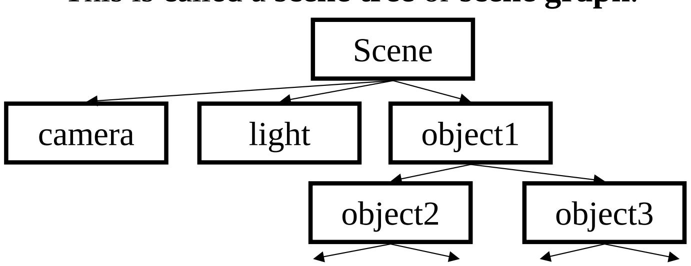
Scene graphs

The idea of hierarchical modeling can be extended to an entire scene, encompassing:

- many different objects
- lights
- camera position



This is called a **scene tree** or **scene graph**.

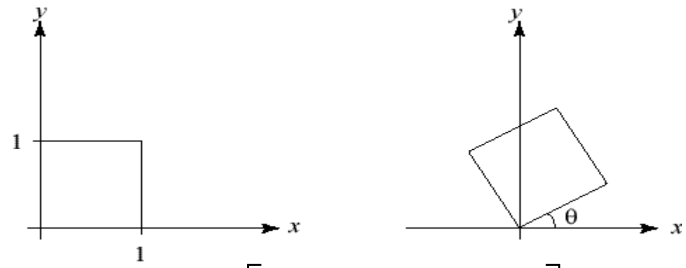


Summary

Here's what you should take home from this lecture:

- How primitives can be instanced and composed to create hierarchical models using geometric transforms.
- How transforms can be thought of as affecting either the geometry, or the coordinate system which it is drawn in.
- How the notion of a model tree or DAG can be extended to entire scenes.

Inverse Rotation



$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$R^{-1}(\theta) = ?$$

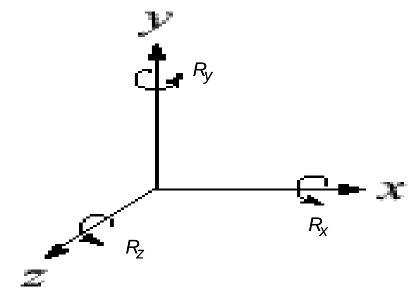
Rotation in 3D

What about the inverses of 3D rotations?

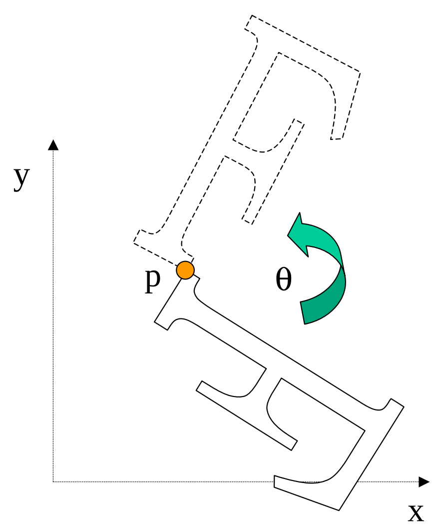
$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

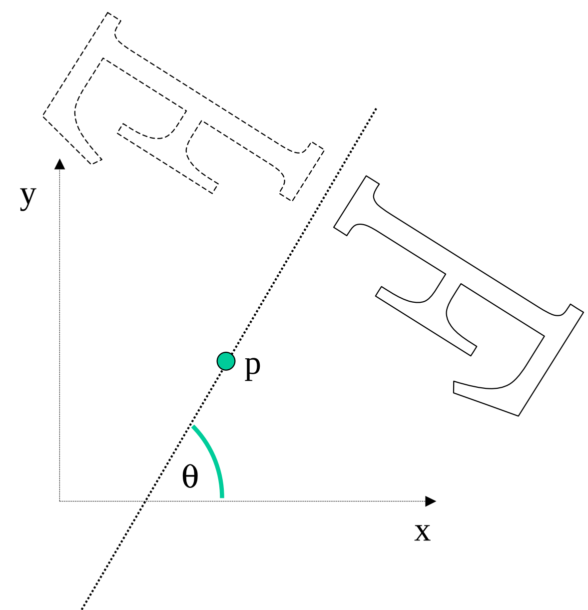
$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



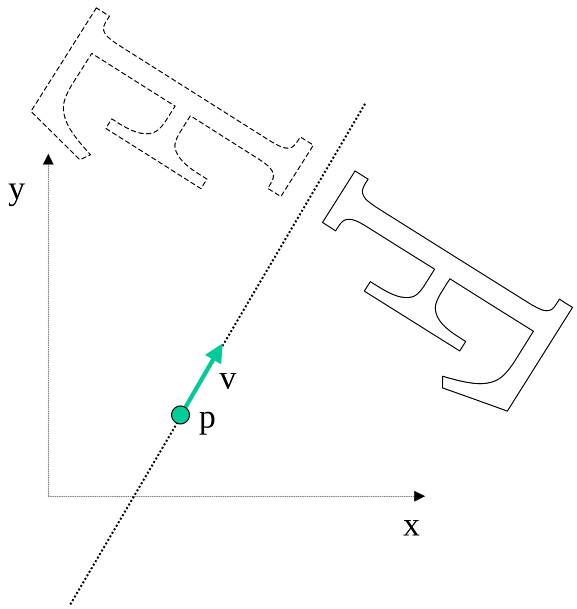
Rotation around arbitrary point



Reflection around arbitrary axis



Reflection around arbitrary axis



Rotation that aligns 3 orthonormal vectors with the principal axes