# Projections

CSE 457, Autumn 2003

Graphics

http://www.cs.washington.edu/education/courses/457/03au/



# Readings and References

Readings

- Sections 5.2.2 – 5.2.4, *3D Computer Graphics*, Watt

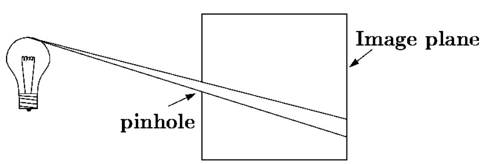
Other References

- Chapter 5.6 and Chapter 6, *Computer Graphics, Principles and Practice*, Foley, van Dam



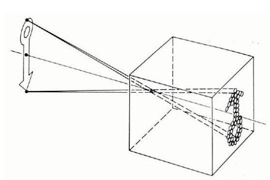
# The pinhole camera

- The first camera - "camera obscura" - known to Aristotle.




Pinhole camera

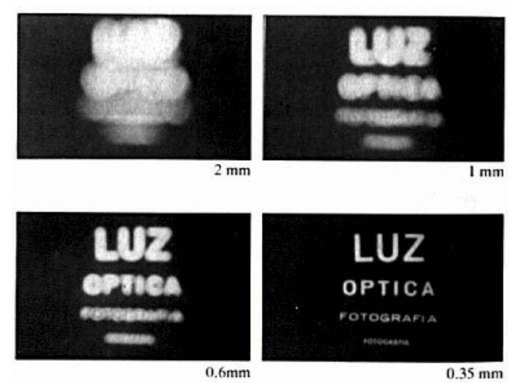
- In 3D, we can visualize the blur induced by the pinhole (a.k.a., **aperture**):



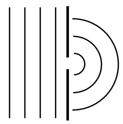
- **Q**: How would we reduce blur?



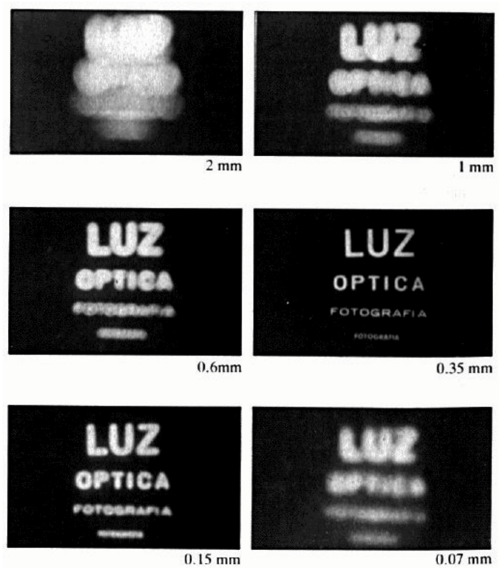
# Shrinking the pinhole




- **Q**: What happens as we continue to shrink the aperture?



Diffraction



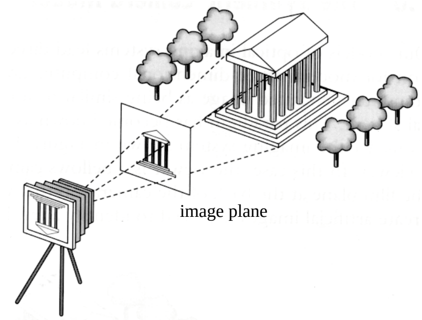

# Imaging with the synthetic camera

- In practice, pinhole cameras require long exposures, can suffer from diffraction effects, and give an inverted image.
- In graphics, none of these physical limitations is a problem.

The image is rendered onto an **image plane** (usually in front of the "camera").

Viewing rays emanate from the **center of projection** (COP) at the center of the pinhole.

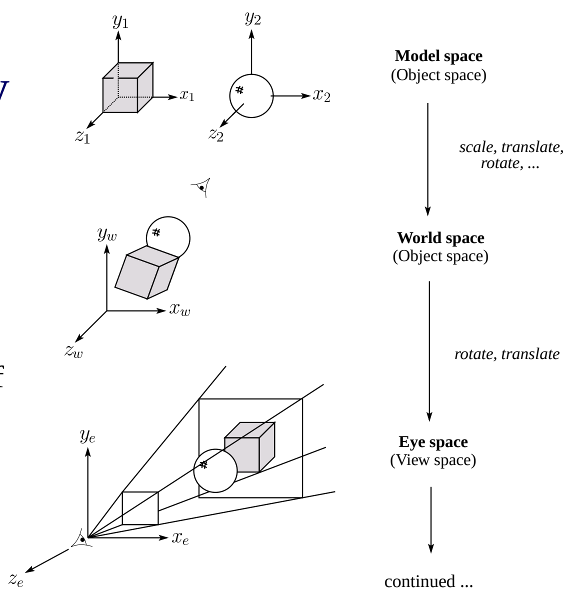
The image of an object point *P* is at the intersection of the viewing ray through *P* and the image plane.

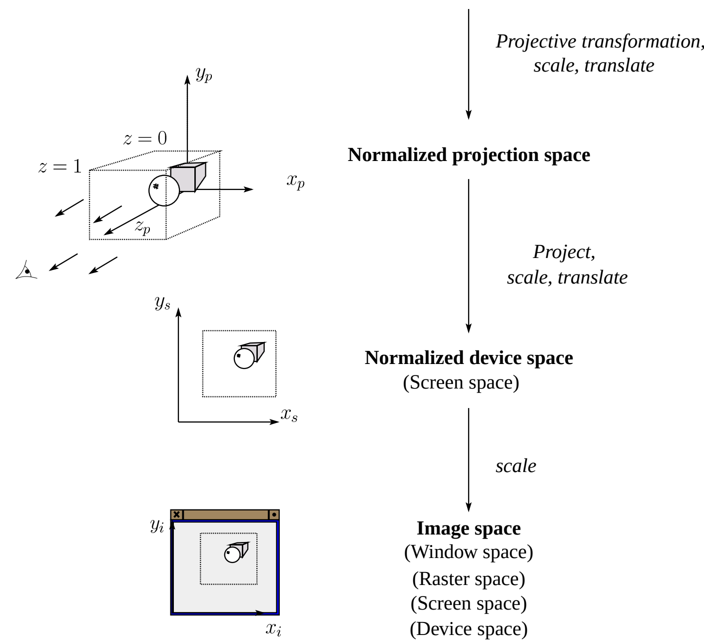





# 3D Geometry Pipeline

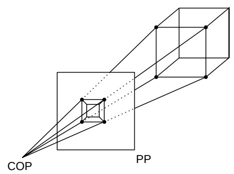
Before being turned into pixels, a piece of geometry goes through a number of transformations...

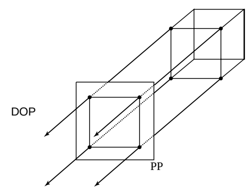


# Projections

- **Projections** transform points in *n*-space to *m*-space, where $m<n$.
- In 3-D, we map points from 3-space to the **projection plane** (PP) (a.k.a., image plane) along **projectors** (a.k.a., viewing rays) emanating from the center of projection (COP):




- There are two basic types of projections:
  - » Perspective – distance from COP to PP finite
  - » Parallel – distance from COP to PP infinite



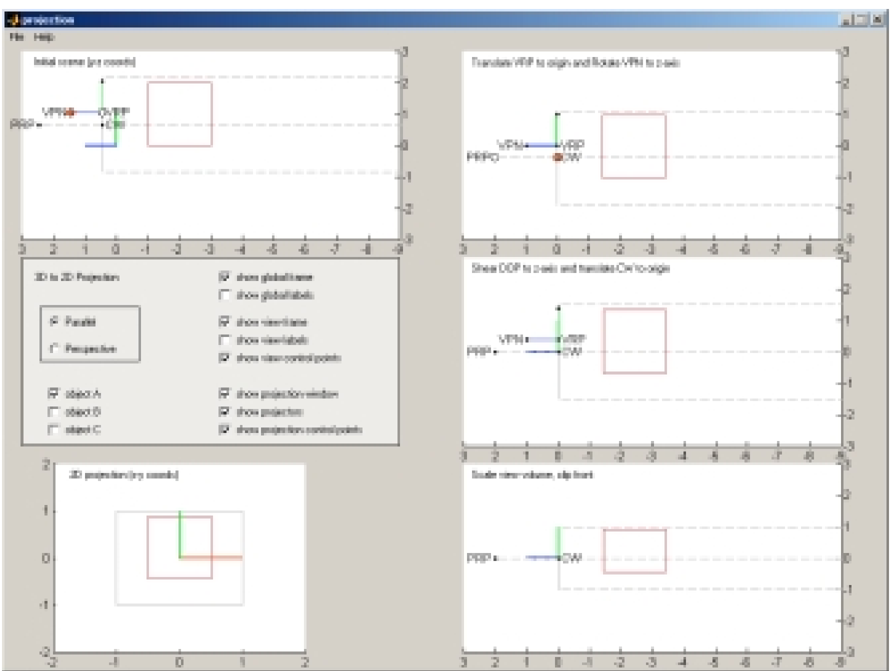
# Parallel projections

- For parallel projections, we specify a **direction of projection** (DOP) instead of a COP.
- There are two types of parallel projections:
  - » **Orthographic projection** – DOP perpendicular to PP
  - » **Oblique projection** – DOP not perpendicular to PP
- Orthographic projections along the z-axis in 3D or to 2D are easy

$$\begin{bmatrix} x' \\ y' \\ k \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & k \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

  - » We can use shear to line things up when doing an oblique projection
  - » We often keep the initial z value around for later use. Why?





# Properties of parallel projection

- Properties of parallel projection:
  - » Not realistic looking
  - » Good for exact measurements
  - » Are actually a kind of affine transformation
    - Parallel lines remain parallel
    - Ratios are preserved
    - Angles not (in general) preserved
  - » Most often used in CAD, architectural drawings, etc., where taking exact measurement is important

# Some oblique projections

Escaping Flatland is one of a series of sculptures by Edward Tufte

http://www.edwardtufte.com/tufte/sculpture

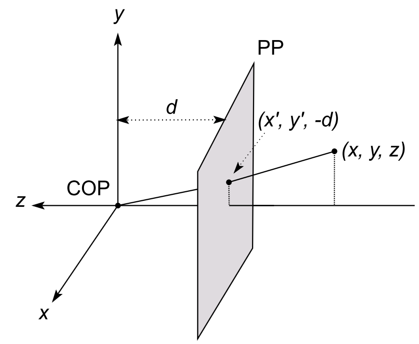
Larry Kagan:Wall Sculpture in Steel and Shadow

http://www.arts.rpi.edu/~kagan/



# Derivation of perspective projection

- Consider the projection of a point onto the projection plane:




By similar triangles, we can compute how much the *x* and *y* coordinates are scaled:

Watt uses a left-handed coordinate system, and he looks down the $+z$ axis, so his image plane is at $+d$.



# Perspective projection

- The perspective projection as a matrix equation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1}{d} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} x \\ y \\ \frac{-z}{d} \end{bmatrix}$$

- But remember we said that affine transformations work with the last coordinate always set to one.
  - » How can we bring this back to w'=1? Divide!
  - » This division step is the "perspective divide."

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{x}{z}d \\ -\frac{y}{z}d \\ 1 \end{bmatrix}$$

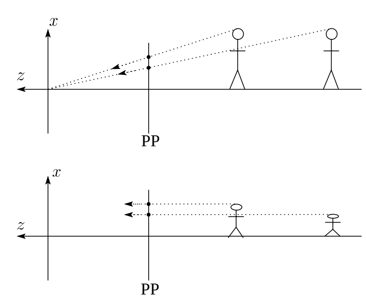
Again, projection implies dropping the *z* coordinate to give a 2D image, but we usually keep it around a little while longer.

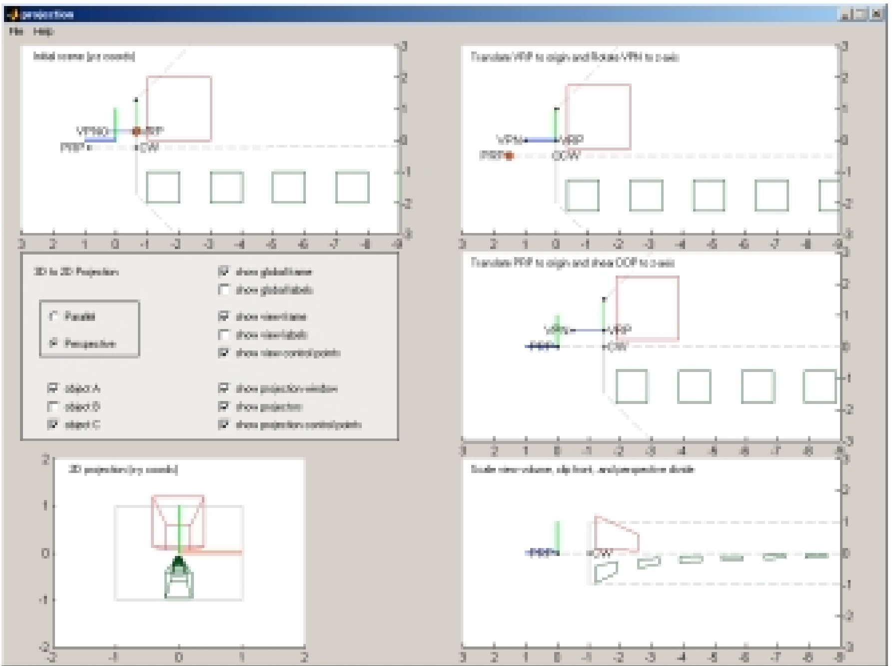


# Projective normalization

- After applying the perspective transformation and dividing by *w*, we are free to do a simple parallel projection to get the 2D image.

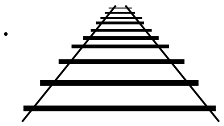
What does this imply about the shape of things after the perspective transformation + divide?





# Vanishing points

- What happens to two parallel lines that are not parallel to the projection plane?
- Think of train tracks receding into the horizon...



- The equation for a line is:

$$\mathbf{l} = \mathbf{p} + t\mathbf{v} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} + t \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}$$

- After perspective transformation we get:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} p_x + tv_x \\ p_y + tv_y \\ -(p_z + tv_z)/d \end{bmatrix}$$



# Vanishing points (cont'd)

- Dividing by $w$:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} -\frac{p_x + tv_x}{p_z + tv_z} d \\ -\frac{p_y + tv_y}{p_z + tv_z} d \\ 1 \end{bmatrix}$$

- Letting $t$ go to infinity:
- We get a point!
- What happens to the line $\mathbf{l} = \mathbf{q} + t\mathbf{v}$?
- Each set of parallel lines intersect at a **vanishing point** on the Projection Plane.
- **Q**: How many vanishing points are there?

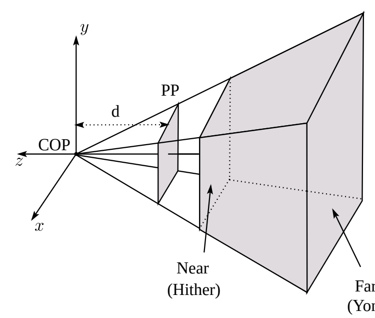


# Clipping and the viewing frustum

- The center of projection and the portion of the projection plane that map to the final image form an infinite pyramid. The sides of the pyramid are **clipping planes**.

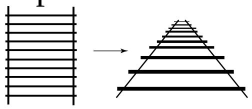
Frequently, additional clipping planes are inserted to restrict the range of depths. These clipping planes are called the **near** and **far** or the **hither** and **yon** clipping planes.

All of the clipping planes bound the **viewing frustum**.



# Properties of perspective projections

- The perspective projection is an example of a **projective transformation**.

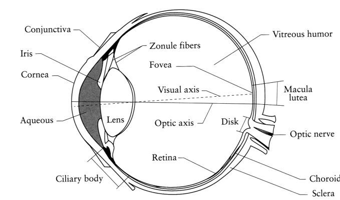


- Some properties of projective transformations:
  - » Lines map to lines
  - » Parallel lines do not necessarily remain parallel
  - » Ratios are not preserved
- An advantage of perspective projection is that size varies inversely with distance – looks realistic.
- A disadvantage is that we can't judge distances as exactly as we can with parallel projections.



# Human vision and perspective

- The human visual system uses a lens to collect light more efficiently, but records perspectively projected images much like a pinhole camera.

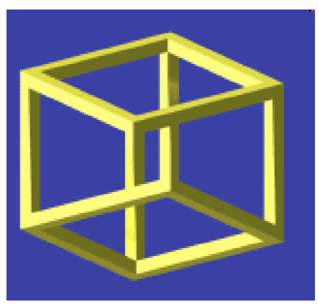



- **Q**: Why did nature give us eyes that perform perspective projections?
  - » How would you construct a vision system that did parallel projections?
- **Q**: Do our eyes "see in 3D"?



# Some View Transformations


http://www.ntv.co.jp/kasoh/past_movie/contents.html



http://www.cs.technion.ac.il/~gershon/EscherForReal/



# Summary

- What to take away from this lecture:
  - » All the boldfaced words.
  - » An appreciation for the various coordinate systems used in computer graphics.
  - » How the perspective transformation works.
  - » How we use homogeneous coordinates to represent perspective projections.
  - » The classification of different types of projections.
  - » The concept of vanishing points
  - » The mathematical properties of projective transformations.