Ray Tracing

CSE 457, Autumn 2003

Graphics

http://www.cs.washington.edu/education/courses/457/03au/

Readings and References

Readings

- Sections 1.3-1.4, 12.1-12.5.1, *3D Computer Graphics*, Watt
- Watt Errata (link on syllabus page and Trace extensions page)

Other References

- A. Glassner. An Introduction to Ray Tracing. 1989.
- T. Whitted. An improved illumination model for shaded display. Communications of the ACM 23(6), 343-349, 1980.
  - » http://portal.acm.org/citation.cfm?id=358882&dl=ACM&coll=GUIDE
- K. Turkowski, “Properties of Surface Normal Transformations,” Graphics Gems, 1990, pp. 539-547.

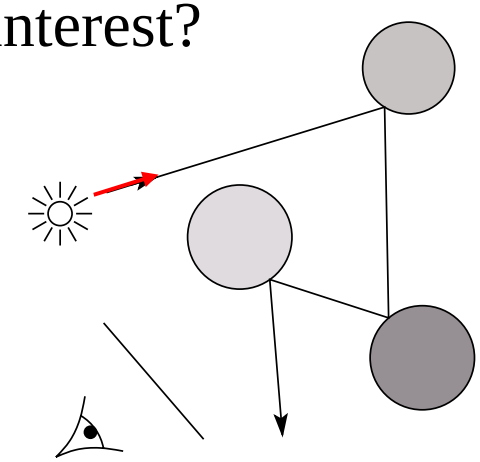
Geometric optics

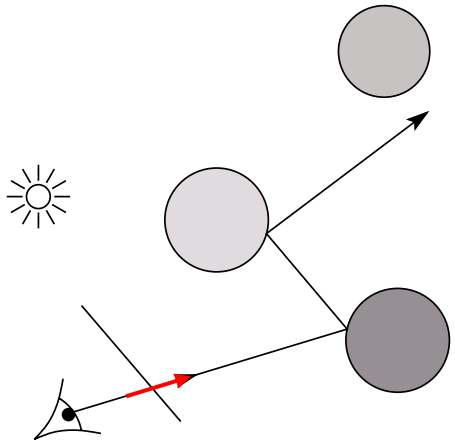
- Modern theories of light treat it as both a wave and a particle.
- We will take a combined and somewhat simpler view of light
  - » the view of **geometric optics**.
- Here are the rules of geometric optics:
  - » Light is a flow of photons with wavelengths. We'll call these flows "light rays."
  - » Light rays travel in straight lines in free space.
  - » Light rays do not interfere with each other as they cross.
  - » Light rays obey the laws of reflection and refraction.
  - » Light rays travel form the light sources to the eye, but the physics is invariant under path reversal (reciprocity).

Eye vs. light ray tracing

Where does a light ray begin its journey of interest?

At the light: light ray tracing
(a.k.a., forward ray tracing
or photon tracing)

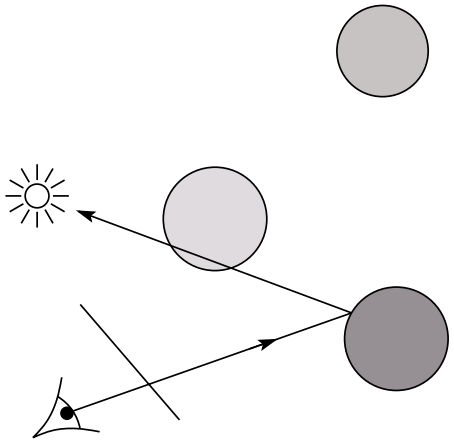




At the eye: eye ray tracing
(a.k.a., backward ray tracing)

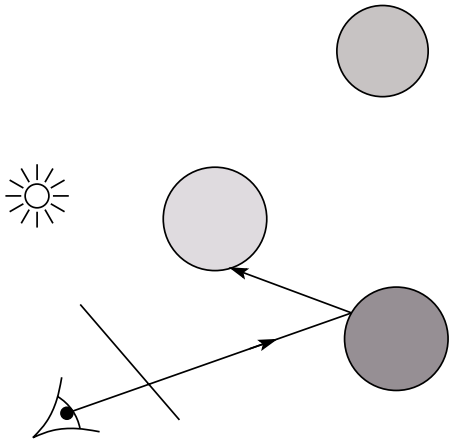
We will generally follow rays
from the eye into the scene.

Precursors to ray tracing



Cast one eye ray, then shade according to light

Local illumination



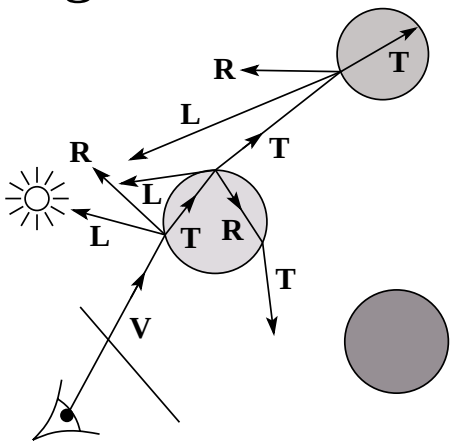
Cast one eye ray + one ray to light

Appel (1968)

Whitted ray-tracing algorithm

In 1980, Turner Whitted introduced ray tracing to the graphics community.

- » Combines eye ray tracing + rays to light
- » Recursively traces rays



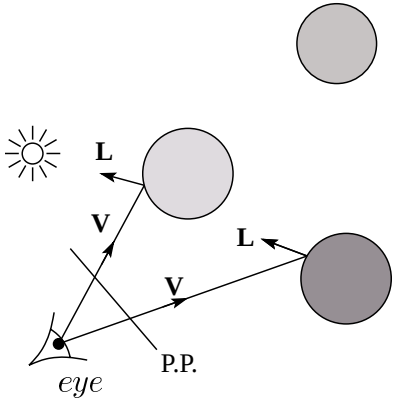
Algorithm:

For each pixel, trace a **primary ray** in direction **V** to the first visible surface.

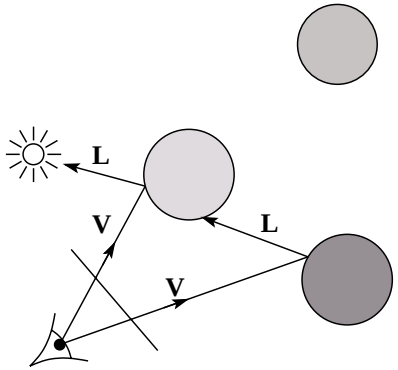
For each intersection, trace **secondary rays**:

- » **Shadow rays** in directions $\mathbf{L_i}$ to light sources
- » **Reflected ray** in direction **R**.
- » **Refracted ray** or **transmitted ray** in direction **T**.

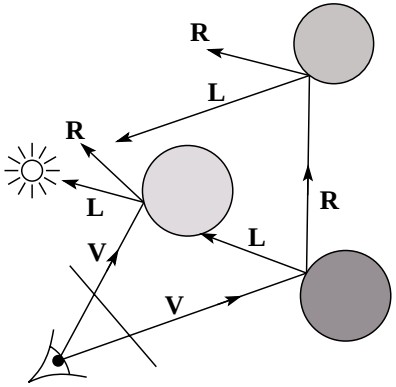
All the Rays



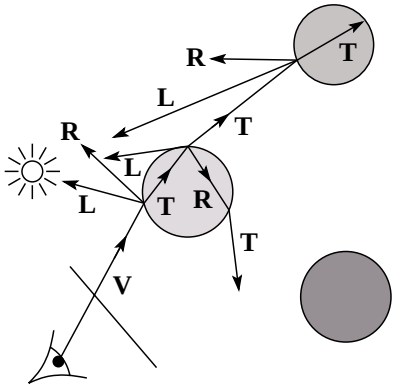
Primary rays



Shadow rays



Reflection rays



Refracted rays

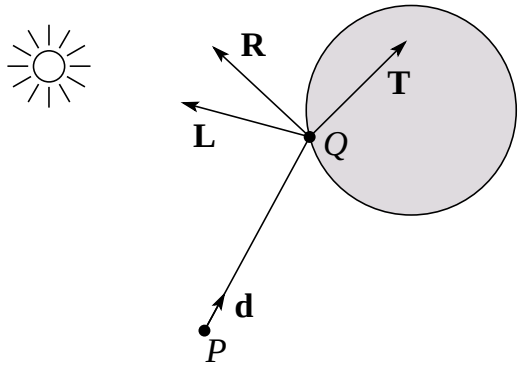
Shading

- A ray is defined by an origin **p** and a unit direction **d** and is parameterized by t: $P + t\mathbf{d}$
- Let $I(P, \mathbf{d})$ be the intensity seen along that ray. Then:

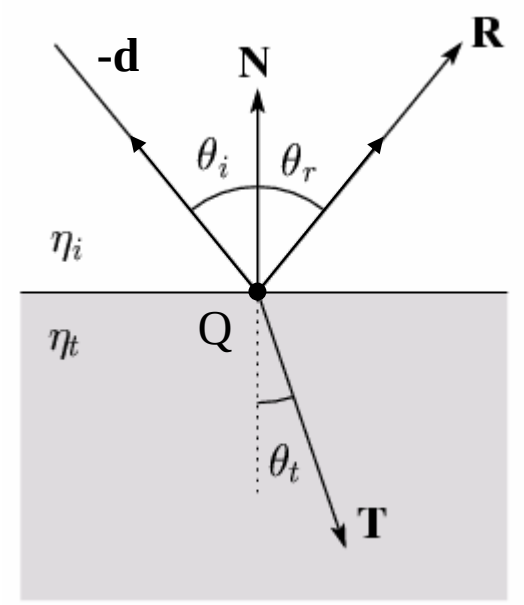
$$I(P, \mathbf{d}) = I_{\text{direct}} + I_{\text{reflected}} + I_{\text{transmitted}}$$

where

  - » I_{direct} is computed from the Phong model using **L**
  - » $I_{\text{reflected}} = k_r I(Q, \mathbf{R})$
  - » $I_{\text{transmitted}} = k_t I(Q, \mathbf{T})$



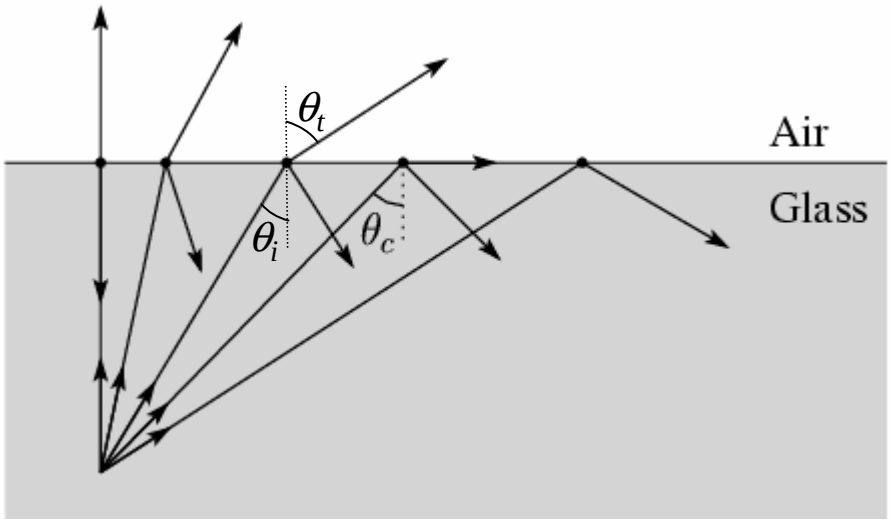
Reflection and transmission



- Law of reflection:
  - » $\theta_i = \theta_r$
- Snell's law of refraction:
  - » $\eta_i \sin\theta_i = \eta_t \sin\theta_t$
  - » where η_i , η_t are **indices of refraction**.

Total Internal Reflection

The equation for the angle of refraction can be computed from Snell's law:



What happens when $\eta_i > \eta_t$?

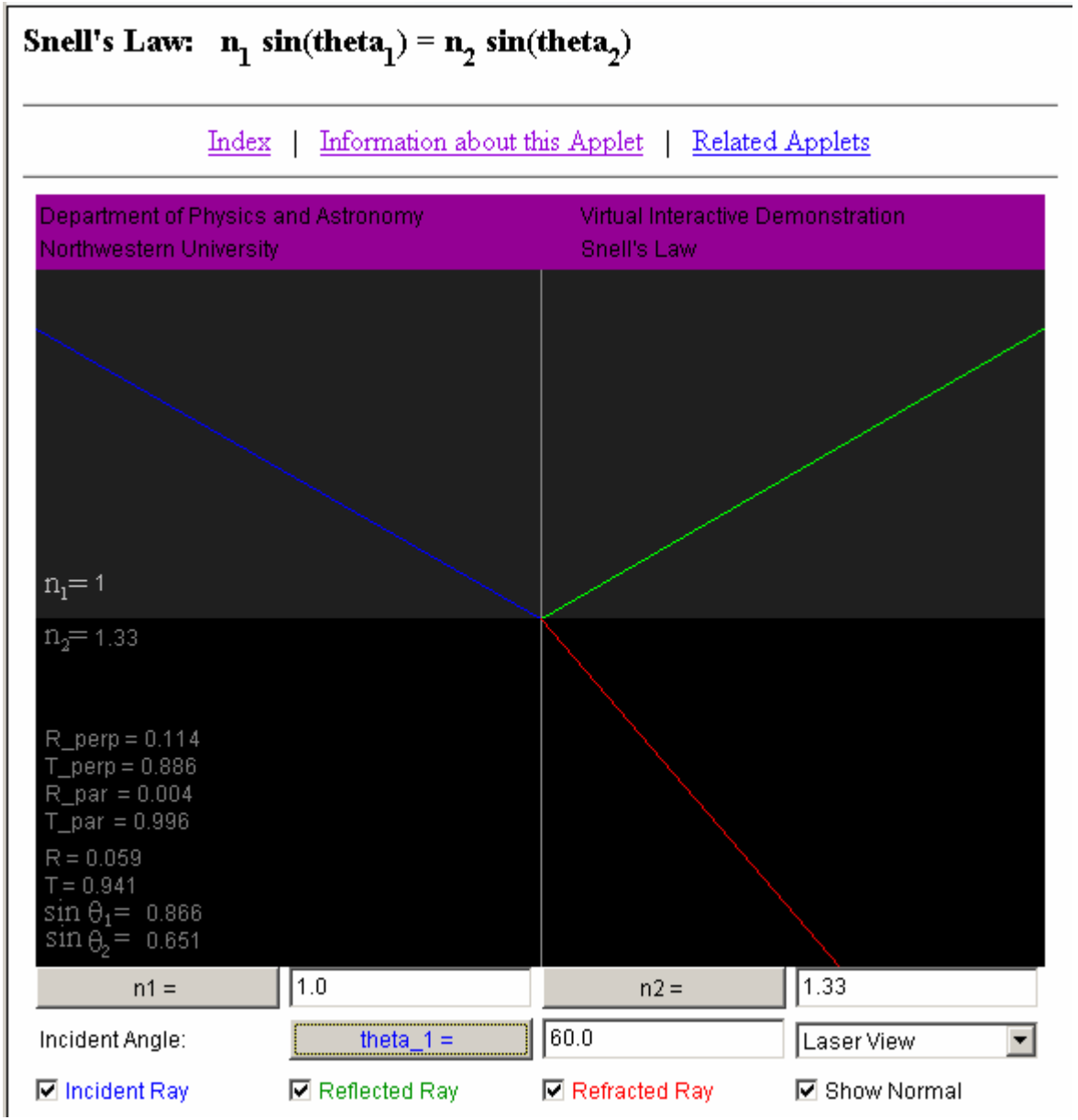
When θ_t is exactly 90°, we say that θ_i has achieved the "critical angle" θ_c.

For $\theta_I > \theta_c$, *no rays are transmitted*, and only reflection occurs, a phenomenon known as "total internal reflection" or TIR.

Refraction

material	*index*
Vacuum	1
Air	1.0003
Water	1.33
Ethyl Alcohol	1.36
Fused Quartz	1.4585
Whale Oil	1.46
Crown Glass	1.52
Salt	1.54
Asphalt	1.635
Heavy Flint Glass	1.65
Diamond	2.42
Lead	2.6

Values come from the CRC Handbook of Chemistry and Physics



Error in Watt!

- In order to compute the refracted direction, it is useful to compute the cosine of the angle of refraction in terms of the incident angle and the ratio of the indices of refraction.
- On page 24 of Watt, he develops a formula for computing this cosine. Notationally, he uses μ instead of η for the index of refraction in the text, but uses η in Figure 1.16, and the angle of incidence is ϕ and the angle of refraction is θ.
- Unfortunately, he makes an error in computing $\cos\theta$.
- The last equation on page 24 should read:

$$\cos\theta = \sqrt{1 - \mu^2(1 - \cos^2\phi)}$$

- See the errata for corrections that you can write into your books.

Ray-tracing pseudocode

We build a ray traced image by casting rays through each of the pixels.

function *traceImage* (scene):
    for each pixel (i,j) in image
        S = *pixelToWorld*(i,j)
        P = **COP**
        d = (S - P)/|| S – P||
        I(i,j) = *traceRay*(scene, P, **d**)
    end for
end function

Ray-tracing pseudocode, traceRay

```
function traceRay(scene, P, d):
    (t, N, mtrl) ← scene.intersect (P, d)
    Q ← ray (P, d) evaluated at t
    I = shade(                              )
    R = reflectDirection(                )
    I ← I + mtrl.k_r ∗ traceRay(scene, Q, R)
    if ray is entering object then
        n_i = index_of_air
        n_t = mtrl.index
    else
        n_i = mtrl.index
        n_t = index_of_air
    if (notTIR (                                    )) then
        T = refractDirection (                          )
        I ← I + mtrl.k_t ∗ traceRay(scene, Q, T)
    end if
    return I
end function
```

Terminating recursion

- **Q**: How do you bottom out of recursive ray tracing?
- Possibilities:

Shading pseudocode

Next, we need to calculate the color returned by the *shade* function.

function *shade*(mtrl, scene, *Q*, **N**, **d**):
    I ← mtrl.k_e + mtrl. k_a * scene->I_a
    for each light source λ **do**:
        atten = λ -> *distanceAttenuation*( ) *
            λ -> *shadowAttenuation*( )
        I ← I + atten*(diffuse term + spec term)
    end for
    return I
end function

Shadow attenuation

Computing a shadow can be as simple as checking to see if a ray makes it to the light source. For a point light source:

function *PointLight*::*shadowAttenuation(*scene, *P)*
    d = (this.position - *P*)**.***normalize*()
    (t, **N**, mtrl) ← scene.*intersect*(*P*, **d**)
    Q ← ray(t)
    if *Q* is before the light source **then**:
        atten = 0
    else
        atten = 1
    end if
    return atten
end function

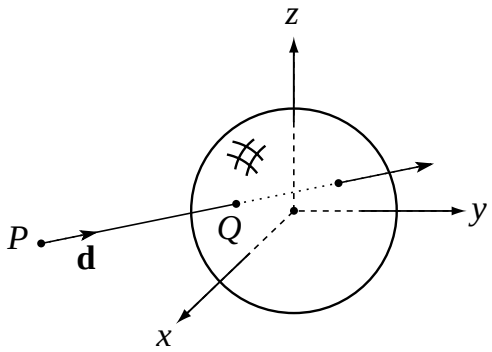
Q: What if there are transparent objects along path to the light source?

Intersecting rays with spheres

Given:

» The coordinates of a point along a ray passing through *P* in the direction **d** are:

$$x = P_x + td_x$$
$$y = P_y + td_y$$
$$z = P_z + td_z$$



» A unit sphere *S* centered at the origin defined by the equation:

Find: The *t* at which the ray intersects *S*.

Intersecting rays with spheres

Solution by substitution:

$$x^2 + y^2 + z^2 - 1 = 0$$

$$(P_x + td_x)^2 + (P_y + td_y)^2 + (P_z + td_z)^2 - 1 = 0$$

$$at^2 + bt + c = 0$$

where $a = d_x^2 + d_y^2 + d_z^2$

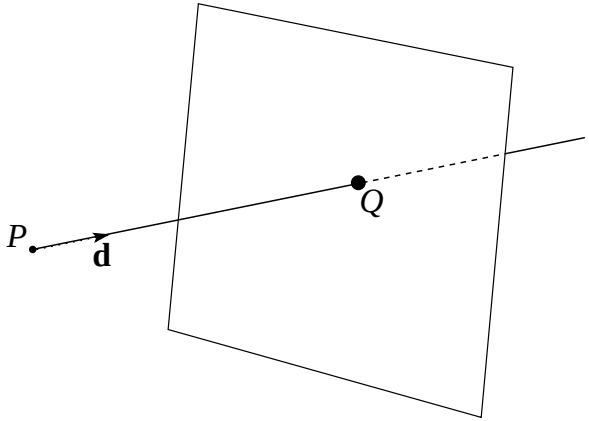
$b = 2(P_x d_x + P_y d_y + P_z d_z)$

$c = P_x^2 + P_y^2 + P_z^2 - 1$

Q: What are the solutions of the quadratic equation in *t* and what do they mean?

Q: What is the normal to the sphere at a point (*x*,*y*,*z*) on the sphere?

Ray-plane intersection



- We can write the equation of a plane as:

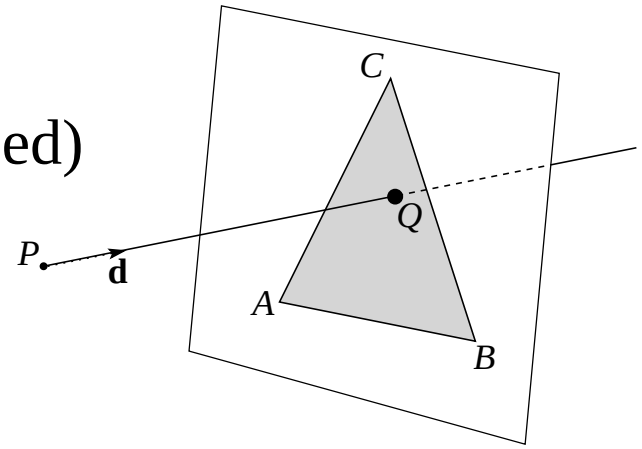
$$ax + by + cz + d = 0$$

- The coefficients *a*, *b*, and *c* form a vector that is normal to the plane, $\mathbf{n} = [a\ b\ c]^T$. Thus, we can re-write the plane equation as:

- We can solve for the intersection parameter (and thus the point):

Ray-triangle intersection

- To intersect with a triangle, we first solve for the equation of its supporting plane.
- How might we compute the (un-normalized) normal?

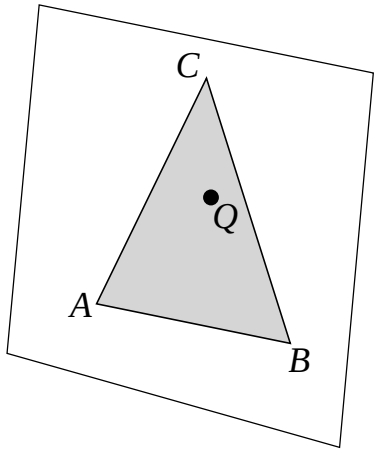


- Given this normal, how would we compute *d*?

- Using these coefficients, we can solve for *Q*. Now, we need to decide if *Q* is inside or outside of the triangle.

3D inside-outside test

- One way to do this “inside-outside test,” is to see if *Q* lies on the left side of each edge as we move counterclockwise around the triangle.



- How might we use cross products to do this?

2D inside-outside test

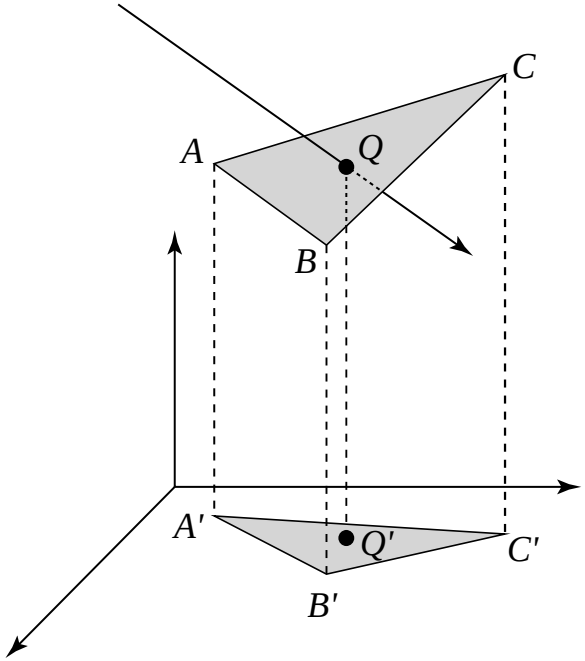
Without loss of generality, we can perform this same test after projecting down a dimension:

If *Q'* is inside of *A'B'C'*, then *Q* is inside of *ABC*.

Why is this projection desirable?

Which axis should you "project away"?

How do you easily select that axis?



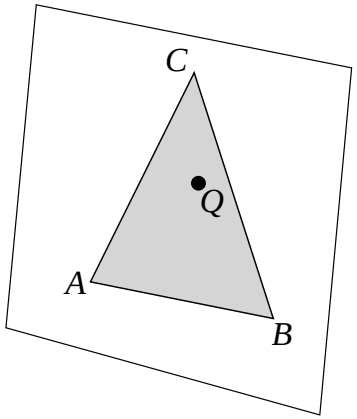
Barycentric coordinates

- It is often useful to represent Q as an **affine combination** of A, B, and C:

$$Q = \alpha A + \beta B + \gamma C$$

- where:

$$\alpha + \beta + \gamma = 1$$

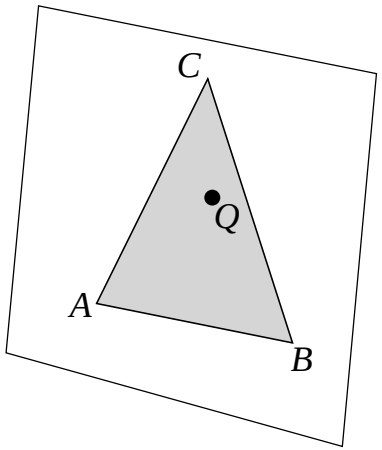


- We call α, β, and γ, the **barycentric coordinates** of Q with respect to A, B, and C.

Barycentric coordinates

- Given a point Q that is inside of triangle ABC, we can solve for Q's barycentric coordinates in a simple way:

$$\alpha = \frac{Area(QBC)}{Area(ABC)}, \beta = \frac{Area(AQC)}{Area(ABC)}, \gamma = \frac{Area(ABQ)}{Area(ABC)}$$



- How can cross products help here?

- In the end, these calculations can be performed in the 2D projection as well!

Interpolating vertex properties

- The barycentric coordinates can also be used to interpolate vertex properties such as:
  - » material properties
  - » texture coordinates
  - » normals
- For example:

$$k_d(Q) = \alpha k_d(A) + \beta k_d(B) + \gamma k_d(C)$$

- Interpolating normals, known as Phong interpolation, gives triangle meshes a smooth shading appearance.

Epsilons

- Due to finite precision arithmetic, we do not always get the exact intersection at a surface.
- **Q**: What kinds of problems might this cause?

- **Q**: How might we resolve this?

Intersecting with xformed geometry

- In many cases, objects such as spheres, cylinders, and boxes will be placed using transformations. What if the object being intersected were transformed by a matrix M?
- Apply M^{-1} to the ray first and intersect in object (local) coordinates

Intersecting with xformed geometry

- The intersected normal is in object (local) coordinates. How do we transform it to world coordinates?

Summary

- What to take home from this lecture:
  - » The meanings of all the boldfaced terms.
  - » Enough to implement basic recursive ray tracing.
  - » How reflection and transmission directions are computed.
  - » How ray--object intersection tests are performed on spheres, planes, and triangles
  - » How barycentric coordinates within triangles are computed
  - » How ray epsilons are used.
  - » How intersections with transformed geometry are done