Orientation

Orientation

- We will define 'orientation' to mean an object's instantaneous rotational configuration
- Think of it as the rotational equivalent of position

Representing Positions

- Cartesian coordinates (x,y,z) are an easy and natural means of representing a position in 3D space
- There are many other alternatives such as polar notation (r,θ,φ) and you can invent others if you want to

Representing Orientations

- Is there a simple means of representing a 3D orientation? (analogous to Cartesian coordinates?)
- Not really.
- There are several popular options though:
  - Euler angles
  - Rotation vectors (axis/angle)
  - 3x3 matrices
  - Quaternions
  - and more…

Euler's Theorem

- Euler's Theorem: Any two independent orthonormal coordinate frames can be related by a sequence of rotations (not more than three) about coordinate axes, where no two successive rotations may be about the same axis.
- Not to be confused with Euler angles, Euler integration, Newton-Euler dynamics, inviscid Euler equations, Euler characteristic…
- Leonard Euler (1707-1783)

Euler Angles

- This means that we can represent an orientation with 3 numbers
- A sequence of rotations around principal axes is called an *Euler Angle Sequence*
- Assuming we limit ourselves to 3 rotations without successive rotations about the same axis, we could use any of the following 12 sequences:

XYZ	XZY	XYX	XZX
YXZ	YZX	YXY	YZY
ZXY	ZYX	ZXZ	ZYZ

Euler Angles

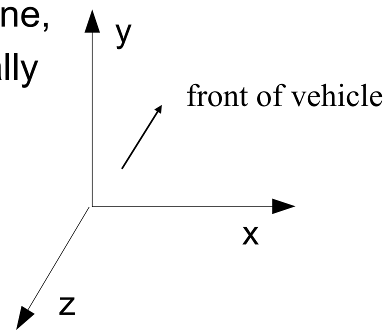
- This gives us 12 redundant ways to store an orientation using Euler angles
- Different industries use different conventions for handling Euler angles (or no conventions)

Using Euler Angles

- To use Euler angles, one must choose which of the 12 representations they want
- There may be some practical differences between them and the best sequence may depend on what exactly you are trying to accomplish

Vehicle Orientation

- Generally, for vehicles, it is most convenient to rotate in roll (z), pitch (x), and then yaw (y)
- In situations where there is a definite ground plane, Euler angles can actually be an intuitive representation



Gimbal Lock

- One potential problem that they can suffer from is 'gimbal lock'
- This results when two axes effectively line up, resulting in a temporary loss of a degree of freedom
- This is related to the singularities in longitude that you get at the north and south poles

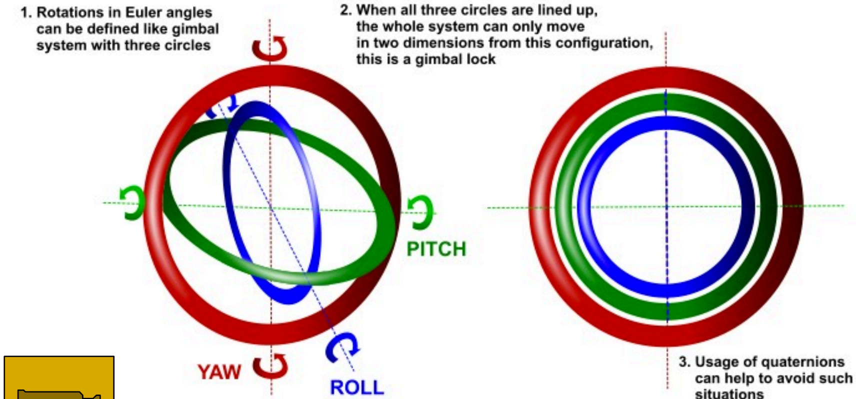
Interpolating Euler Angles

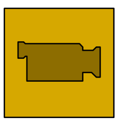
- One can simply interpolate between the three values independently
- This will result in the interpolation following a different path depending on which of the 12 schemes you choose
- This may or may not be a problem, depending on your situation
- Interpolating near the 'poles' can be problematic
- Note: when interpolating angles, remember to check for crossing the +180/-180 degree boundaries

Euler Angles

- Euler angles are used in a lot of applications, but they tend to require some rather arbitrary decisions
- They also do not interpolate in a consistent way (but this isn't always bad)
- They can suffer from Gimbal lock and related problems
- There is no simple way to concatenate rotations
- Conversion to/from a matrix requires several trigonometry operations
- They are compact (requiring only 3 numbers)

Gimbal Lock





Solution? Quaternions

- Discovered by Sir Hamilton in 1843
- Preferred rotation operator in chemistry, robotics, space shuttle controls, 3D games, VR, etc…
- Advantages
  - Represents "pure" attitude or rotation
  - No mathematical singularities
  - Operations are computationally easy
  - Smooth and easy interpolation → great for animation
- Disadvantages
  - Completely unintuitive → fortunately not a real issue

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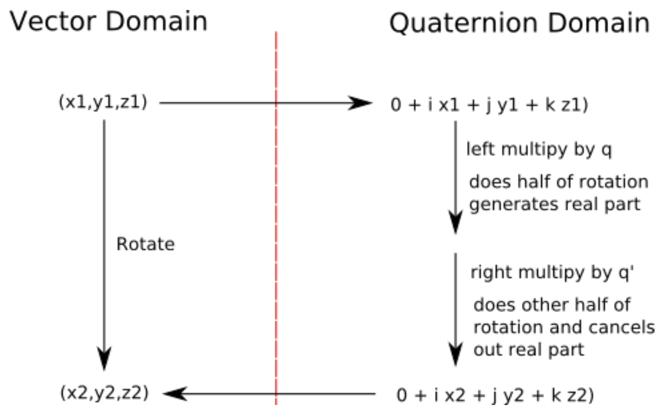
Solution? Quaternions



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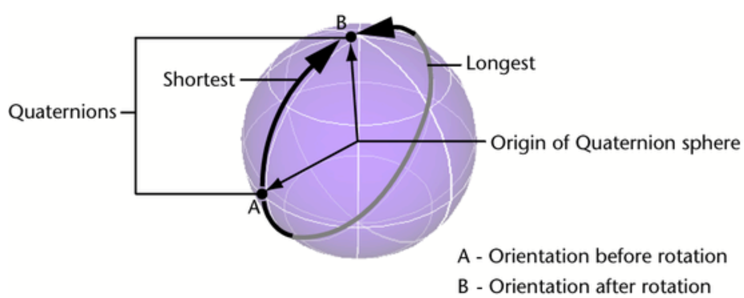
Solution? Quaternions



Quaternion rotation interpolation

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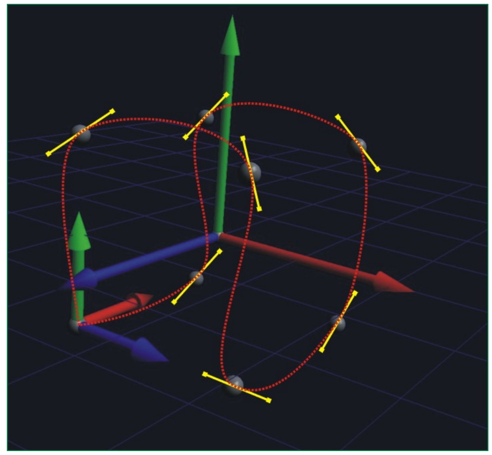
Quaternions & Interpolation

- Quaternions are very suitable for animating attitude
  - Linear interpolation
  - SLERP interpolation
- Equally suitable for animating a camera (or more camera's)
  - Camera is just another object
- Position
  - Linear interpolation
  - Splines
- Other aspects
  - Scaling
  - Morphing



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Motion Fusion: MPU-6050