

Method II: Calculation!

Probability: function assigning any assertion (sentence) a number in range  $[0, \dots, 1]$ .

- must satisfy axioms of probability
  - $0 \leq P(a) \leq 1$
  - $P(\text{true}) = 1$
  - $P(\text{false}) = 0$
  - $P(a \vee b) = P(a) + P(b) - P(a \& b)$

What is it?

- a degree of belief
- a summary of historical statistics
- a property of an object (e.g. a die)

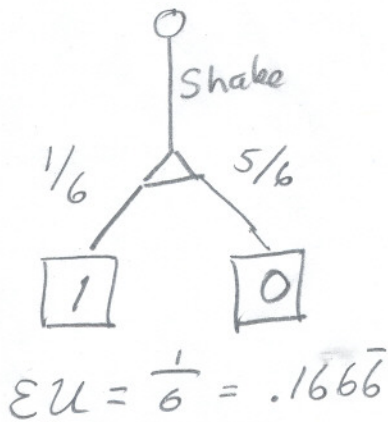
RESULT(action) = random variable ranging over possible worlds

Utility: assigns each possible world a real number (reward)

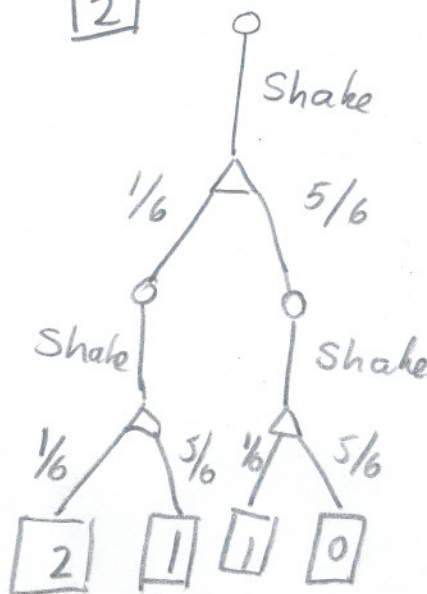
Expected Utility of an action =  $\sum_s P(\text{RESULT}(a)=s) U(s)$

Draw game trees!

1



2

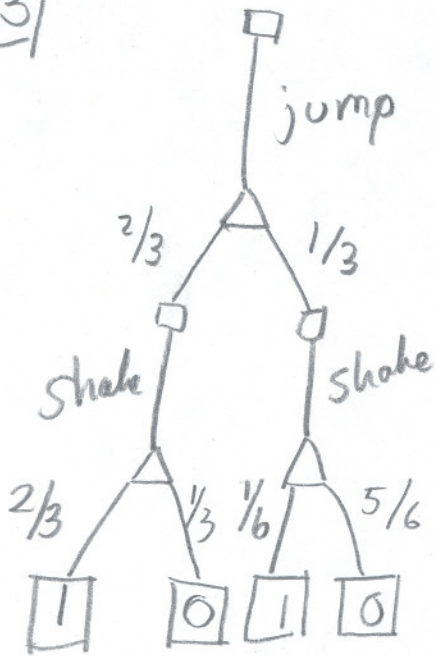


$$\mathcal{E}U = 2 \left(\frac{1}{6}\right) \left(\frac{1}{6}\right) + 1 \left(\frac{5}{6}\right) \left(\frac{1}{6}\right) + 1 \left(\frac{5}{6}\right) \left(\frac{1}{6}\right) =$$

$$\frac{2}{36} + \frac{5}{36} + \frac{5}{36} = \frac{12}{36} = \frac{1}{3}$$

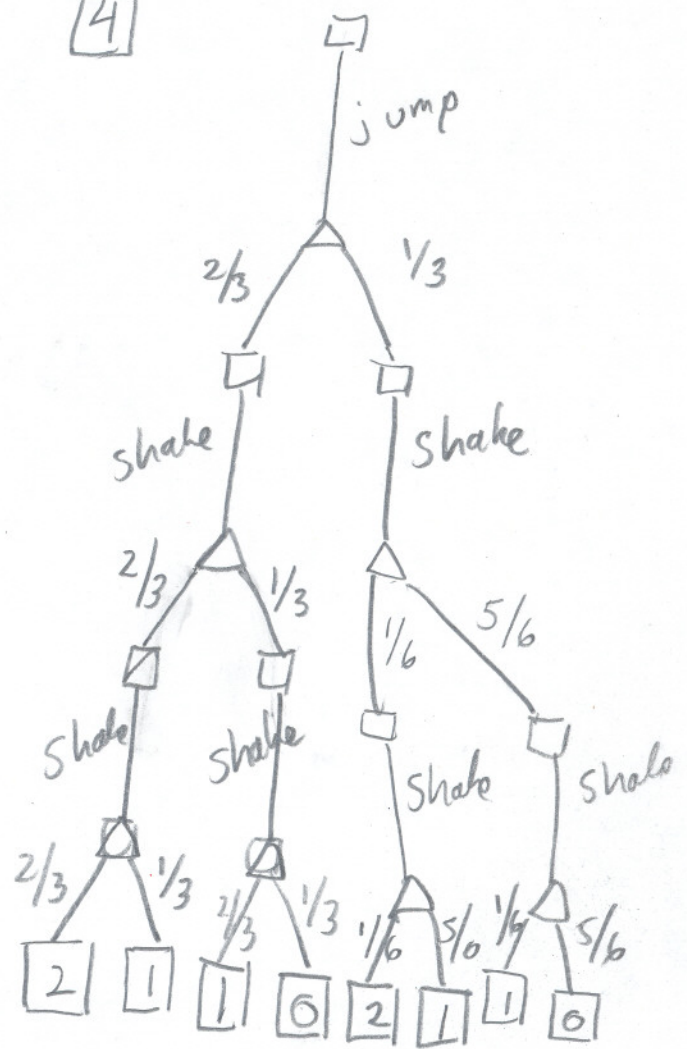
$$= .33\bar{3}$$

3



$$\begin{aligned}
 EU &= 1 \left( \frac{2}{3} \right) \left( \frac{2}{3} \right) + \\
 &\quad 1 \left( \frac{1}{3} \right) \left( \frac{1}{6} \right) \\
 &= \frac{4}{9} + \frac{1}{18} = \frac{9}{18} = \frac{1}{2} \\
 &= .5
 \end{aligned}$$

4



$$\begin{aligned}
 EU &= \\
 &2 \left( \frac{2}{3} \right) \left( \frac{2}{3} \right) \left( \frac{2}{3} \right) + 1 \left( \frac{1}{3} \right) \left( \frac{2}{3} \right) \left( \frac{2}{3} \right) \\
 &+ 1 \left( \frac{2}{3} \right) \left( \frac{1}{3} \right) \left( \frac{2}{3} \right) + \\
 &+ 2 \left( \frac{1}{6} \right) \left( \frac{1}{6} \right) \left( \frac{1}{3} \right) + 1 \left( \frac{5}{6} \right) \left( \frac{1}{6} \right) \left( \frac{1}{3} \right) \\
 &+ 1 \left( \frac{1}{6} \right) \left( \frac{5}{6} \right) \left( \frac{1}{3} \right) \\
 &= \frac{16}{27} + \frac{4}{27} + \frac{4}{27} + \frac{2}{108} + \frac{5}{108} + \frac{5}{108} \\
 &= \frac{24}{27} + \frac{12}{108} = 1 \quad (\star)
 \end{aligned}$$