CSE 473

Chapter 9
Reasoning with First-Order Logic

What's on our menu today?

- Reasoning with FOL
  - Unification
  - Forward/Backward Chaining
  - Resolution
  - Compilation to SAT

Motivation for Unification

- What if we want to use modus ponens?

  Propositional Logic:

  $$\frac{a \wedge b, \quad a \wedge b \Rightarrow c}{c}$$

- In First-Order Logic?

  Monkey(x) ⇒ Curious(x)
  Monkey(George)
  ____
  ????

- Must "***unify***" x with George:

  Need to substitute {x/George} in Monkey(x) ⇒ Curious(x) to infer Curious(George)

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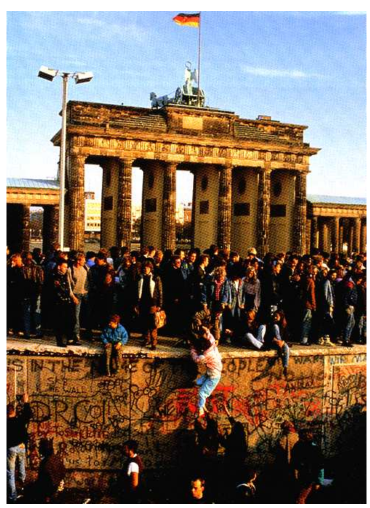
What is Unification?



CITY PRICES London Herald LATE EDITION

BERLIN WALL TUMBLES

'Beginning of the End' for Communism

Breaching the Wall

Not this kind of unification…

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What is Unification?

- Match up expressions by *finding variable values that make the expressions identical*
- Unify(x, y) returns most general unifier (MGU). Examples:

  Unify(city(x), city(kent)) returns {x/kent}

  Unify(PokesInTheEyes(Moe,x), PokesInTheEyes(y,z))

  returns {y/Moe,x/z}

  - {y/Moe,x/Moe,z/Moe} possible but not MGU
  - MGU places fewest restrictions on values of variables

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Unification and Substitution

Unification produces a mapping from variables to values (e.g., {x/kent,y/seattle})

Substitution: Subst(mapping,sentence) returns new sentence with variables replaced by values

Subst({x/kent,y/seattle }, connected(x, y)) returns connected(kent, seattle)

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Unification Examples I

- Unify(road(x, kent), road(seattle, y))
  Returns {x / seattle, y / kent}
  When substituted in both expressions, the resulting expressions match:
  Each is **(road(seattle, kent))**
- Unify(road(x, x), road(seattle, kent))
  Not possible – Fails!
  x can't be seattle and kent at the same time!

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Unification Examples II

- Unify(f(g(x, dog), y)), f(g(cat, y), dog)
  {x / cat, y / dog}
- Unify(f(g(x)), f(x))
  Fails: no substitution makes them identical.
  E.g. {x / g(x) } yields f(g(g(x))) and f(g(x)) which are not identical!
- Thus: A variable value may not ***contain*** itself in a substitution
  Directly or indirectly

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Unification Examples III

- Unify(f(g(cat, y), y), f(x, dog))
  { x / g(cat, dog), y / dog}
- Unify(f(g(y)), f(x))
  {x / g(y)}
- Back to curious monkeys:

  Monkey(x) ⇒ Curious(x)
  Monkey(George)
  ———————————————
  Curious(George)

  Unify and then use modus ponens =
  generalized modus ponens
  ("Lifted" version of modus ponens)

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Inference I: Forward Chaining

- The algorithm:
  Start with the KB
  Add any fact you can generate with GMP (i.e., unify expressions and use modus ponens)
  Repeat until: goal reached or generation halts.
- Sound? Complete? Decidable?
  Yes; yes for definite KB; no (see p. 283 in text)
- Speed concerns? Inefficiencies due to:
  Unification via exhaustive pattern matching; premise rechecking; irrelevant fact generation.
  (see p. 283-287 for strategies to increase speed)

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Inference II: Backward Chaining

- The algorithm:
  - Start with KB and goal.
  - Find all rules whose *results* unify with goal:
    - Add the *premises* of these rules to the goal list
    - Remove the corresponding result from the goal list
  - Stop when:
    - Goal list is empty (SUCCEED) or
    - Progress halts (FAIL)

Inference III: Resolution

[Robinson 1965]

$$\{ (p \vee q), (\neg p \vee r \vee s) \} \vdash_R (q \vee r \vee s)$$

Recall Propositional Case:
- **Literal in one clause**
- **Its negation in the other**
- **Result is disjunction of *other* literals**

First-Order Resolution

[Robinson 1965]

$\{ (p(x) \vee q(A), \quad (\neg p(B) \vee r(x) \vee s(y)) \}$

$\vdash_R$

$(q(A) \vee r(B) \vee s(y))$

Substitute MGU {x/B} in all literals

- **Literal in one clause**
- **Negation of *something which unifies* in other**
- **Result is disjunction of all other literals with substitution based on MGU**

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Inference using First-Order Resolution

- As before, use "proof by contradiction"
  To show KB $\models \alpha$, show KB $\wedge \neg\alpha$ unsatisfiable
- Method
  Let S = KB $\wedge \neg$goal
  Convert S to clausal form
  - Standardize apart variables
  - Move quantifiers to front, skolemize to remove $\exists$
  - Replace $\Rightarrow$ with $\vee$ and $\neg$
  - Demorgan's laws to get CNF (ands-of-ors)

  Resolve clauses in S until empty clause (unsatisfiable) or no new clauses added

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First-Order Resolution Example

- Given

  ∀x man(x) ⇒ human(x)

  ∀x woman(x) ⇒ human(x)

  ∀x singer(x) ⇒ man(x) ∨ woman(x)

  singer(Diddy)

- Prove

  human(Diddy)

CNF representation (list of clauses):

[¬m(x),h(x)] [¬w(y), h(y)] [¬s(z),m(z),w(z)] [s(D)] [¬h(D)]

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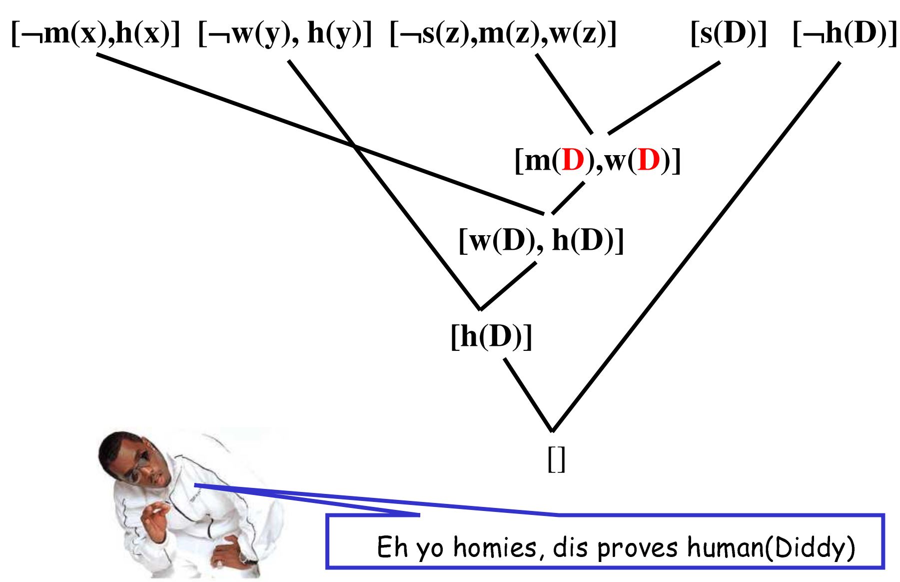
FOL Resolution Example



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FOL Resolution Example

- Much More Difficult Exercise:
  Prove human(MJ)

What about me?

FOL Resolution Example 2

Given

$\forall x\ \exists y\ \text{Twin}(x) \Rightarrow \text{Twin}(y)$

Twin(Ashley)

Prove

Twin(Diddy)

$\forall x\ \text{Twin}(x) \Rightarrow \text{Twin}(F(x))$ **Skolemization**

[($\neg$T(x), T(F(x))) (T(A)) ($\neg$T(D))]

(T(F(A)))

(T(F(F(A))))

(T(F(F(F(A)))))

...

May not terminate!

Inference IV: Compilation to Prop. Logic

- Sentence S:

  $\forall_{city}\ a,b$ Connected(a,b)
- Universe

  Cities: seattle, tacoma, enumclaw
- Equivalent propositional formula?

Cst ∧ Cse ∧ Cts ∧ Cte ∧ Ces ∧ Cet

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Compilation to Prop. Logic (cont)

- Sentence S:

  $\exists_{city}\ c$ Biggest(c)
- Universe

  Cities: seattle, tacoma, enumclaw
- Equivalent propositional formula?

Bs ∨ Bt ∨ Be

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Compilation to Prop. Logic (cont again)

- Universe
  - Cities: seattle, tacoma, enumclaw
  - Firms: IBM, Microsoft, Boeing
- First-Order formula

  $\forall_{firm}\, f \;\; \exists_{city}\, c \quad \text{HeadQuarters}(f, c)$
- Equivalent propositional formula

  $[\ (HQis \vee HQit \vee HQie) \wedge$
  $(HQms \vee HQmt \vee HQme) \wedge$
  $(HQbs \vee HQbt \vee HQbe)\ ]$

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Hey!

- You said FO Inference is semi-decidable
- But you compiled it to SAT

  Which is NP Complete
- So now we can always do the inference?!?

  (might take exponential time but still decidable?)
- Something seems wrong here….????

  Something to ponder over the weekend…

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