CSE 473

Chapter 3
Problem Solving using Search

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From The New Yorker Book of Technology Cartoons.

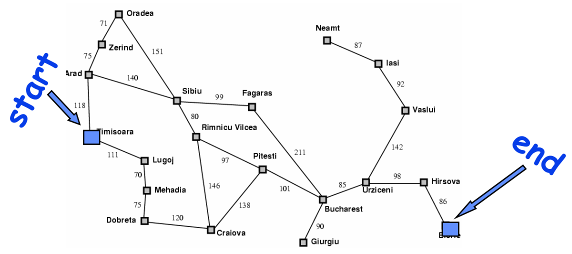
"First, they do an on-line search"

Example: The 8-puzzle

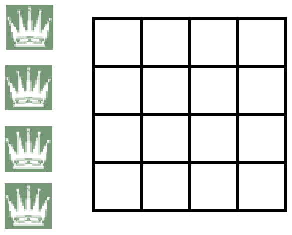
1	2	3
8		4
7	6	5

1	2	3
4	5	6
7	8	

Example: Route Planning

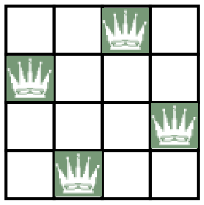


Example: N Queens



4 Queens

Example: N Queens



4 Queens

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State-Space Search Problems

General problem:

Given a *start state*, find a path to a *goal state*

- **Can test if a state is a goal**
- **Given a state, can generate its *successor* states**

Variants:

- **Find any path *vs.* a least-cost path**
- **Goal is completely specified, task is just to find the path**
  - **Route planning**
- **Path doesn't matter, only finding the goal state**
  - **8 puzzle**

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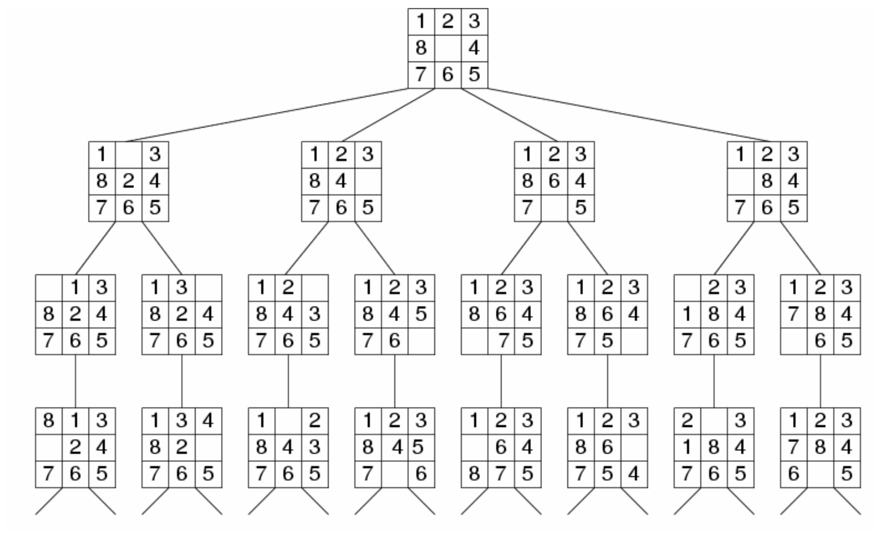
Tree Representation of 8-Puzzle Problem Space



Implementation: general tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem] applied to STATE(node) succeeds return node
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

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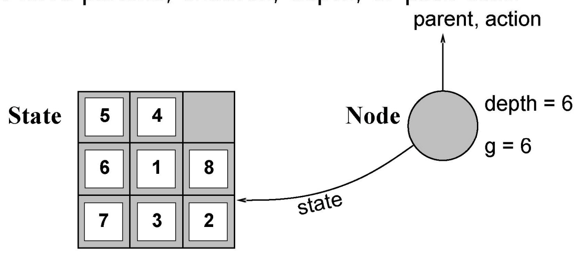
      if GOAL-TEST[problem] applied to STATE(node) succeeds return node
      fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND( node, problem) returns a set of nodes
   successors ← the empty set
   for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
      s ← a new NODE
      PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
      PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
      DEPTH[s] ← DEPTH[node] + 1
      add s to successors
   return successors
```

Implementation: states vs. nodes

A *state* is a (representation of) a physical configuration
A *node* is a data structure constituting part of a search tree
includes *parent*, *children*, *depth*, *path cost* $g(x)$
States do not have parents, children, depth, or path cost!



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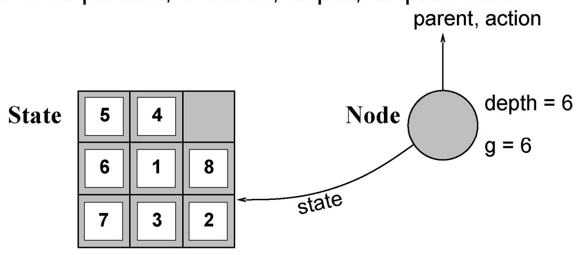


The EXPAND function creates new nodes, filling in the various fields and using the SUCCESSORFN of the problem to create the corresponding states.

Search strategies

A strategy is defined by picking the *order of node expansion*

Strategies are evaluated along the following dimensions:
completeness—does it always find a solution if one exists?
time complexity—number of nodes generated/expanded
space complexity—maximum number of nodes in memory
optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of
b—maximum branching factor of the search tree
d—depth of the least-cost solution
m—maximum depth of the state space (may be ∞)

Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

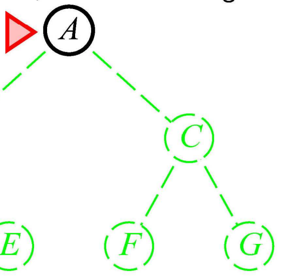
Iterative deepening search

Breadth-first search

Expand shallowest unexpanded node

Implementation:

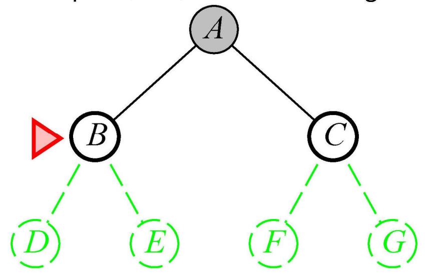
$fringe$ is a FIFO queue, i.e., new successors go at end



Breadth-first search

Expand shallowest unexpanded node

Implementation:
fringe is a FIFO queue, i.e., new successors go at end



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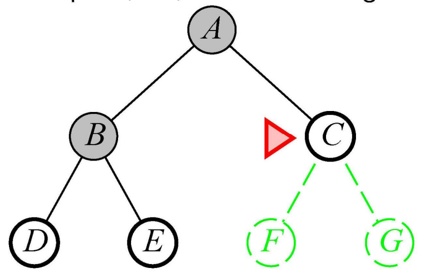

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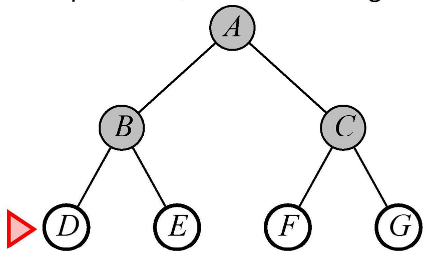

Properties of breadth-first search

Complete??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal?? Yes (if cost $= 1$ per step); not optimal in general

Space is the big problem for BFS.

Example: b = 10, 10,000 nodes/sec, 1KB/node

d = 2 Ł 1100 nodes, 0.11 secs, 1MB

d = 4 Ł 111,100 nodes, 11 secs, 106 MB

d = 8 Ł 10^9 nodes, 31 hours, 1 TB

Uniform-cost search

Expand least-cost unexpanded node

Implementation:
    fringe = queue ordered by path cost

Equivalent to breadth-first if step costs all equal

Complete?? Yes, if step cost $\geq \epsilon$

Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
where C^* is the cost of the optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$

Optimal?? Yes—nodes expanded in increasing order of $g(n)$

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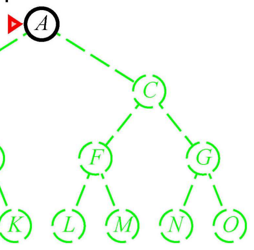
Depth-first search

Expand deepest unexpanded node

Implementation:
    fringe = LIFO queue, i.e., put successors at front



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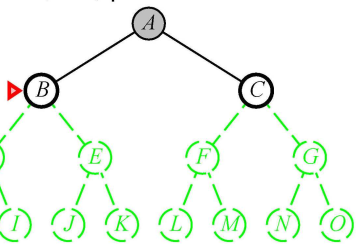

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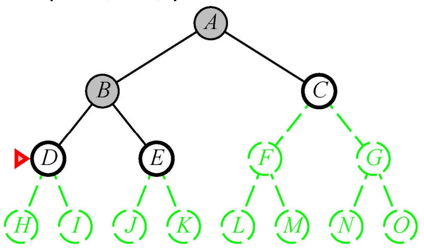

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Depth-first search

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Implementation:
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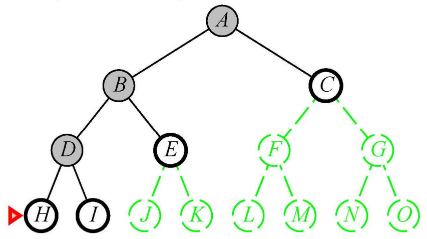

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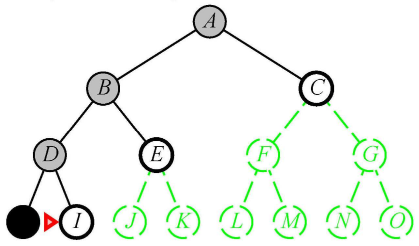

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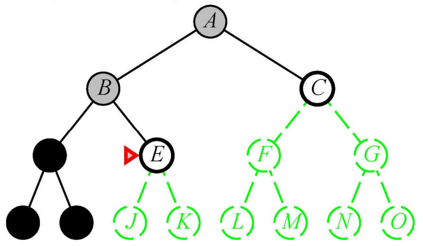

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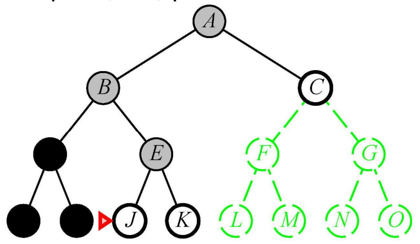

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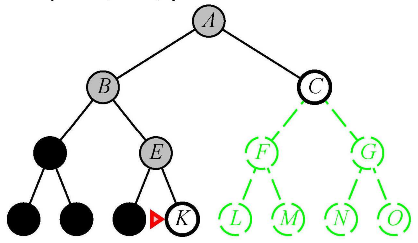

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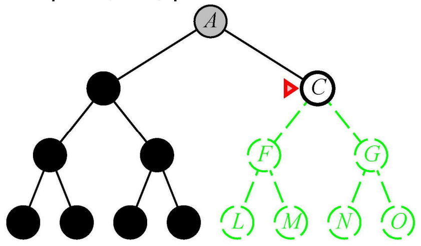

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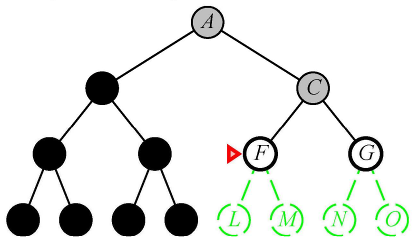

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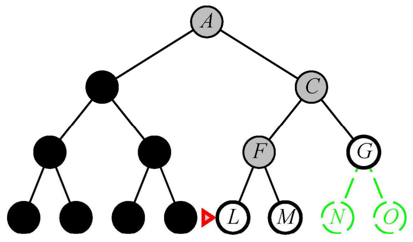

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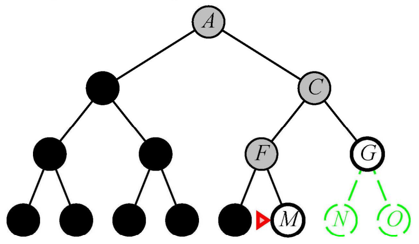

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Properties of depth-first search

Complete??

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Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time??

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Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space??

38

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal??

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Properties of depth-first search

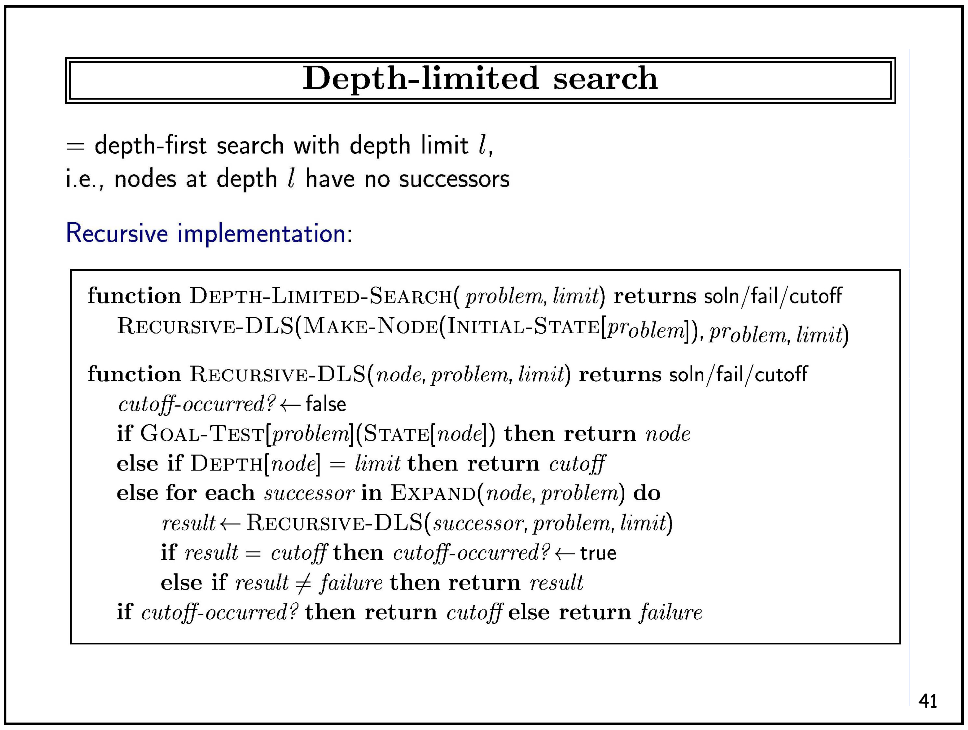
Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal?? No

40



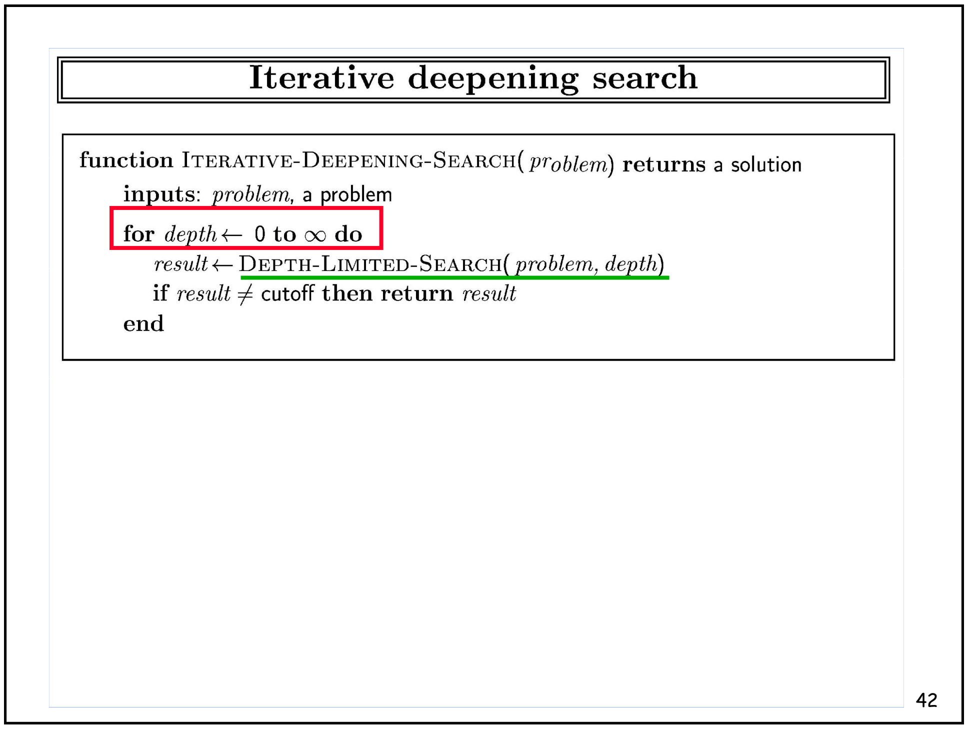
Depth-limited search

= depth-first search with depth limit l,
i.e., nodes at depth l have no successors

Recursive implementation:

```
function Depth-Limited-Search( problem, limit) returns soln/fail/cutoff
   Recursive-DLS(Make-Node(Initial-State[problem]), problem, limit)

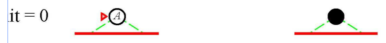
function Recursive-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if Goal-Test[problem](State[node]) then return node
   else if Depth[node] = limit then return cutoff
   else for each successor in Expand(node, problem) do
      result ← Recursive-DLS(successor, problem, limit)
      if result = cutoff then cutoff-occurred? ← true
      else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```



Iterative deepening search

```
function Iterative-Deepening-Search( problem) returns a solution
   inputs: problem, a problem
   for depth ← 0 to ∞ do
      result ← Depth-Limited-Search( problem, depth)
      if result ≠ cutoff then return result
   end
```

Iterative deepening search $l = 0$



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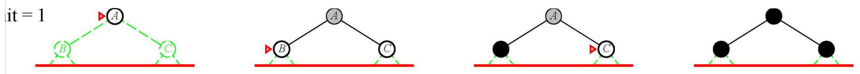
Iterative deepening search $l = 1$



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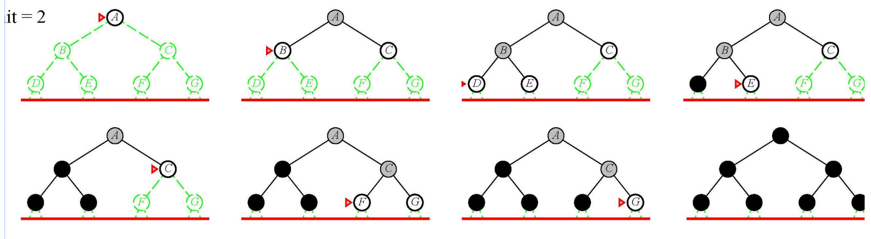
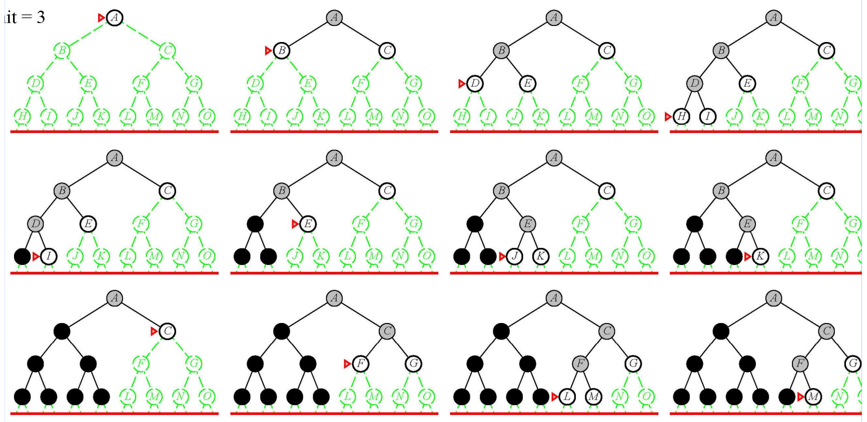
Iterative deepening search $l = 2$



Iterative deepening search $l = 3$



Properties of iterative deepening search

Complete??

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Properties of iterative deepening search

Complete?? Yes

Time??

48

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space??

49

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal??

50

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? Yes, if step cost $= 1$

Can be modified to explore uniform-cost tree

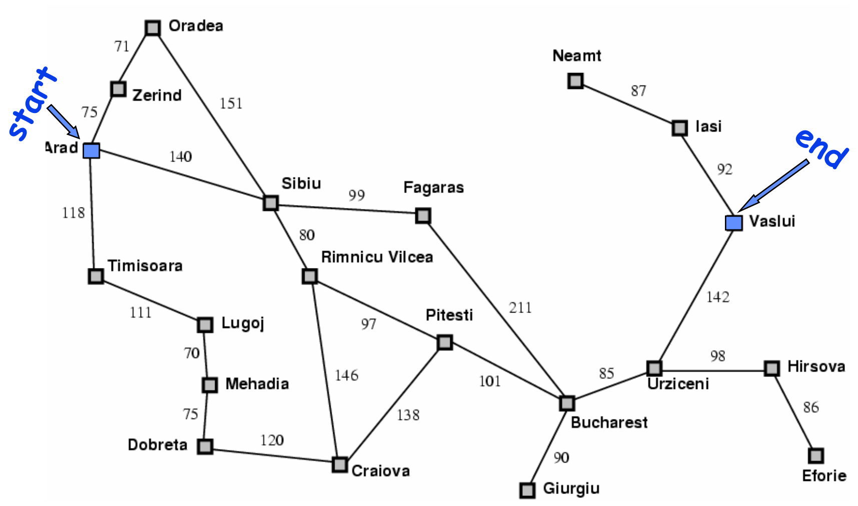
Increasing path-cost limits instead of depth limits

This is called Iterative lengthening search (exercise 3.11)

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes*	No	No	Yes

Forwards *vs.* Backwards



Problem: Find the shortest route

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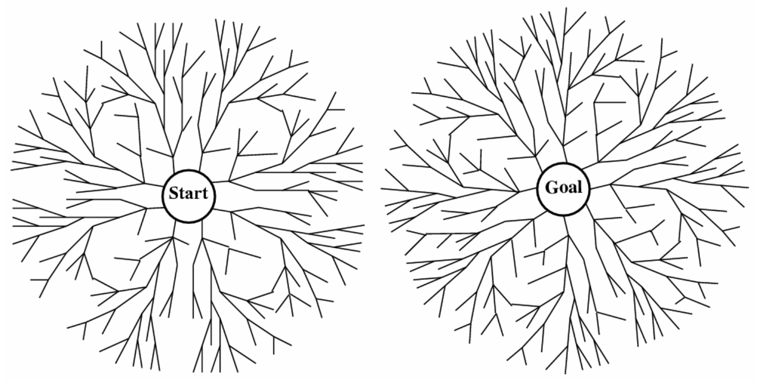
Bidirectional Search



Motivation: $b^{d/2} + b^{d/2} << b^d$

Can use breadth-first search or uniform-cost search

Hard for implicit goals e.g., goal = “checkmate” in chess

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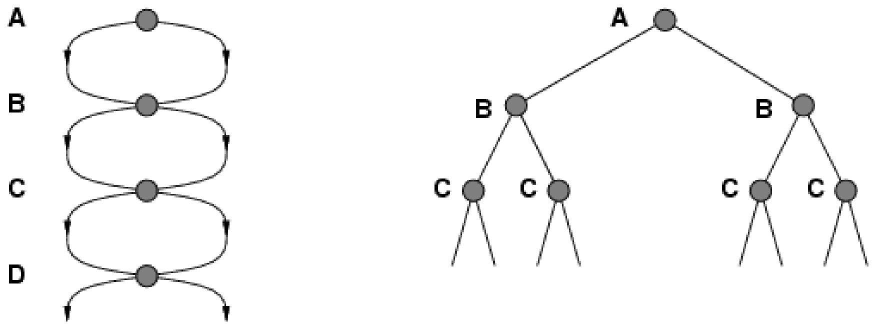
Repeated States

Failure to detect repeated states can turn a linear problem into an exponential one! (e.g., repeated states in 8 puzzle)



Graph search algorithm: Store expanded nodes in a set called *closed* and only add new nodes to the fringe

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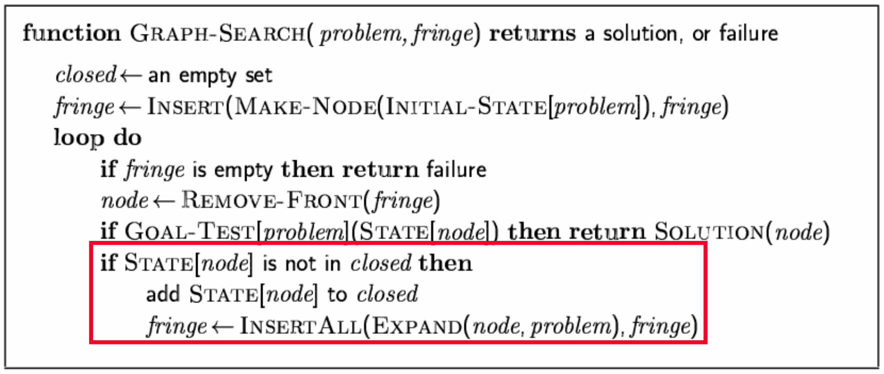
Graph Search



```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
   closed ← an empty set
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node ← REMOVE-FRONT(fringe)
       if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
       if STATE[node] is not in closed then
           add STATE[node] to closed
           fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

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Can we do better?

All these methods are slow (blind)

Solution use problem-specific knowledge to guide search ("heuristic function")
"informed search" (next lecture)

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