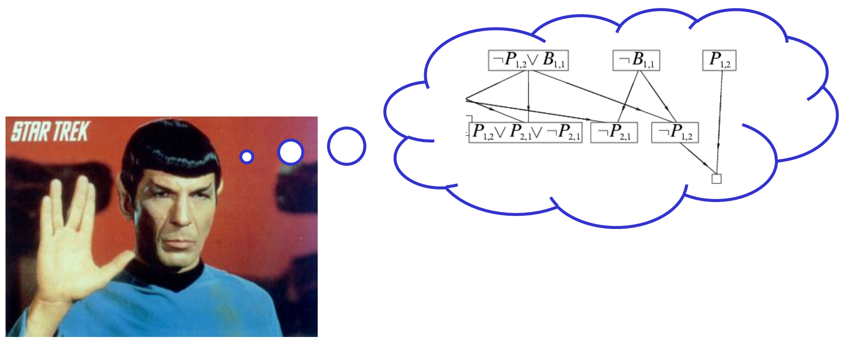
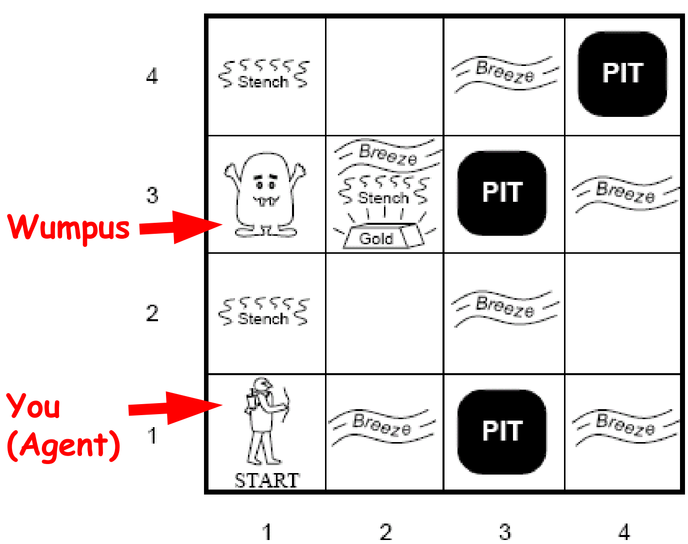
CSE 473

Chapter 7

Inference Techniques for Logical Reasoning



Recall: Wumpus World



Recall: Wumpus KB

Knowledge Base (KB) includes the following sentences:

- Statements currently known to be true:

  $\neg P_{1,1}$
  $\neg B_{1,1}$
  $B_{2,1}$

- Properties of the world: E.g., "Pits cause breezes in adjacent squares"

  $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
  $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
  (and so on for all squares)

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 $\boxed{A}$ B OK	3,1 P?	4,1

Is there no pit in [1,2]?

KB $\models \neg P_{1,2}$?

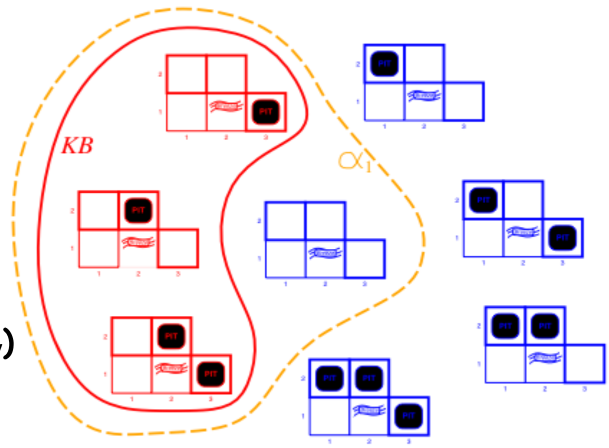
Recall from last time:

m **is a model of a sentence α if** **α is true in *m***

M(α) **is the set of <u>all models</u> of α**

KB $\models \alpha$ (KB "entails" α) iff *M(KB)* $\subseteq$ *M(α)*

$\alpha_1 = \neg P_{1,2}$



M(KB) $\subseteq$ ***M(α_1)***

KB = wumpus-world rules + observations

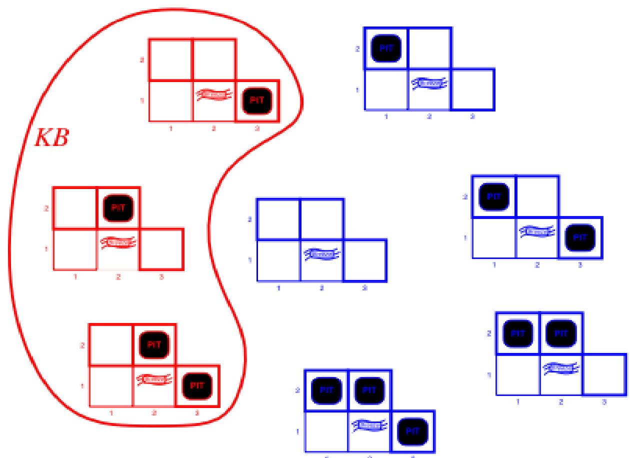
Inference by Truth Table Enumeration

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	$\neg P_{1,2}$
false	*false*	*false*	*false*	*false*	*false*	*false*	*false*	*true*
false	*false*	*false*	*false*	*false*	*false*	*true*	*false*	*true*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*false*	*true*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*false*	*true*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*false*

In all models in which KB is true, $\neg P_{1,2}$ is also true
Therefore, KB $\models \neg P_{1,2}$

Another Example

Is there a pit in [2,2]?



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Inference by Truth Table Enumeration

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*

$P_{2,2}$ is false in a model in which KB is true
Therefore, KB $\not\models P_{2,2}$

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Inference by TT Enumeration

- Algorithm: Depth-first enumeration of all models (see Fig. 7.10 in text for pseudocode)
- - Algorithm is sound & complete

- For n symbols:
- time complexity = $O(2^n)$, space = $O(n)$

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Concepts for Other Techniques: Logical Equivalence

Two sentences are logically equivalent iff they are true in the same models: $\alpha \equiv \beta$ iff $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

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Concepts for Other Techniques: Validity and Satisfiability

- A sentence is *valid* if it is true in *all* models (a tautology)
  e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$
- Validity is connected to inference via the Deduction Theorem:
  $KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid
- A sentence is *satisfiable* if it is true in *some* model
  e.g., $A \vee B$, C
- A sentence is *unsatisfiable* if it is true in no models
  e.g., $A \wedge \neg A$
- Satisfiability is connected to inference via the following: $KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable (proof by contradiction)

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Inference/Proof Techniques

- Two kinds (roughly):

  Model checking
  - Truth table enumeration (always exponential in *n*)
  - Efficient backtracking algorithms,
    e.g., Davis-Putnam-Logemann-Loveland (DPLL)
  - Local search algorithms (sound but incomplete)
    e.g., randomized hill-climbing (WalkSAT)

  Successive application of inference rules
  - Generate new sentences from old in a sound way
  - Proof = a sequence of inference rule applications
  - Use inference rules as *successor function* in a standard search algorithm

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Inference Technique I: Resolution

Terminology:

Literal = proposition symbol or its negation

E.g., A, $\neg A$, B, $\neg B$, etc.

Clause = disjunction of literals

E.g., $(B \vee \neg C \vee \neg D)$

Resolution assumes sentences are in

Conjunctive Normal Form (CNF):

sentence = conjunction of clauses

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Conversion to CNF

E.g., $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$
2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.
   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$
3. Move $\neg$ inwards using de Morgan's rules and double-negation:
   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$
4. Apply distributivity law ($\wedge$ over $\vee$) and flatten:
   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

   This is in CNF – Done!

Resolution motivation

There is a pit in [1,3] or
There is a pit in [2,2]

There is no pit in [2,2]

There is a pit in [1,3]

More generally,

$$\frac{l_1 \vee \ldots \vee l_k, \qquad \neg l_i}{l_1 \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k}$$

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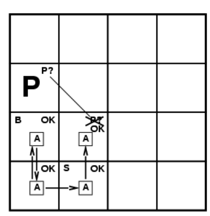
Inference Technique: Resolution

- General Resolution inference rule (for CNF):

$$\frac{l_1 \vee \ldots \vee l_k, \qquad m_1 \vee \ldots \vee m_n}{l_1 \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n}$$

  where l_i and m_j are complementary literals.

  E.g., $$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$



- Resolution is sound for propositional logic

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Resolution

Soundness of resolution inference rule
(Recall logical equivalence $A \Rightarrow B \equiv \neg A \lor B$)

$$\neg(l_1 \lor \ldots \lor l_{i-1} \lor l_{i+1} \lor \ldots \lor l_k) \Rightarrow l_i$$
$$\neg m_j \Rightarrow (m_1 \lor \ldots \lor m_{j-1} \lor m_{j+1} \lor \ldots \lor m_n)$$

$$\neg(l_i \lor \ldots \lor l_{i-1} \lor l_{i+1} \lor \ldots \lor l_k) \Rightarrow (m_1 \lor \ldots \lor m_{j-1} \lor m_{j+1} \lor \ldots \lor m_n)$$

(since $l_i = \neg m_j$)

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Resolution algorithm

- To show **KB** $\models \alpha$, use proof by contradiction, i.e., show $KB \land \neg\alpha$ unsatisfiable

```
function PL-RESOLUTION(KB, α) returns true or false
   clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
   new ← { }
   loop do
      for each Ci, Cj in clauses do
         resolvents ← PL-RESOLVE(Ci, Cj)
         if resolvents contains the empty clause then return true
         new ← new ∪ resolvents
      if new ⊆ clauses then return false
      clauses ← clauses ∪ new
```

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Resolution example

Given no breeze in [1,1], prove there's no pit in [1,2]

$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$ and $\alpha = \neg P_{1,2}$

Resolution: Convert to CNF and show KB $\wedge \neg \alpha$ is unsatisfiable

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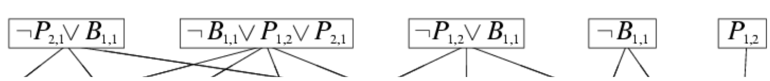
Resolution example



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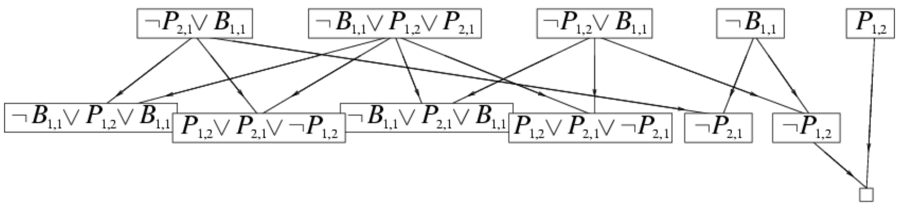
Resolution example



Empty clause
(i.e., KB $\wedge \neg \alpha$ unsatisfiable)

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Inference Technique II: Forward/Backward Chaining

- Require sentences to be in **Horn Form**:
  - KB = conjunction of Horn clauses
  - Horn clause =
    - proposition symbol or
    - "(conjunction of symbols) $\Rightarrow$ symbol"
    - (i.e. clause with at most 1 positive literal)
  - E.g., KB = $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$
- F/B chaining based on "Modus Ponens" rule:

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

  Complete for Horn clauses
- Very natural and linear time complexity in size of KB

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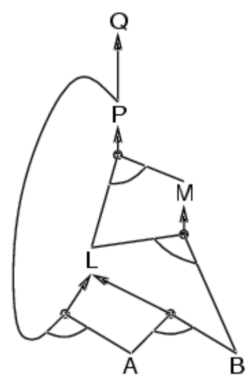
Forward chaining

- Idea: fire any rule whose premises are satisfied in *KB*, add its conclusion to *KB*, until query q is found

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

Query = "Is Q true?"



AND-OR Graph

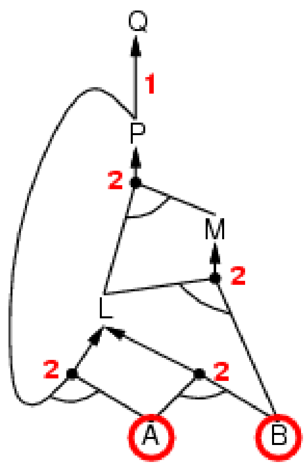
Forward chaining algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known to be true

   while agenda is not empty do
      p ← POP(agenda)
      unless inferred[p] do
         inferred[p] ← true
         for each Horn clause c in whose premise p appears do
            decrement count[c]
            if count[c] = 0 then do
               if HEAD[c] = q then return true
               PUSH(HEAD[c], agenda)
   return false
```

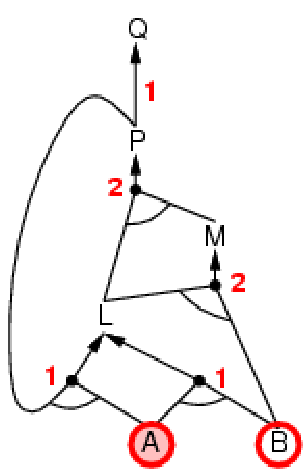
Forward chaining is sound & complete for Horn KB

Forward chaining example

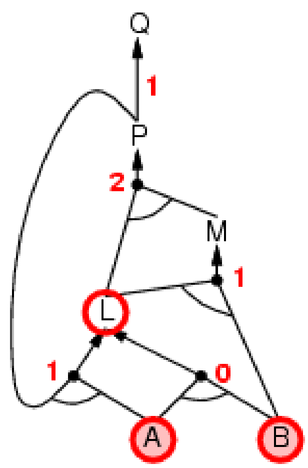


Query = Q
(i.e. "Is Q true?")

Forward chaining example



Forward chaining example



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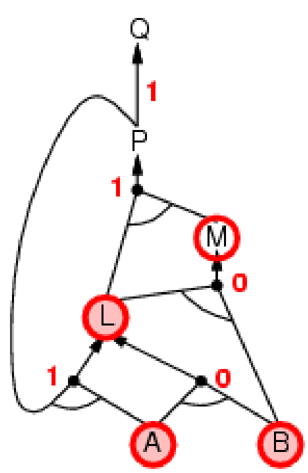
Forward chaining example



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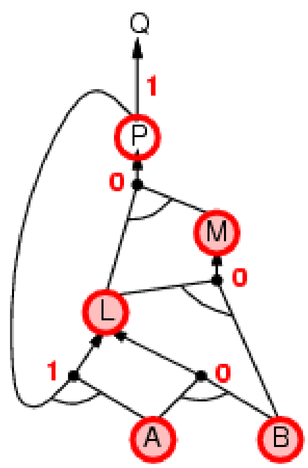
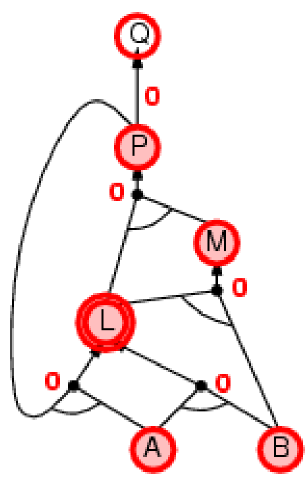
Forward chaining example



Forward chaining example



Backward chaining

Idea: work backwards from the query q:

to prove q by BC,

check if q is known already, or

prove by BC all premises of some rule concluding q

Avoid loops: check if new subgoal is already on goal stack

Avoid repeated work: check if new subgoal

1. has already been proved true, or
2. has already failed

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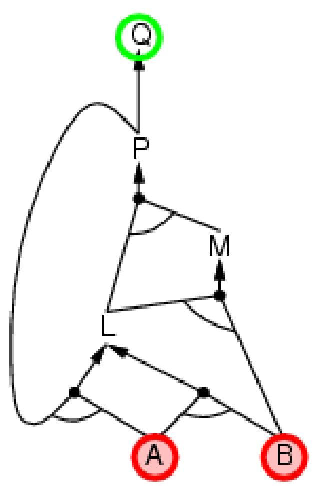
Backward chaining example



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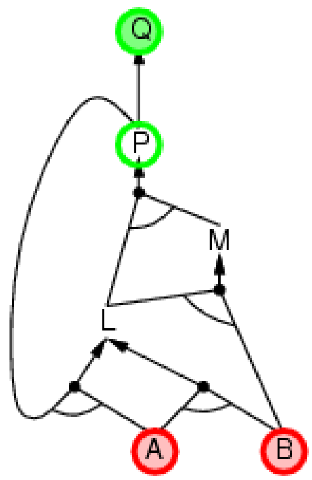
Backward chaining example



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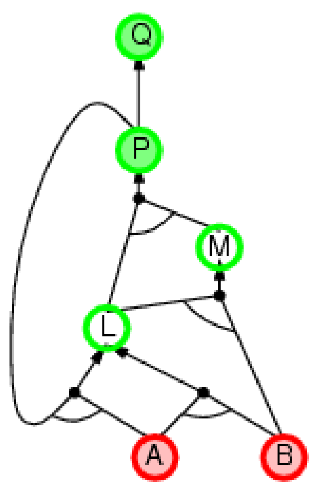
Backward chaining example



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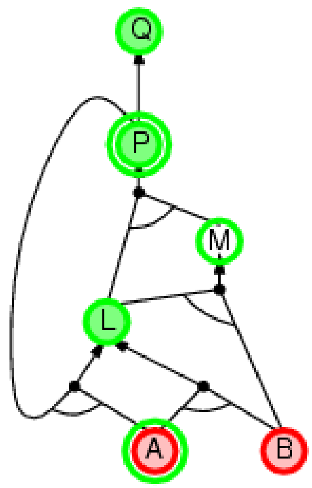
Backward chaining example



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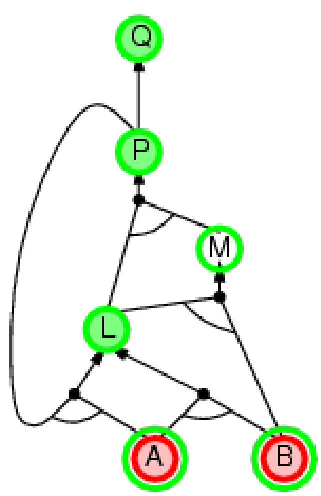
Backward chaining example



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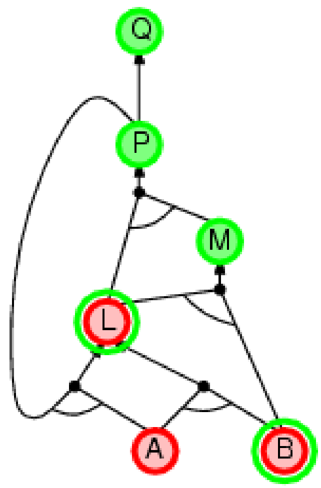
Backward chaining example



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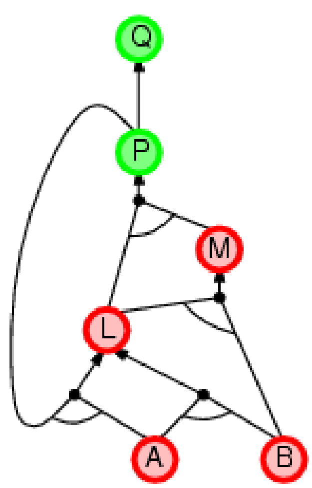
Backward chaining example



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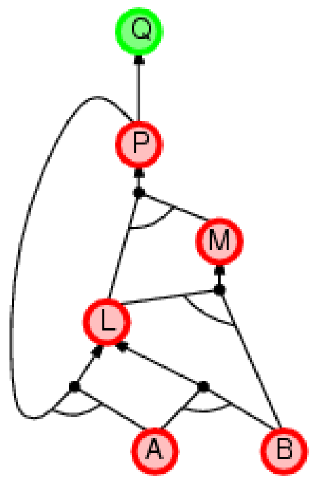
Backward chaining example



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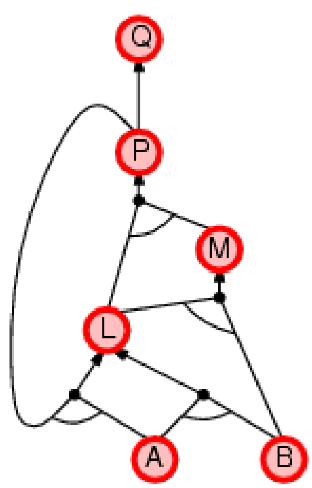
Backward chaining example



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Forward vs. backward chaining

- FC is data-driven, automatic, unconscious processing,
  e.g., object recognition, routine decisions
- FC may do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving,
  e.g., How do I get an A in this class?
  e.g., What is my best exit strategy out of the classroom?
  e.g., How can I impress my date tonight?
- Complexity of BC can be much less than linear in size of KB

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The DPLL algorithm

Determine if an input propositional logic sentence (in CNF) is satisfiable.

Improvements over truth table enumeration:

1. Early termination
   A clause is true if any literal is true.
   A sentence is false if any clause is false.

2. Pure symbol heuristic
   Pure symbol: always appears with the same "sign" in all clauses.
   e.g., In the three clauses $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$, A and B are pure, C is impure.
   Make a pure symbol literal true.

3. Unit clause heuristic
   Unit clause: only one literal in the clause
   The only literal in a unit clause must be true.

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The DPLL algorithm

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*
  inputs: s, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of s
  symbols ← a list of the proposition symbols in s
  return DPLL(*clauses*, *symbols*, [ ])

---

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

  if every clause in *clauses* is true in *model* **then return** *true*
  if some clause in *clauses* is false in *model* **then return** *false*
  P, *value* ← FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)
  if P is non-null **then return** DPLL(*clauses*, *symbols*–P, [P = *value*|*model*])
  P, *value* ← FIND-UNIT-CLAUSE(*clauses*, *model*)
  if P is non-null **then return** DPLL(*clauses*, *symbols*–P, [P = *value*|*model*])
  P ← FIRST(*symbols*); *rest* ← REST(*symbols*)
  return DPLL(*clauses*, *rest*, [P = *true*|*model*]) **or**
      DPLL(*clauses*, *rest*, [P = *false*|*model*])

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Next Time

- WalkSAT
- Logical Agents: Wumpus
- First-Order Logic
- To Do:
  - Project #2
  - Finish Chapter 7
  - Start Chapter 8

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