CSE 473

Lecture 24
(Chapter 18)

Decision Trees

Goal: Learn the function "PlayTennis?" from example data

Input Attributes *Output*

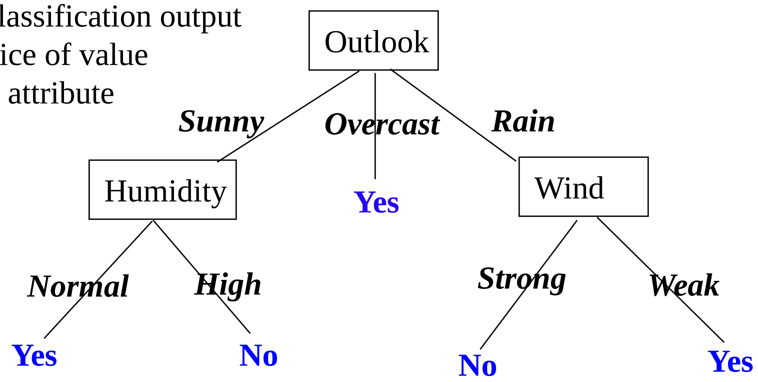
Day	Outlook	Humid	Wind	PlayTennis? "yes" (y) or "no" (n)
d1	s	h	w	n
d2	s	h	s	n
d3	o	h	w	y
d4	r	h	w	y
d5	r	n	w	y
d6	r	n	s	y
d7	o	n	s	y
d8	s	h	w	n
d9	s	n	w	y
d10	r	n	w	y
d11	s	n	s	y
d12	o	h	s	y
d13	o	n	w	y
d14	r	h	s	n

- Outlook = sunny, overcast, or rain
- Humidity = high, or normal
- Wind = weak or strong

A Decision Tree for the Same Data

Decision Tree for "PlayTennis?"

Leaves = classification output
Arcs = choice of value for parent attribute



Decision tree equivalent to logical statment in disjunctive normal form

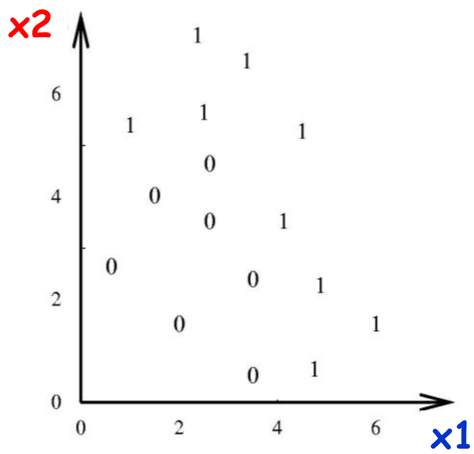
PlayTennis ⇔ (Sunny ∧ Normal) ∨ Overcast ∨ (Rain ∧ Weak)

Decision Trees

- **Input:** Set of **attributes** describing an object or situation
- **Output:** Predicted output value for the input
- Decision tree is consistent if it produces the correct output on all training examples
- Input and output can be **discrete or continuous**

Example: Decision Tree for Continuous Values

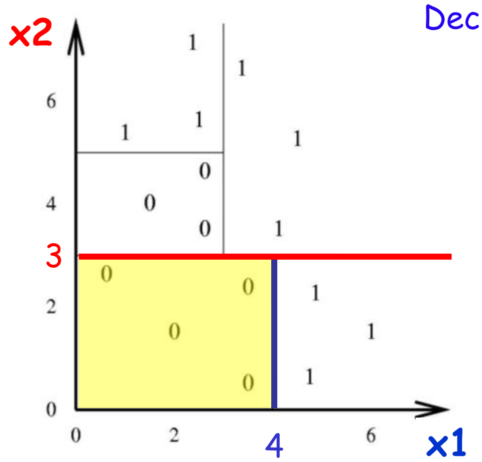
Input: Continuous-valued attributes (x1,x2)
Output: 0 or 1



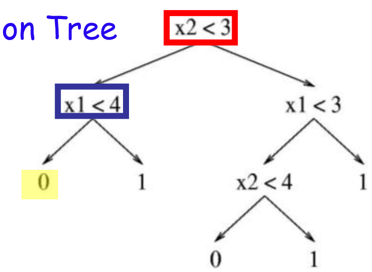
How do we branch on attribute values x1 and x2 to partition the space and generate correct outputs?

Example: Classification of Continuous Valued Inputs

Decision trees divide the feature space into axis-parallel rectangles, and label each rectangle with one of the K classes.



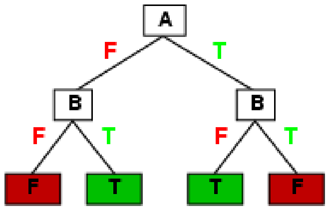
Decision Tree



Expressiveness of Decision Trees

- Decision trees can express any function of the input attributes.
- E.g., Boolean functions, truth table row = path to leaf:

A	B	A xor B
F	F	F
F	T	T
T	F	T
T	T	F



- Trivially, there is a consistent decision tree for any training set with one path to leaf for each example
  - But most likely won't generalize to new examples
- Prefer to find more compact decision trees

Learning Decision Trees

- Example: When should I wait for a table at a restaurant?

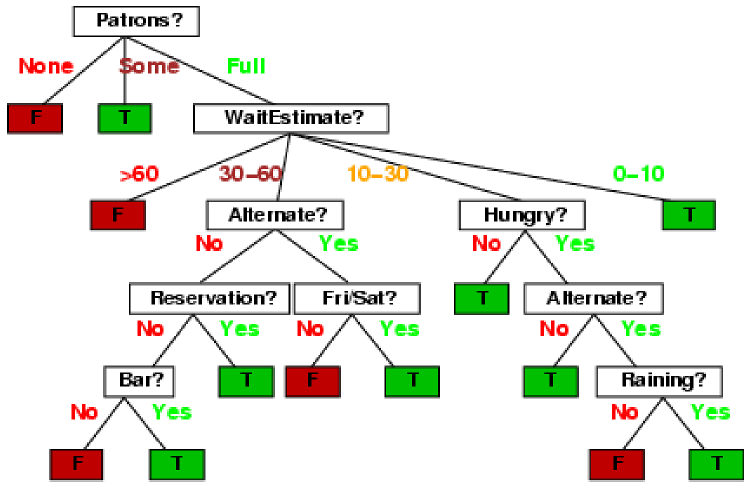
Learning Decision Trees

- Example: When should I wait for a table at a restaurant?

- Attributes (features) relevant to *Wait?* decision:
  1. Alternate: is there an alternative restaurant nearby?
  2. Bar: is there a comfortable bar area to wait in?
  3. Fri/Sat: is today Friday or Saturday?
  4. Hungry: are we hungry?
  5. Patrons: number of people in the restaurant (None, Some, Full)
  6. Price: price range ($, $$, $$$)
  7. Raining: is it raining outside?
  8. Reservation: have we made a reservation?
  9. Type: kind of restaurant (French, Italian, Thai, Burger)
  10. WaitEstimate: estimated waiting time (0-10, 10-30, 30-60, >60)

A “personal” decision tree

- A decision tree for *Wait?* based on personal “rules of thumb”:



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Input Data for Learning

- Past examples when I did/did not wait for a table:

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*Wait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

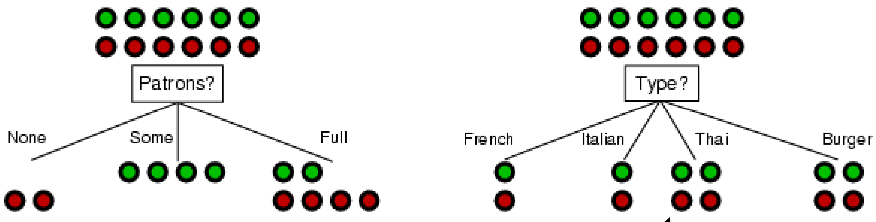
Decision Tree Learning

- Aim: Find a small tree *consistent* with training examples
- Idea: (recursively) choose "most significant" attribute as root of (sub)tree

```
function DTL(examples, attributes, default) returns a decision tree
   if examples is empty then return default
   else if all examples have the same classification then return the classification
   else if attributes is empty then return MODE(examples)
   else
      best ← CHOOSE-ATTRIBUTE(attributes, examples)
      tree ← a new decision tree with root test best
      for each value v_i of best do
         examples_i ← {elements of examples with best = v_i}
         subtree ← DTL(examples_i, attributes − best, MODE(examples))
         add a branch to tree with label v_i and subtree subtree
      return tree
```

Choosing an attribute to split on

- Idea: a good attribute should reduce uncertainty
  - E.g., splits the examples into subsets that are (ideally) "all positive" (T) or "all negative“ (F)



For *Type?*, to wait or not to wait is still at 50%

- *Patrons?* is a better choice

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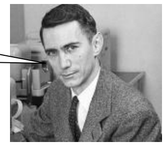
Reduce uncertainty?
How do you quantify uncertainty?

http://a.espncdn.com/media/ten/2006/0306/photo/g_mcenroe_195.jpg

Use information theory!



- Entropy measures the amount of uncertainty in a probability distribution

- Entropy (or information content in bits) of an answer to a question with *n* possible answers $v_1, \ldots, v_n$:

$$I(P(v_1), \ldots, P(v_n)) = \Sigma_{i=1}^{n} -P(v_i) \log_2 P(v_i)$$

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Using information theory

- Suppose we have p examples with Wait = True (positive) and n examples with Wait = false (negative).

- Our best estimate of the probabilities of Wait = true or false is given by:

$$P(true) \approx p / p + n$$
$$p(false) \approx n / p + n$$

- Hence the entropy (in bits) is given by:

$$I\left(\frac{p}{p+n}, \frac{n}{p+n}\right) = -\frac{p}{p+n}\log_2 \frac{p}{p+n} - \frac{n}{p+n}\log_2 \frac{n}{p+n}$$

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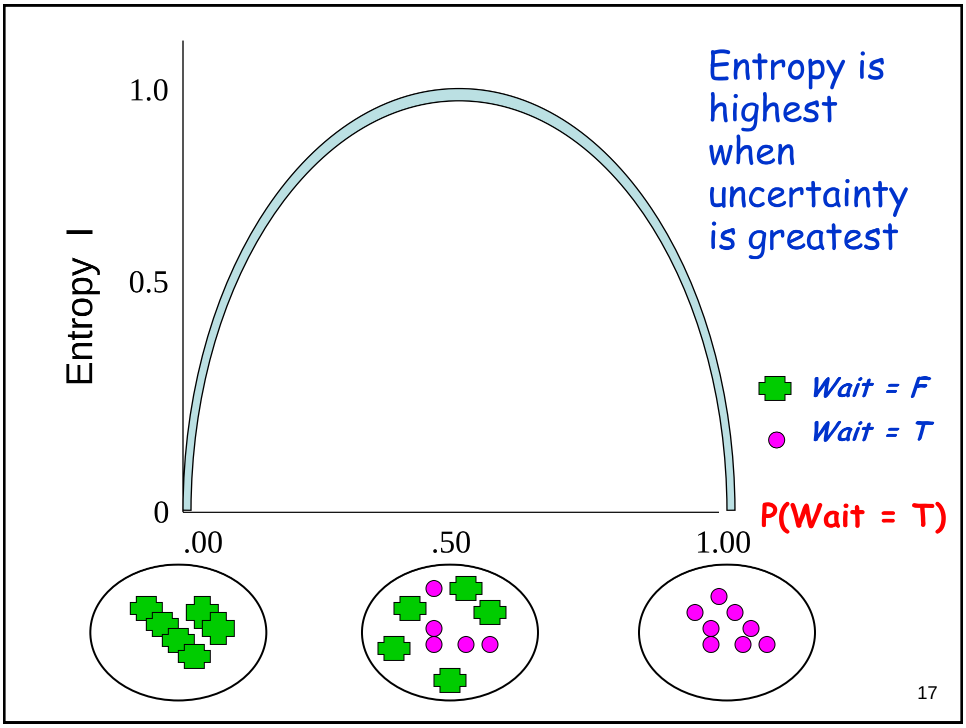

Choosing an attribute to split on

- Idea: a good attribute should reduce uncertainty and result in "gain in information"
- How much information do we gain if we disclose the value of some attribute?
- Answer:

  uncertainty before – uncertainty after

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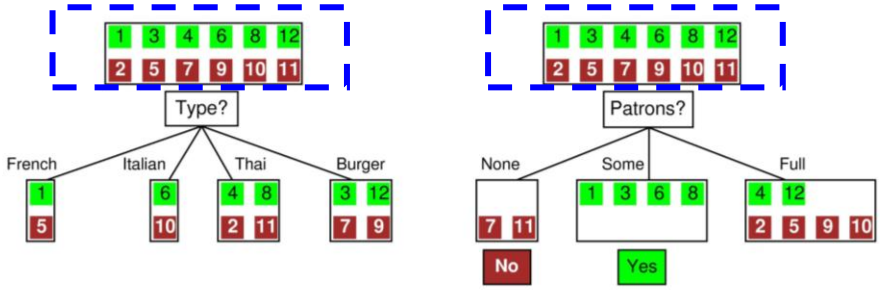
Back at the Restaurant



Before choosing an attribute:

Entropy = - 6/12 log(6/12) – 6/12 log(6/12)

= - log(1/2) = log(2) = 1 bit

There is "1 bit of information to be discovered"

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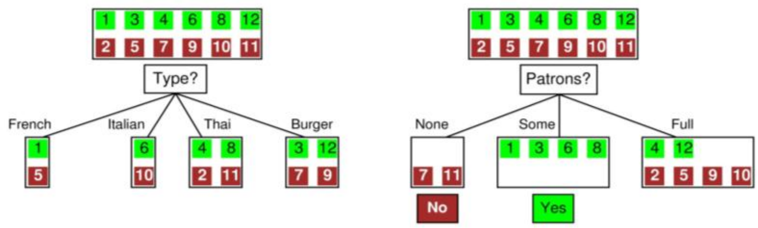
Back at the Restaurant



If we choose Type: Along "French": entropy = 1 bit.

Information gain = 1-1 = 0. (same for other branches)

If we choose Patrons:

In branches "None" and "Some", entropy = 0

For "Full", entropy = -2/6 log(2/6)-4/6 log(4/6) = 0.92

Info gain = (1-0) or (1-0.92) bits > 0 in both cases

So choosing Patrons gains more information!

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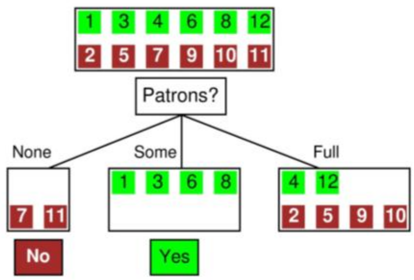
Combining entropy across branches

- Compute average entropy
- Weight entropies according to probability of branches
  2/12 times we entered "None"
  so weight for "None" = 1/6
  "Some" has weight: 4/12 = 1/3
  "Full" has weight: 6/12 = ½



2/12 4/12 6/12

$$AvgEntropy = \sum_{i=1}^{n} \frac{p_i + n_i}{p+n} Entropy\left(\frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i}\right)$$

Sum over all *n* branches

weight for each branch

entropy for each branch

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Information gain

- Information Gain (IG) = reduction in entropy from using attribute A:

  IG(A) = Entropy before choosing
  – AvgEntropy after choosing A

- When constructing each level of decision tree, choose attribute with largest IG

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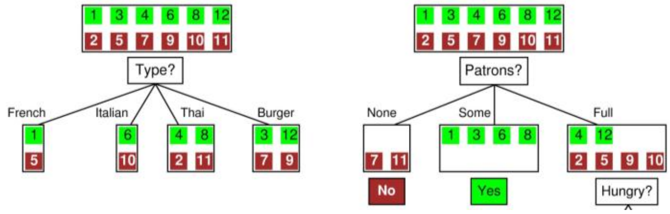
Information gain in our example



$$IG(Patrons) = 1 - [\frac{2}{12} I(0,1) + \frac{4}{12} I(1,0) + \frac{6}{12} I(\frac{2}{6}, \frac{4}{6})] = .541 \text{ bits}$$

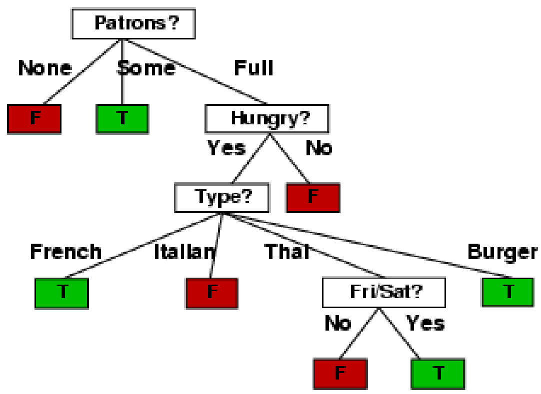
$$IG(Type) = 1 - [\frac{2}{12} I(\frac{1}{2}, \frac{1}{2}) + \frac{2}{12} I(\frac{1}{2}, \frac{1}{2}) + \frac{4}{12} I(\frac{2}{4}, \frac{2}{4}) + \frac{4}{12} I(\frac{2}{4}, \frac{2}{4})] = 0 \text{ bits}$$

Patrons has highest IG of all attributes

$\Rightarrow$ DTL algorithm chooses *Patrons* as the root

Learned Decision Tree for "Wait?"

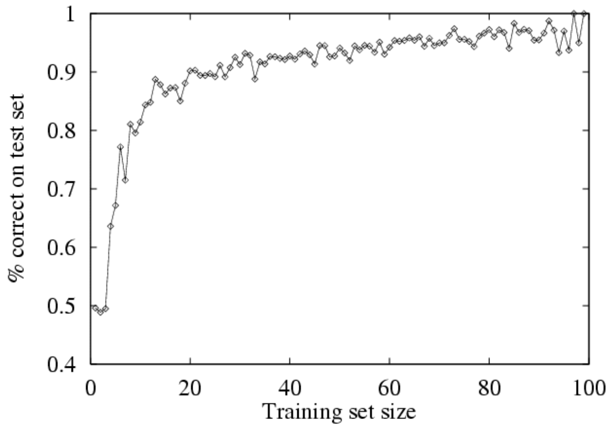
- Decision tree learned from the 12 examples:



- Substantially simpler than "rules-of-thumb" tree
  - more complex hypothesis not justified by small amount of data

Performance Evaluation

- How do we know that the learned tree $h \approx$ *true f* ?
- Answer: Try *h* on a new test set of examples
- Learning curve = % correct on test set as a function of training set size



Generalization

- How do we know the classifier function we have learned is good?
  - Look at generalization error on test data
    - Method 1: Split data into separate training and test sets (the "hold out" method)
    - Method 2: Cross-Validation

Cross-validation

- K-fold cross-validation:
  - Divide data into K subsets of equal size
  - Train learning algorithm K times, leaving out one of the subsets. Compute error on left-out subset
  - Report average error over all subsets
- Leave-1-out cross-validation:
  - Train on all but 1 data point, test on that data point; repeat for each point
  - Report average error over all points

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Next Time

- Other classification methods
  - Nearest Neighbor
  - Support Vector Machines
- To Do:
  - Project 4
  - Read Chapter 18

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