CSE 473

Lecture 5

Heuristics

Last Time: A* Search

- Use an evaluation function f(n) for node n.
  - *f(n)* = estimated total cost of path thru *n* to goal
- $f(n) = g(n) + h(n)$
  - *g(n)* = cost so far to reach *n*
  - *h(n)* = estimated cost from *n* to goal
- Always choose the node from frontier that has the lowest f value.
  - Frontier = priority queue

A* vs. Uniform Cost Search vs. Dijkstra

- All three are optimal but differ in search strategy, time/space complexity, and goals
- A* uses *$f(n) = g(n) + h(n)$* to find shortest path to a single goal
- Uniform cost search uses *$f(n) = g(n)$* to find shortest path to a single goal
- Dijkstra's algorithm also uses *$f(n) = g(n)$* but finds shortest paths to *all nodes*

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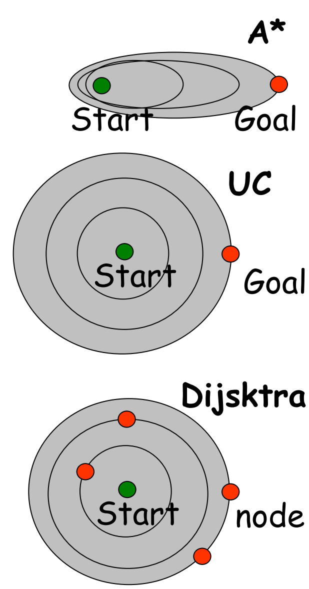
A* vs. Uniform Cost Search vs. Dijkstra

- A* expands mainly toward the goal with the help of the heuristic function
- Uniform-cost and Dijkstra expand in all directions
- A* can be more efficient if the heuristic is good



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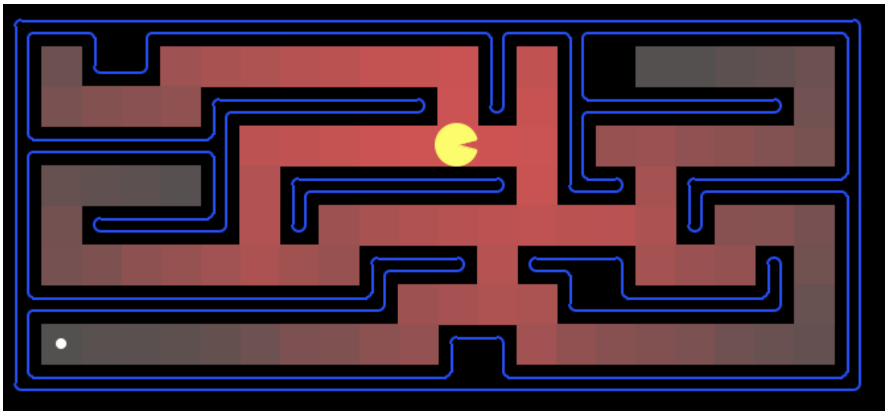
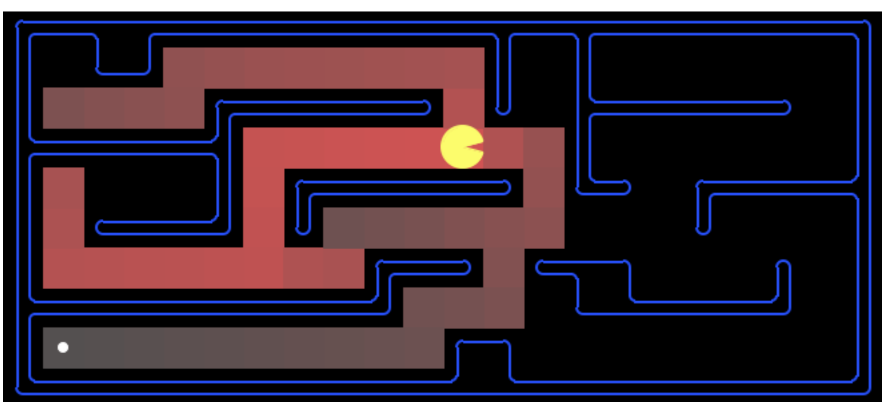
Uniform Cost Pac-Man

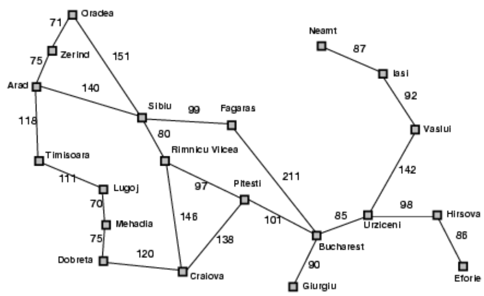


A* Pac-Man with Manhattan distance heuristic



Recall: Admissible Heuristics

- A* uses f(x) = g(x) + h(x)
- g: cost so far
- h: underestimate of remaining costs



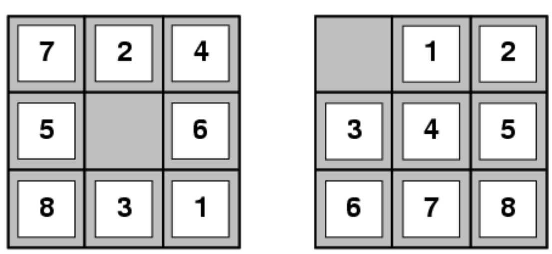
e.g., h_{SLD} is an admissible heuristic for the route finding problem

Proved last time: If h(n) admissible, A* optimal

More heuristic functions

For the 8-puzzle (get to goal state with smallest # of moves), what are some heuristic functions?

- $h_1(n)$ = ?
- $h_2(n)$ = ?



Start State Goal State

Example heuristic functions

Examples:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance (no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

S

	1	2
3	4	5
6	7	8

Goal State

G

- <u>$h_1(S)$ = ?</u>
- <u>$h_2(S)$ = ?</u>

Example heuristics

Examples:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance (no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- <u>$h_1(S)$ = ?</u> 8
- <u>$h_2(S)$ = ?</u> 3+1+2+2+2+3+3+2 = 18

- *Are these admissible heuristics?*

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 **dominates** h_1
- h_2 is better for search (why?)

  Getting closer to the actual cost to goal

- Does one dominate the other for:

  $h_1(n)$ = number of misplaced tiles

  $h_2(n)$ = total Manhattan distance

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Dominance

- For 8-puzzle heuristics h_1 and h_2, typical search costs (average number of nodes expanded for solution depth d):

- *d=12* IDS = 3,644,035 nodes

  $A^*(h_1)$ = 227 nodes

  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes to fit in memory

  $A^*(h_1)$ = 39,135 nodes

  $A^*(h_2)$ = 1,641 nodes

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For many problems, A* can still require too much memory

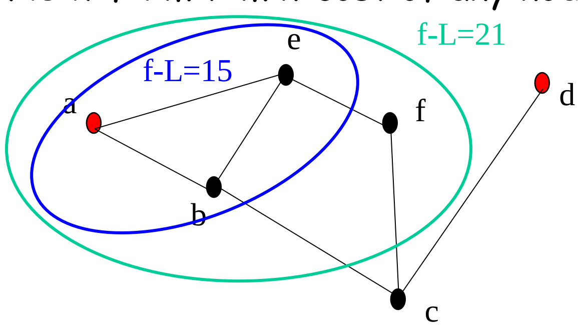
Iterative-Deepening A* (IDA*)

- Less memory required compared to A*
- Like iterative-deepening search, but...
- Depth bound modified to be an **f-limit**

  Start with limit = h(start)

  Prune any node if f(node) > f-limit

  Next f-limit=min-cost of any node pruned



That's cool yo but how do you derive heuristic functions?

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Relaxed Problems

- Derive admissible heuristic from **exact** cost of a solution to a **relaxed** version of problem

For route planning, what is a relaxed problem?

Relax requirement that car has to stay on road
→ Straight Line Distance becomes optimal cost

- Cost of optimal soln to relaxed problem ≤ cost of optimal soln for real problem

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Heuristics for eight puzzle

7	2	3
5	1	6
8	4	

→

1	2	3
4	5	6
7	8	

start **goal**

- What can we relax?

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Heuristics for eight puzzle

7	2	3
5	1	6
8	4	

→

1	2	3
4	5	6
7	8	

Original: Tile can move from location A to B if A is horizontally or vertically next to B *and* B is blank

Relaxed 1: Tile can move from any loc A to any loc B
Cost = h_1 = number of misplaced tiles

Relaxed 2: Tile can move from loc A to loc B if A is horizontally or vertically next to B
Cost = h_2 = total Manhattan distance

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Need for Better Heuristics

Performance of h_2 (Manhattan Distance Heuristic)

8 Puzzle	< 1 second
15 Puzzle	1 minute
24 Puzzle	65000 years

Can we do better?

Adapted from Richard Korf presentation

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Creating New Heuristics

- Given admissible heuristics h_1, h_2, …, h_m, none of them dominating any other, how to choose the best?
- Answer: No need to choose only one! Use:

  $h(n) = \max \{h_1(n), h_2(n), \ldots, h_n(n)\}$

  - h is admissible (why?)
  - h dominates each individual h_i (by construction)

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Pattern Databases [Culberson & Schaeffer 1996]

- Idea: Use solution cost of a subproblem as heuristic. For 8-puzzle: pick any subset of tiles
  - E.g., 3 tiles
- Precompute a table
  - Compute optimal cost of solving just these tiles
    - This is a lower bound on actual cost with all tiles
  - For all possible configurations of these tiles
    - Could be several million
  - Use breadth first search back from goal state
    - State = position of just these tiles (& blank)
- Admissible heuristic h_{DB} for complete state = cost of corresponding sub-problem state in database

Adapted from Richard Korf presentation 21

Combining Multiple Databases

- Repeat for another subset of tiles
  - Precompute multiple tables
- How to combine table values?
  - Use the *max* trick!
- E.g. Optimal solutions to Rubik's cube
  - First found w/ IDA* using pattern DB heuristics
  - Multiple DBs were used (diff subsets of cubies)
  - Most problems solved optimally in 1 day
  - Compare with ***574,000 years*** for IDS

Adapted from Richard Korf presentation 22

Drawbacks of Standard Pattern DBs

- Since we can only take ***max***
  Diminishing returns on additional DBs
- Would like to be able to ***add*** values
  - But not exceed the actual solution cost (admissible)
  - How?

Disjoint Pattern DBs

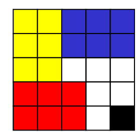
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

- Partition tiles into disjoint sets
  For each set, precompute table
  Don't count moves of tiles not in set
  - This makes sure costs are disjoint
  - Can be added without overestimating!
  - E.g. 8 tile DB has 519 million entries
  - And 7 tile DB has 58 million
- During search
  Look up costs for each set in DB
  Add values to get heuristic function value

  Manhattan distance is a special case of this idea where each set is a single tile

Performance of Disjoint PDBs

- 15 Puzzle: 2000x speedup *vs* Manhattan dist
  IDA* with the two DBs solves 15 Puzzles optimally in 30 milliseconds
- 24 Puzzle: 12 millionx speedup *vs* Manhattan
  IDA* can solve random instances in 2 days
  Uses DBs for 4 disjoint sets as shown
  Each DB has 128 million entries
  Without PDBs: 65,000 years



Adapted from Richard Korf presentation

Next Time

- Local search
- Gaming search and searching for Games
- To do: Project #1, Reading