CSE 473: Artificial Intelligence

Machine Learning: Naive Bayes and Perceptron

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Many slides over the course adapted from Dan Klein.

Example: Spam Filter

- Input: email
- Output: spam/ham
- Setup:
  - Get a large collection of example emails, each labeled “spam” or “ham”
  - Note: someone has to hand label all this data!
  - Want to learn to predict labels of new, future emails
- Features: The attributes used to make the ham / spam decision
  - Words: FREE!
  - Text Patterns: $dd, CAPS
  - Non-text: SenderInContacts
  - …

Dear Sir.

First, I must solicit your confidence in this transaction, this is by virture of its nature as being utterly confidencial and top secret. …

TO BE REMOVED FROM FUTURE MAILINGS, SIMPLY REPLY TO THIS MESSAGE AND PUT "REMOVE" IN THE SUBJECT.

99 MILLION EMAIL ADDRESSES FOR ONLY $99

Ok, Iknow this is blatantly OT but I'm beginning to go insane. Had an old Dell Dimension XPS sitting in the corner and decided to put it to use, I know it was working pre being stuck in the corner, but when I plugged it in, hit the power nothing happened.

Example: Digit Recognition

- Input: images / pixel grids
- Output: a digit 0-9
- Setup:
  - Get a large collection of example images, each labeled with a digit
  - Note: someone has to hand label all this data!
  - Want to learn to predict labels of new, future digit images

- Features: The attributes used to make the digit decision
  - Pixels: (6,8)=ON
  - Shape Patterns: NumComponents, AspectRatio, NumLoops
  - …

0

1

2

1

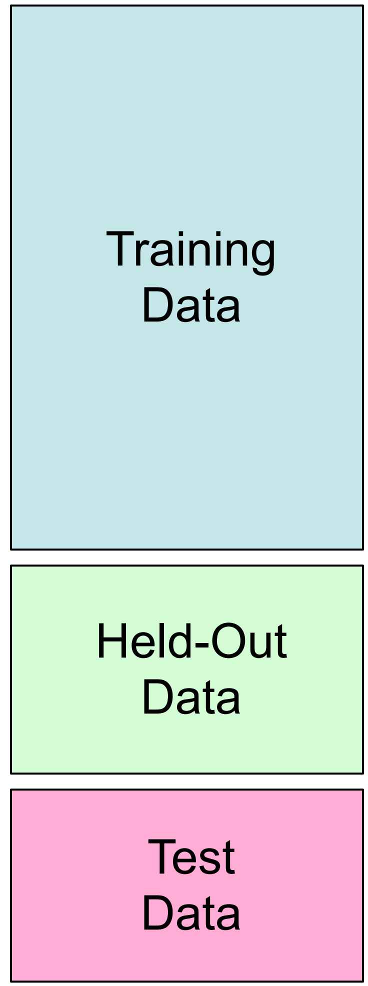
??

Other Classification Tasks

- In classification, we predict labels y (classes) for inputs x
- Examples:
  - Spam detection (input: document, classes: spam / ham)
  - OCR (input: images, classes: characters)
  - Medical diagnosis (input: symptoms, classes: diseases)
  - Automatic essay grader (input: document, classes: grades)
  - Fraud detection (input: account activity, classes: fraud / no fraud)
  - Customer service email routing
  - … many more
- Classification is an important commercial technology!

Important Concepts

- Data: labeled instances, e.g. emails marked spam/ham
  - Training set
  - Held out set
  - Test set
- Features: attribute-value pairs which characterize each x
- Experimentation cycle
  - Learn parameters (e.g. model probabilities) on training set
  - (Tune hyperparameters on held-out set)
  - Very important: never “peek” at the test set!
- Evaluation
  - Compute accuracy of test set
  - Accuracy: fraction of instances predicted correctly
- Overfitting and generalization
  - Want a classifier which does well on *test* data
  - Overfitting: fitting the training data very closely, but not generalizing well



Bayes Nets for Classification

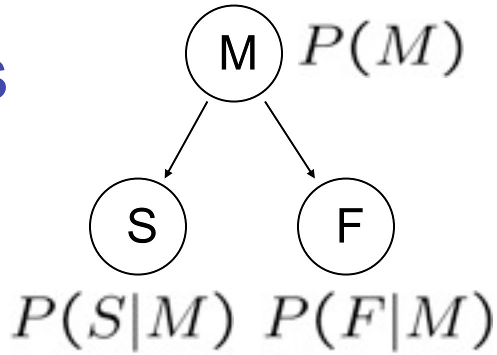
- One method of classification:
  - Use a probabilistic model!
  - Features are observed random variables F_i
  - Y is the query variable
  - Use probabilistic inference to compute most likely Y

$$y = \text{argmax}_y \;\; P(y|f_1 \ldots f_n)$$

- You already know how to do this inference

Simple Classification

- Simple example: two binary features



$$P(m|s,f)$$ direct estimate

$$P(m|s,f) = \frac{P(s,f|m)P(m)}{P(s,f)}$$ Bayes estimate (no assumptions)

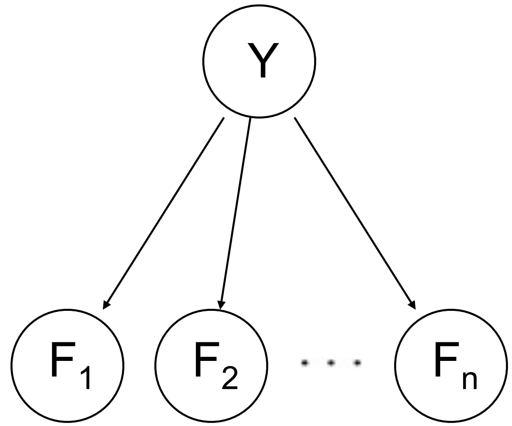
$$P(m|s,f) = \frac{P(s|m)P(f|m)P(m)}{P(s,f)}$$ Conditional independence

$$+ \quad \begin{cases} P(+m,s,f) = P(s|+m)P(f|+m)P(+m) \\ P(-m,s,f) = P(s|-m)P(f|-m)P(-m) \end{cases}$$

General Naïve Bayes

- A general *naive Bayes* model:

$$P(\mathsf{Y}, \mathsf{F}_1 \dots \mathsf{F}_n) = P(\mathsf{Y}) \prod_i P(\mathsf{F}_i|\mathsf{Y})$$



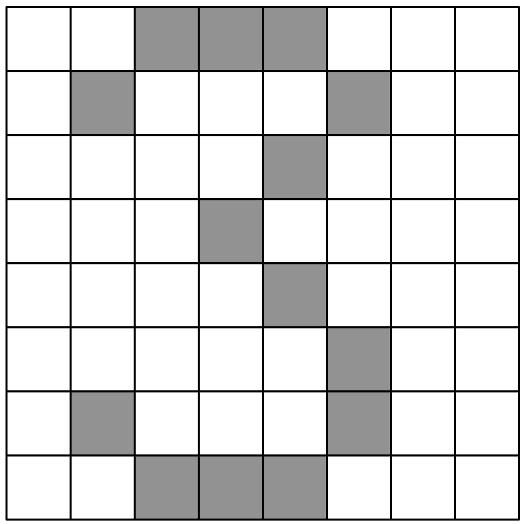
- We only specify how each feature depends on the class
- Total number of parameters is *linear* in *n*

General Naïve Bayes

- What do we need in order to use naïve Bayes?
  - Inference (you know this part)
    - Start with a bunch of conditionals, P(Y) and the $P(F_i|Y)$ tables
    - Use standard inference to compute $P(Y|F_1 \ldots F_n)$
    - Nothing new here
  - Estimates of local conditional probability tables
    - P(Y), the prior over labels
    - $P(F_i|Y)$ for each feature (evidence variable)
    - These probabilities are collectively called the *parameters* of the model and denoted by θ
    - Up until now, we assumed these appeared by magic, but…
    - …they typically come from training data: we'll look at this now

A Digit Recognizer

- Input: pixel grids



- Output: a digit 0-9

Naïve Bayes for Digits

- Simple version:
  - One feature F_{ij} for each grid position <i,j>
  - Possible feature values are on / off, based on whether intensity is more or less than 0.5 in underlying image
  - Each input maps to a feature vector, e.g.

$$\rightarrow \langle F_{0,0} = 0 \;\; F_{0,1} = 0 \;\; F_{0,2} = 1 \;\; F_{0,3} = 1 \;\; F_{0,4} = 0 \;\; \ldots F_{15,15} = 0 \rangle$$

  - Here: lots of features, each is binary valued
- Naïve Bayes model:

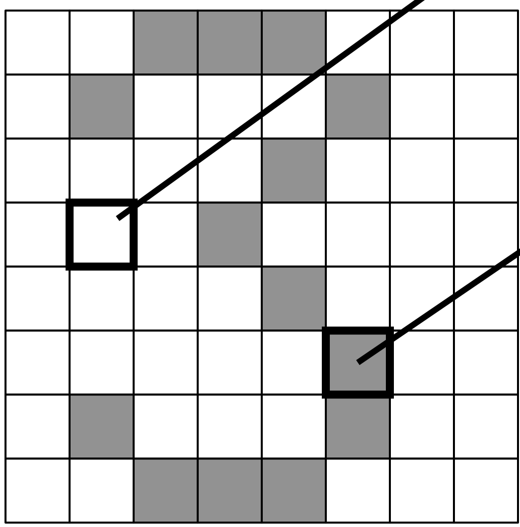
$$P(Y|F_{0,0} \ldots F_{15,15}) \propto P(Y) \prod_{i,j} P(F_{i,j}|Y)$$

- What do we need to learn?

Examples: CPTs

$P(Y)$

1	0.1
2	0.1
3	0.1
4	0.1
5	0.1
6	0.1
7	0.1
8	0.1
9	0.1
0	0.1



$P(F_{3,1} = on|Y)$

1	0.01
2	0.05
3	0.05
4	0.30
5	0.80
6	0.90
7	0.05
8	0.60
9	0.50
0	0.80

$P(F_{5,5} = on|Y)$

1	0.05
2	0.01
3	0.90
4	0.80
5	0.90
6	0.90
7	0.25
8	0.85
9	0.60
0	0.80

Parameter Estimation

- Estimating distribution of random variables like X or X | Y
- *Elicitation:* ask a human!
  - Usually need domain experts, and sophisticated ways of eliciting probabilities (e.g. betting games)
  - Trouble calibrating
- *Empirically:* use training data
  - For each outcome x, look at the *empirical rate* of that value:

$$P_{\mathrm{ML}}(x) = \frac{\mathrm{count}(x)}{\text{total samples}}$$

r g g

$$P_{\mathrm{ML}}(\mathrm{r}) = 1/3$$

  - This is the estimate that maximizes the *likelihood of the data*

$$L(x, \theta) = \prod_i P_\theta(x_i)$$

A Spam Filter

- Naïve Bayes spam filter
- Data:
  - Collection of emails, labeled spam or ham
  - Note: someone has to hand label all this data!
  - Split into training, held-out, test sets
- Classifiers
  - Learn on the training set
  - (Tune it on a held-out set)
  - Test it on new emails

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Naïve Bayes for Text

- Bag-of-Words Naïve Bayes:
  - Predict unknown class label (spam vs. ham)
  - Assume evidence features (e.g. the words) are independent
  - Warning: subtly different assumptions than before!

- Generative model

$$P(C, W_1 \ldots W_n) = P(C) \prod_i P(W_i|C)$$

Word at position i, not i^{th} word in the dictionary!

- Tied distributions and bag-of-words
  - Usually, each variable gets its own conditional probability distribution P(F|Y)
  - In a bag-of-words model
    - Each position is identically distributed
    - All positions share the same conditional probs P(W|C)
    - Why make this assumption?

Example: Spam Filtering

- Model: $P(C, W_1 \ldots W_n) = P(C) \prod_i P(W_i|C)$
- What are the parameters?

$P(C)$

ham	0.66
spam	0.33

$P(W|\text{spam})$

the	0.0156
to	0.0153
and	0.0115
of	0.0095
you	0.0093
a	0.0086
with	0.0080
from	0.0075
...	

$P(W|\text{ham})$

the	0.0210
to	0.0133
of	0.0119
2002	0.0110
with	0.0108
from	0.0107
and	0.0105
a	0.0100
...	

- Where do these come from?

Spam Example

Word	P(w|spam)	P(w|ham)	Tot Spam	Tot Ham
(prior)	0.33333	0.66666	-1.1	-0.4

P(spam | w) = 98.9

Example: Overfitting

$P(\text{features}, C = 2)$

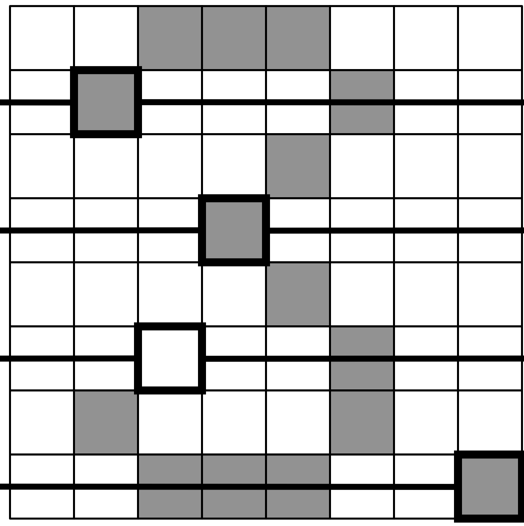
$P(C = 2) = 0.1$

$P(\text{on}|C = 2) = 0.8$

$P(\text{on}|C = 2) = 0.1$

$P(\text{off}|C = 2) = 0.1$

$P(\text{on}|C = 2) = 0.01$



$P(\text{features}, C = 3)$

$P(C = 3) = 0.1$

$P(\text{on}|C = 3) = 0.8$

$P(\text{on}|C = 3) = 0.9$

$P(\text{off}|C = 3) = 0.7$

$P(\text{on}|C = 3) = 0.0$

2 wins!!

Generalization and Overfitting

- Relative frequency parameters will overfit the training data!
  - Just because we never saw a 3 with pixel (15,15) on during training doesn't mean we won't see it at test time
  - Unlikely that every occurrence of "minute" is 100% spam
  - Unlikely that every occurrence of "seriously" is 100% ham
  - What about all the words that don't occur in the training set at all?
  - In general, we can't go around giving unseen events zero probability

- As an extreme case, imagine using the entire email as the only feature
  - Would get the training data perfect (if deterministic labeling)
  - Wouldn't *generalize* at all
  - Just making the bag-of-words assumption gives us some generalization, but isn't enough

- To generalize better: we need to smooth or regularize the estimates

Estimation: Smoothing

- Problems with maximum likelihood estimates:
  - If I flip a coin once, and it's heads, what's the estimate for P(heads)?
  - What if I flip 10 times with 8 heads?
  - What if I flip 10M times with 8M heads?

- Basic idea:
  - We have some prior expectation about parameters (here, the probability of heads)
  - Given little evidence, we should skew towards our prior
  - Given a lot of evidence, we should listen to the data

Estimation: Smoothing

- Relative frequencies are the maximum likelihood estimates

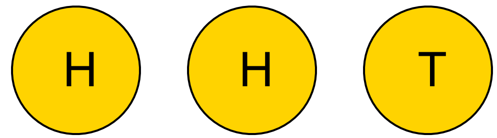
$$\theta_{ML} = \arg\max_{\theta} P(\mathbf{X}|\theta) = \arg\max_{\theta} \prod_i P_\theta(X_i) \quad \Rightarrow \quad P_{\mathrm{ML}}(x) = \frac{\mathrm{count}(x)}{\mathrm{total\ samples}}$$

- In Bayesian statistics, we think of the parameters as just another random variable, with its own distribution

$$\theta_{MAP} = \arg\max_{\theta} P(\theta|\mathbf{X}) = \arg\max_{\theta} P(\mathbf{X}|\theta)P(\theta)/P(\mathbf{X}) = \arg\max_{\theta} P(\mathbf{X}|\theta)P(\theta) \quad \Rightarrow \quad ????$$

Estimation: Laplace Smoothing

- Laplace's estimate:
  - Pretend you saw every outcome once more than you actually did



$$P_{LAP}(x) = \frac{c(x)+1}{\sum_x [c(x)+1]}$$

$$= \frac{c(x)+1}{N+|X|}$$

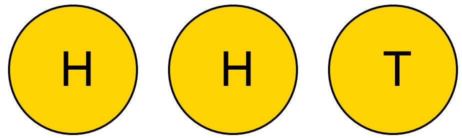
$$P_{ML}(X) =$$

$$P_{LAP}(X) =$$

  - Can derive this as a MAP estimate with *Dirichlet priors* (Bayesian justfication)

Estimation: Laplace Smoothing

- Laplace's estimate (extended):
  - Pretend you saw every outcome k extra times



$$P_{LAP,k}(x) = \frac{c(x) + k}{N + k|X|}$$

$$P_{LAP,0}(X) =$$

$$P_{LAP,1}(X) =$$

  - What's Laplace with k = 0?
  - k is the strength of the prior

$$P_{LAP,100}(X) =$$

- Laplace for conditionals:
  - Smooth each condition independently:

$$P_{LAP,k}(x|y) = \frac{c(x,y) + k}{c(y) + k|X|}$$

Estimation: Linear Interpolation

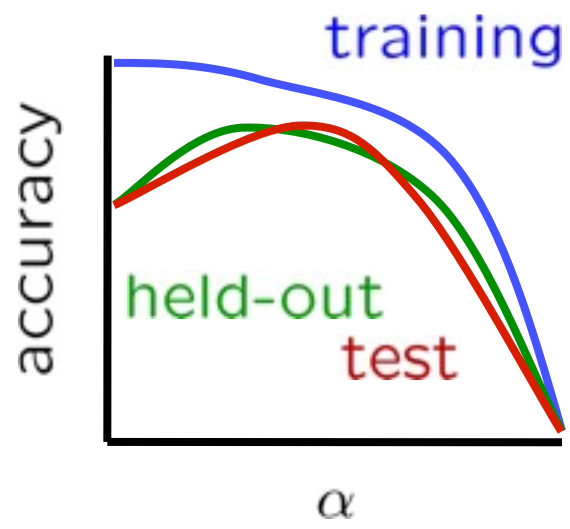
- In practice, Laplace often performs poorly for P(X|Y):
  - When |X| is very large
  - When |Y| is very large
- Another option: linear interpolation
  - Also get P(X) from the data
  - Make sure the estimate of P(X|Y) isn't too different from P(X)

$$P_{LIN}(x|y) = \alpha \hat{P}(x|y) + (1.0 - \alpha)\hat{P}(x)$$

  - What if α is 0? 1?

Tuning on Held-Out Data

- Now we've got two kinds of unknowns
  - Parameters: the probabilities P(Y|X), P(Y)
  - Hyperparameters, like the amount of smoothing to do: k, α
- Where to learn?
  - Learn parameters from training data
  - Must tune hyperparameters on different data
    - Why?
  - For each value of the hyperparameters, train and test on the held-out data
  - Choose the best value and do a final test on the test data



Baselines

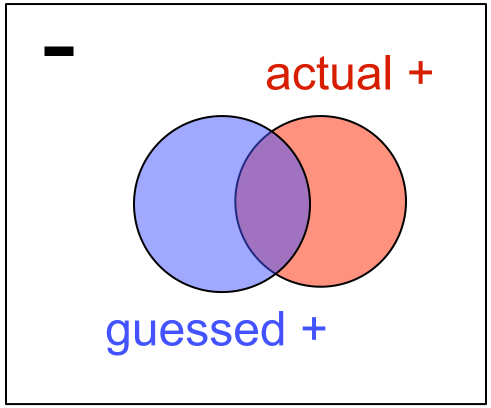
- First step: get a baseline
  - Baselines are very simple “straw man” procedures
  - Help determine how hard the task is
  - Help know what a “good” accuracy is

- Weak baseline: most frequent label classifier
  - Gives all test instances whatever label was most common in the training set
  - E.g. for spam filtering, might label everything as ham
  - Accuracy might be very high if the problem is skewed
  - E.g. calling everything “ham” gets 66%, so a classifier that gets 70% isn’t very good…

- For real research, usually use previous work as a (strong) baseline

Precision vs. Recall

- Let's say we want to classify web pages as homepages or not
  - In a test set of 1K pages, there are 3 homepages
  - Our classifier says they are all non-homepages
  - 99.7 accuracy!
  - Need new measures for rare positive events



- Precision: fraction of guessed positives which were actually positive

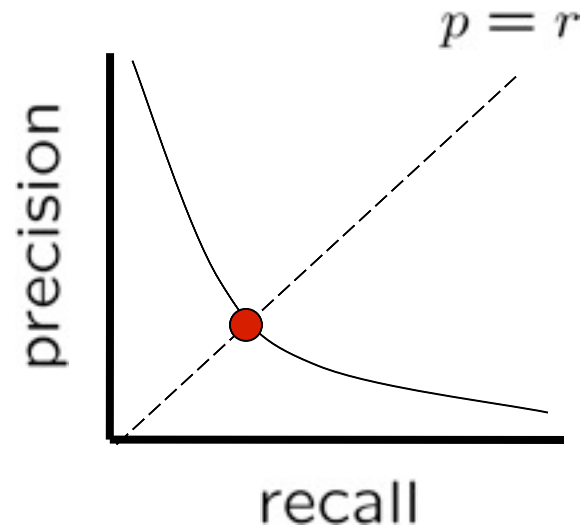
- Recall: fraction of actual positives which were guessed as positive

- Say we detect 5 spam emails, of which 2 were actually spam, and we missed one
  - Precision: 2 correct / 5 guessed = 0.4
  - Recall: 2 correct / 3 true = 0.67

- Which is more important in customer support email automation?

Precision vs. Recall

- Precision/recall tradeoff
  - Often, you can trade off precision and recall
  - Only works well with weakly calibrated classifiers



- To summarize the tradeoff:
  - Break-even point: precision value when p = r
  - F-measure: harmonic mean of p and r:

$$F_1 = \frac{2}{1/p + 1/r}$$

Errors, and What to Do

- Examples of errors

```
Dear GlobalSCAPE Customer,

GlobalSCAPE has partnered with ScanSoft to offer you the
latest version of OmniPage Pro, for just $99.99* - the
regular list price is $499! The most common question we've
received about this offer is - Is this genuine? We would like
to assure you that this offer is authorized by ScanSoft, is
genuine and valid. You can get the . . .
```

```
. . . To receive your $30 Amazon.com promotional certificate,
click through to

  http://www.amazon.com/apparel

and see the prominent link for the $30 offer. All details are
there. We hope you enjoyed receiving this message. However,
if you'd rather not receive future e-mails announcing new
store launches, please click . . .
```

What to Do About Errors?

- Need more features– words aren't enough!
  - Have you emailed the sender before?
  - Have 1K other people just gotten the same email?
  - Is the sending information consistent?
  - Is the email in ALL CAPS?
  - Do inline URLs point where they say they point?
  - Does the email address you by (your) name?

- Can add these information sources as new variables in the NB model

- Next class we'll talk about classifiers which let you easily add arbitrary features more easily

Summary

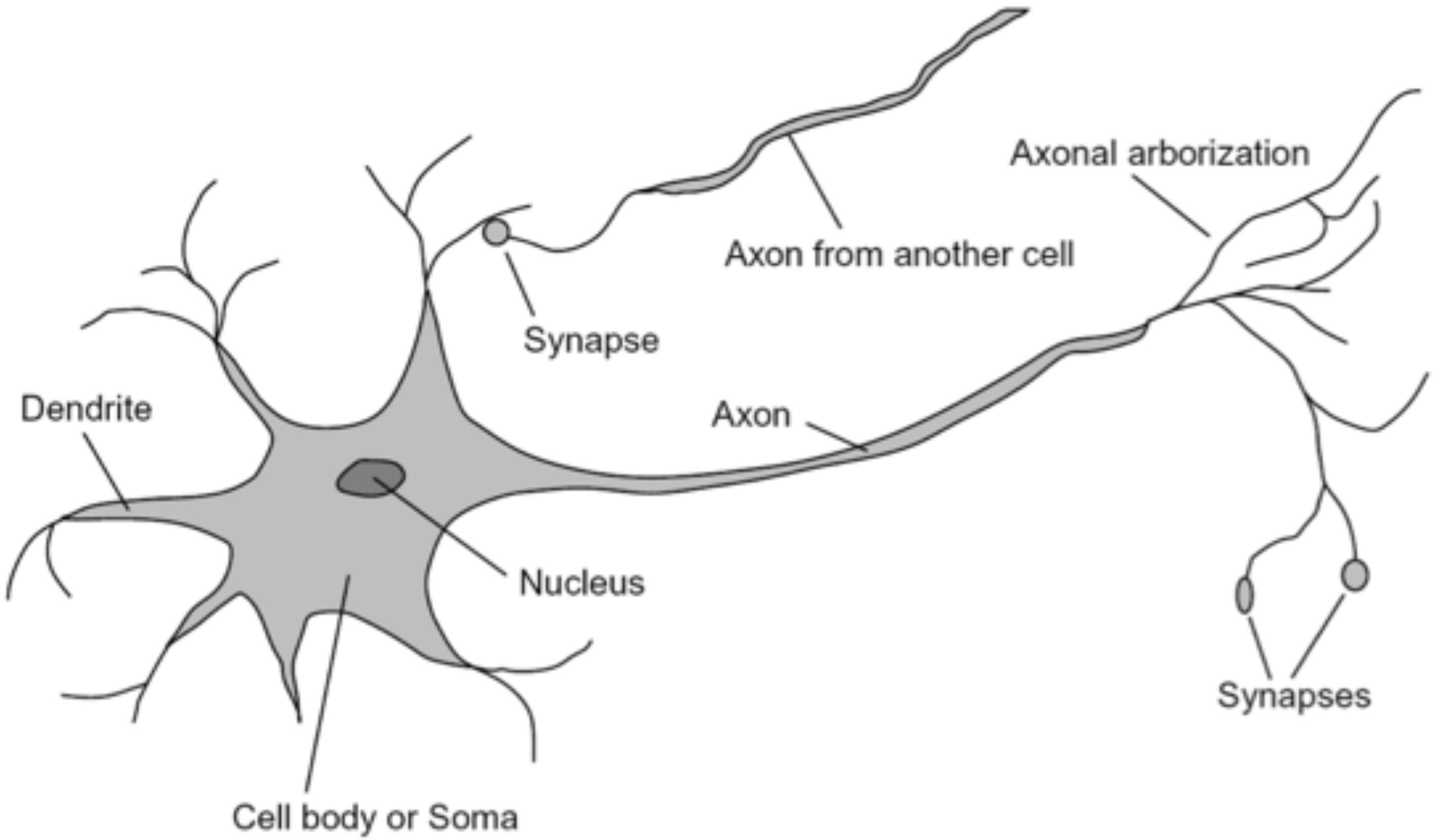
- Bayes rule lets us do diagnostic queries with causal probabilities
- The naïve Bayes assumption takes all features to be independent given the class label
- We can build classifiers out of a naïve Bayes model using training data
- Smoothing estimates is important in real systems
- Classifier confidences are useful, when you can get them

Generative vs. Discriminative

- Generative classifiers:
  - E.g. naïve Bayes
  - A joint probability model with evidence variables
  - Query model for causes given evidence
- Discriminative classifiers:
  - No generative model, no Bayes rule, often no probabilities at all!
  - Try to predict the label Y directly from X
  - Robust, accurate with varied features
  - Loosely: mistake driven rather than model driven

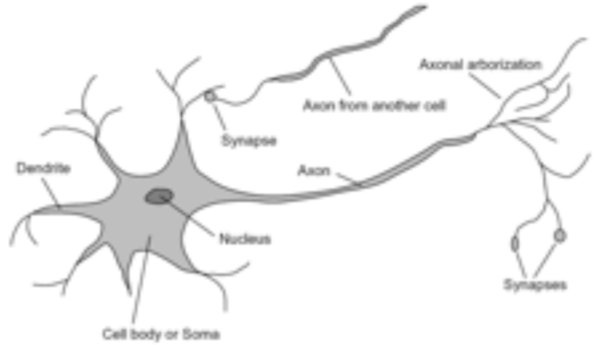
Some (Simplified) Biology

- Very loose inspiration: human neurons



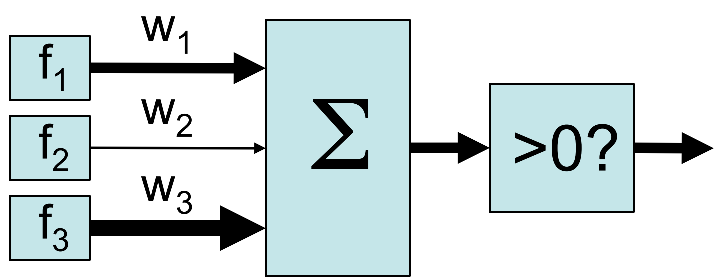
Linear Classifiers

- Inputs are feature values
- Each feature has a weight
- Sum is the activation



$$\text{activation}_w(x) = \sum_i w_i \cdot f_i(x) = w \cdot f(x)$$

- If the activation is:
  - Positive, output +1
  - Negative, output -1



Example: Spam

- Imagine 4 features (spam is "positive" class):
  - free (number of occurrences of "free")
  - money (occurrences of "money")
  - BIAS (intercept, always has value 1)

$$w \cdot f(x)$$

$$\sum_i w_i \cdot f_i(x)$$

x	$f(x)$	w
"free money"	BIAS : 1 free : 1 money : 1 ...	BIAS : -3 free : 4 money : 2 ...

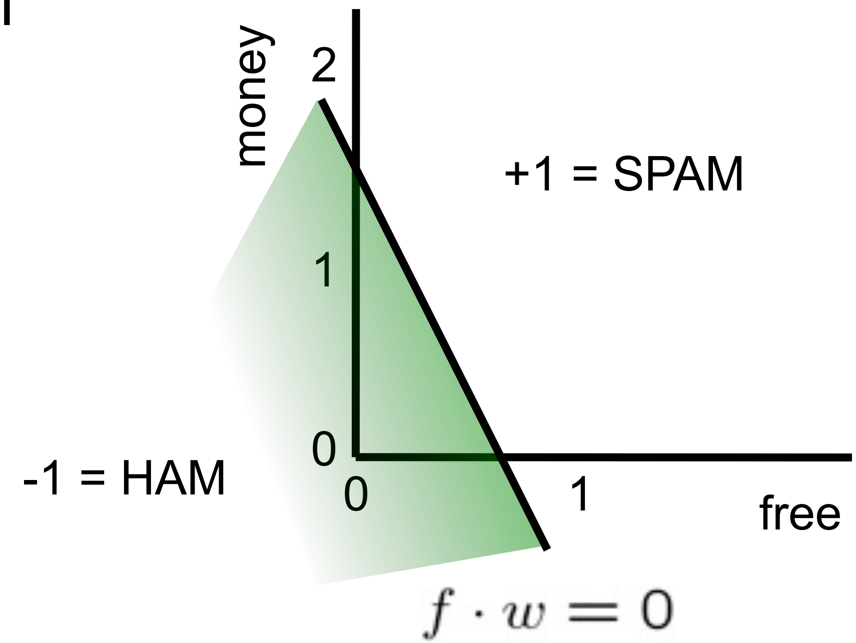
$$\begin{aligned} &(1)(-3) \quad + \\ &(1)(4) \quad + \\ &(1)(2) \quad + \\ &\ldots \\ &= 3 \end{aligned}$$

Binary Decision Rule

- In the space of feature vectors
  - Examples are points
  - Any weight vector is a hyperplane
  - One side corresponds to Y=+1
  - Other corresponds to Y=-1

w

```
BIAS  : -3
free  :  4
money :  2
...
```



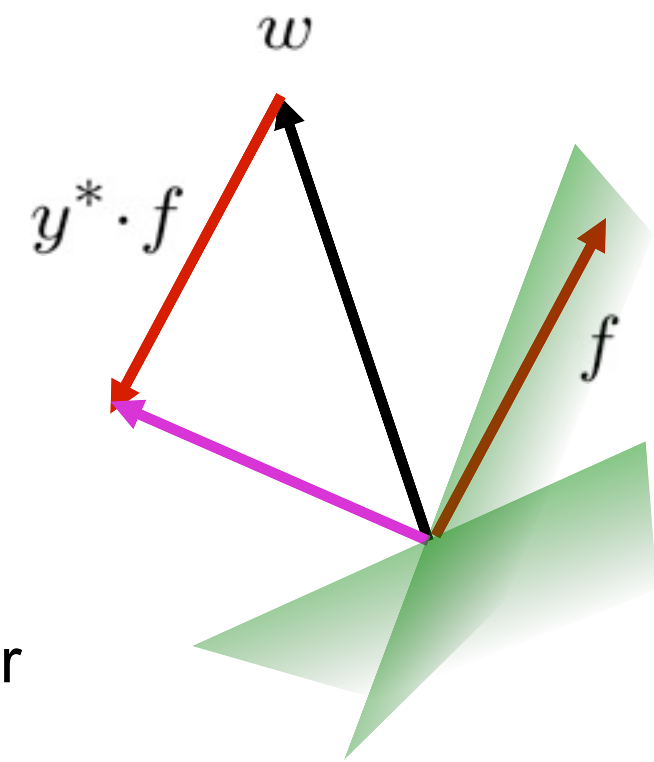
Binary Perceptron Algorithm

- Start with zero weights
- For each training instance:
  - Classify with current weights

$$y = \begin{cases} +1 & \text{if } w \cdot f(x) \geq 0 \\ -1 & \text{if } w \cdot f(x) < 0 \end{cases}$$

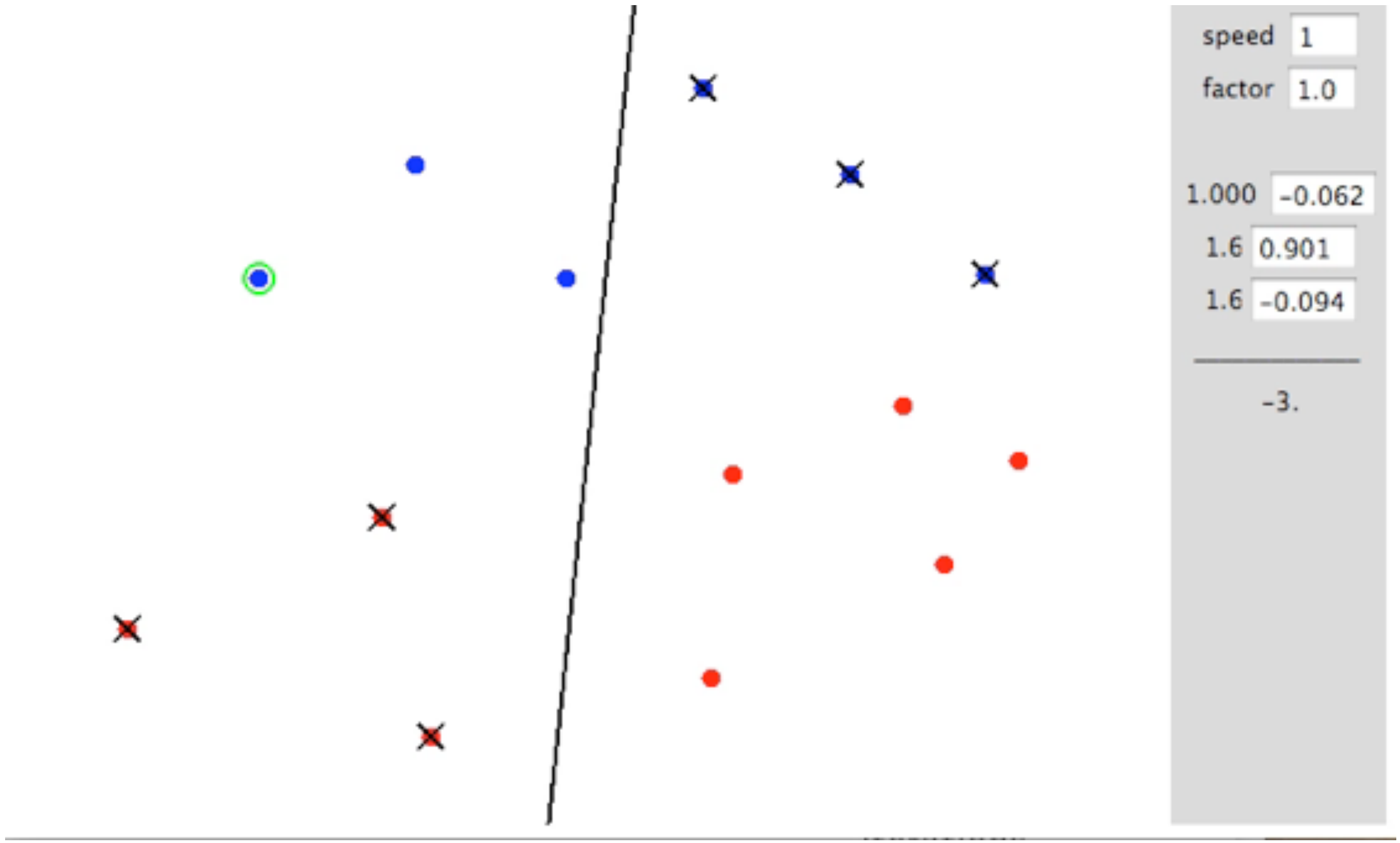
  - If correct (i.e., y=y*), no change!
  - If wrong: adjust the weight vector by adding or subtracting the feature vector. Subtract if y* is -1.

$$w = w + y^* \cdot f$$



Examples: Perceptron

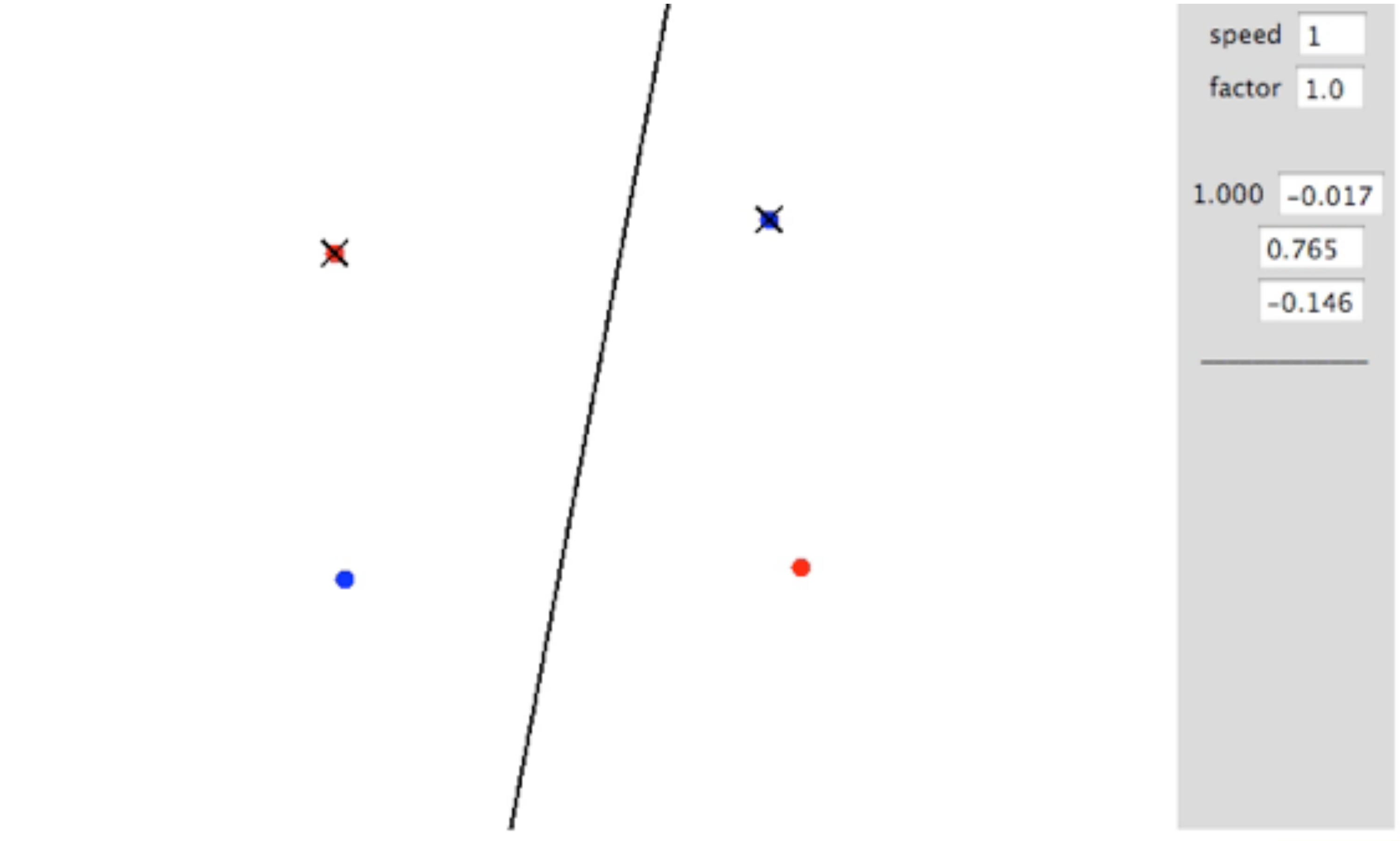
- Separable Case



http://isl.ira.uka.de/neuralNetCourse/2004/VL_11_5/Perceptron.html

Examples: Perceptron

- Inseparable Case



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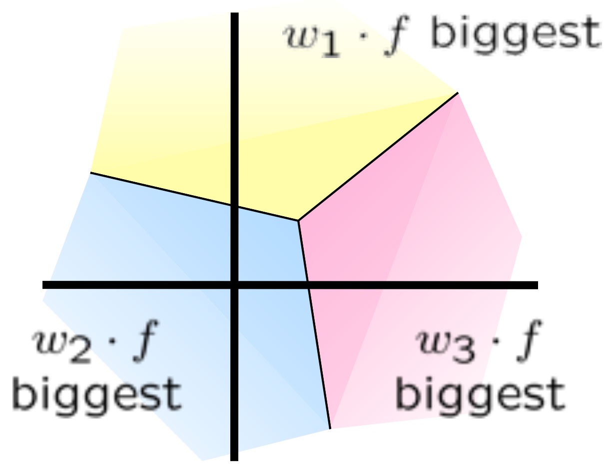
Multiclass Decision Rule

- If we have more than two classes:
  - Have a weight vector for each class: w_y
  - Calculate an activation for each class



$$\text{activation}_w(x, y) = w_y \cdot f(x)$$

  - Highest activation wins

$$y = \arg\max_y \; (\text{activation}_w(x, y))$$

Example

"win the vote"

"win the election"

"win the game"

w_{SPORTS}	$w_{POLITICS}$	w_{TECH}
BIAS :	BIAS :	BIAS :
win :	win :	win :
game :	game :	game :
vote :	vote :	vote :
the :	the :	the :
...	...	...

Example

"win the vote"

```
BIAS  :  1
win   :  1
game  :  0
vote  :  1
the   :  1
...
```

w_{SPORTS}

```
BIAS  : -2
win   :  4
game  :  4
vote  :  0
the   :  0
...
```

$w_{POLITICS}$

```
BIAS  :  1
win   :  2
game  :  0
vote  :  4
the   :  0
...
```

w_{TECH}

```
BIAS  :  2
win   :  0
game  :  2
vote  :  0
the   :  0
...
```

The Multi-class Perceptron Alg.

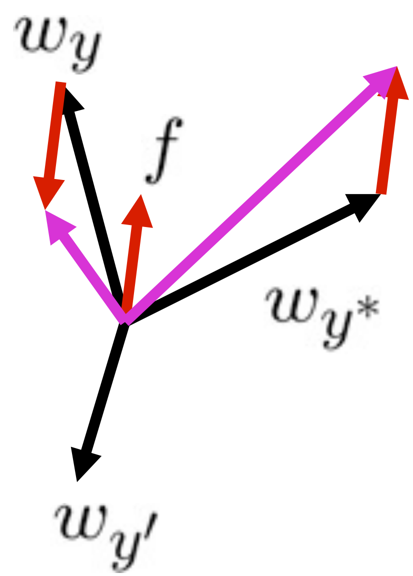
- Start with zero weights
- Iterate training examples
  - Classify with current weights

$$\begin{aligned} y &= \arg\max_y \; w_y \cdot f(x) \\ &= \arg\max_y \; \sum_i w_{y,i} \cdot f_i(x) \end{aligned}$$

  - If correct, no change!
  - If wrong: lower score of wrong answer, raise score of right answer

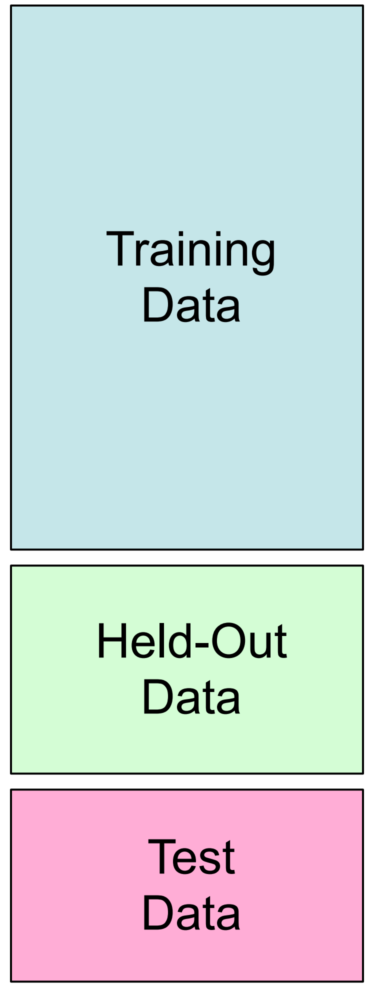
$$w_y = w_y - f(x)$$

$$w_{y^*} = w_{y^*} + f(x)$$



Mistake-Driven Classification

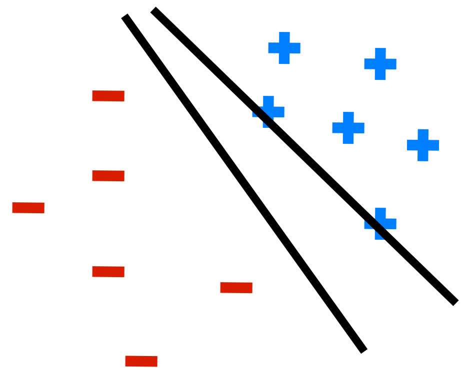
- For Naïve Bayes:
  - Parameters from data statistics
  - Parameters: probabilistic interpretation
  - Training: one pass through the data
- For the perceptron:
  - Parameters from reactions to mistakes
  - Parameters: discriminative interpretation
  - Training: go through the data until held-out accuracy maxes out



Properties of Perceptrons

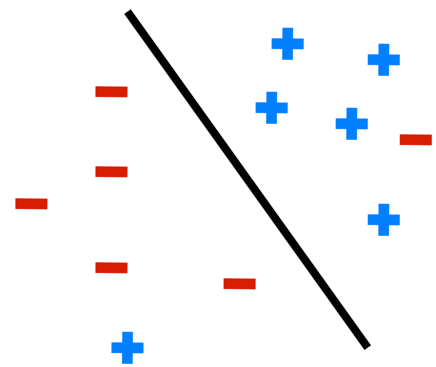
- Separability: some parameters get the training set perfectly correct

Separable



- Convergence: if the training is separable, perceptron will eventually converge (binary case)

Non-Separable

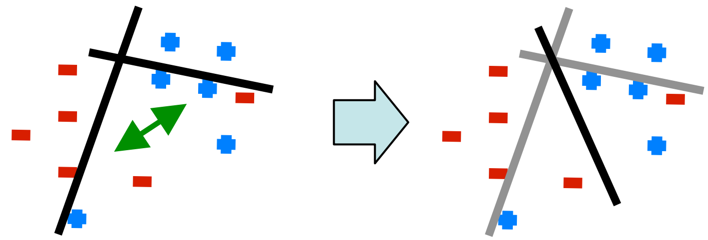


- Mistake Bound: the maximum number of mistakes (binary case) related to the *margin* or degree of separability

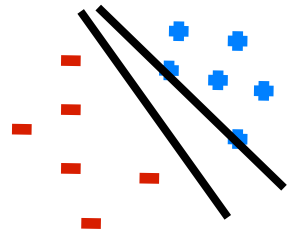
$$\text{mistakes} < \frac{k}{\delta^2}$$

Problems with the Perceptron

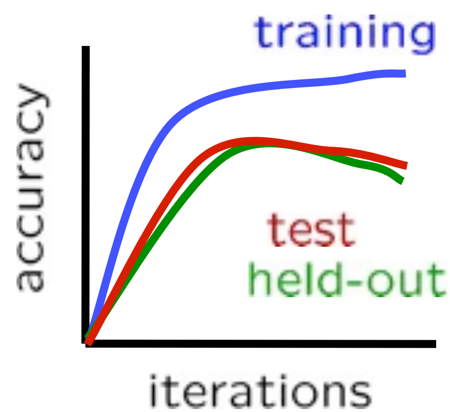
- Noise: if the data isn't separable, weights might thrash
  - Averaging weight vectors over time can help (averaged perceptron)



- Mediocre generalization: finds a "barely" separating solution



- Overtraining: test / held-out accuracy usually rises, then falls
  - Overtraining is a kind of overfitting



Fixing the Perceptron

- Idea: adjust the weight update to mitigate these effects

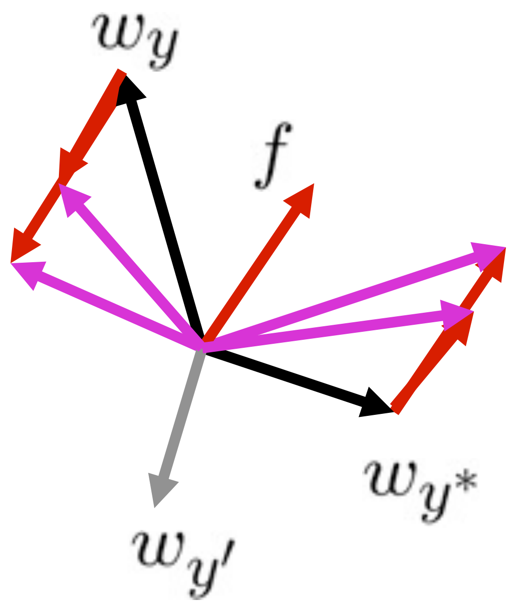
- MIRA*: choose an update size that fixes the current mistake…
- … but, minimizes the change to w

$$\min_w \frac{1}{2} \sum_y ||w_y - w'_y||^2$$

$$w_{y^*} \cdot f(x) \geq w_y \cdot f(x) + 1$$

- The +1 helps to generalize

* Margin Infused Relaxed Algorithm



Guessed y instead of y^* on example x with features $f(x)$

$$w_y = w'_y - \tau f(x)$$
$$w_{y^*} = w'_{y^*} + \tau f(x)$$

Minimum Correcting Update

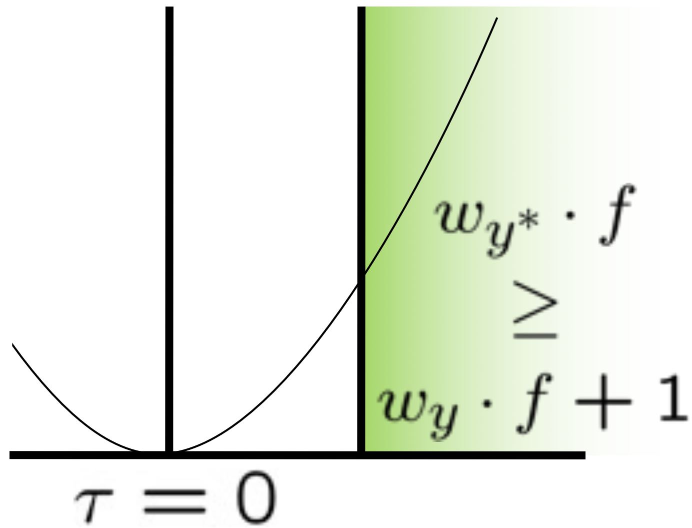
$$\min_{w} \frac{1}{2}\sum_{y} ||w_y - w'_y||^2$$

$$w_{y^*} \cdot f \geq w_y \cdot f + 1$$

$$w_y = w'_y - \tau f(x)$$
$$w_{y^*} = w'_{y^*} + \tau f(x)$$

$$\min_{\tau} ||\tau f||^2$$

$$w_{y^*} \cdot f \geq w_y \cdot f + 1$$



$$(w'_{y^*} + \tau f) \cdot f = (w'_y - \tau f) \cdot f + 1$$

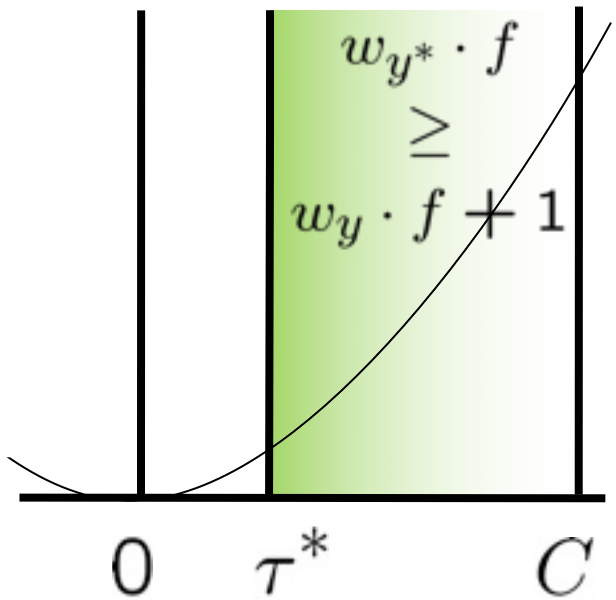
$$\tau = \frac{(w'_y - w'_{y^*}) \cdot f + 1}{2 f \cdot f}$$

min not τ=0, or would not have made an error, so min will be where equality holds

Maximum Step Size

- In practice, it's also bad to make updates that are too large
  - Example may be labeled incorrectly
  - You may not have enough features
  - Solution: cap the maximum possible value of τ with some constant C

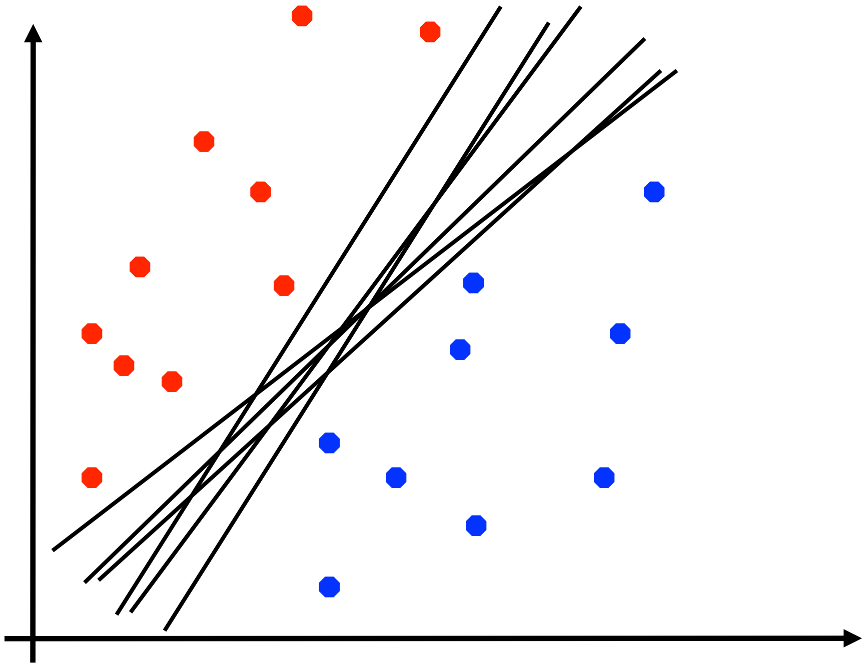
$$\tau^* = \min\left(\frac{(w'_y - w'_{y^*}) \cdot f + 1}{2f \cdot f}, C\right)$$



  - Corresponds to an optimization that assumes non-separable data
  - Usually converges faster than perceptron
  - Usually better, especially on noisy data

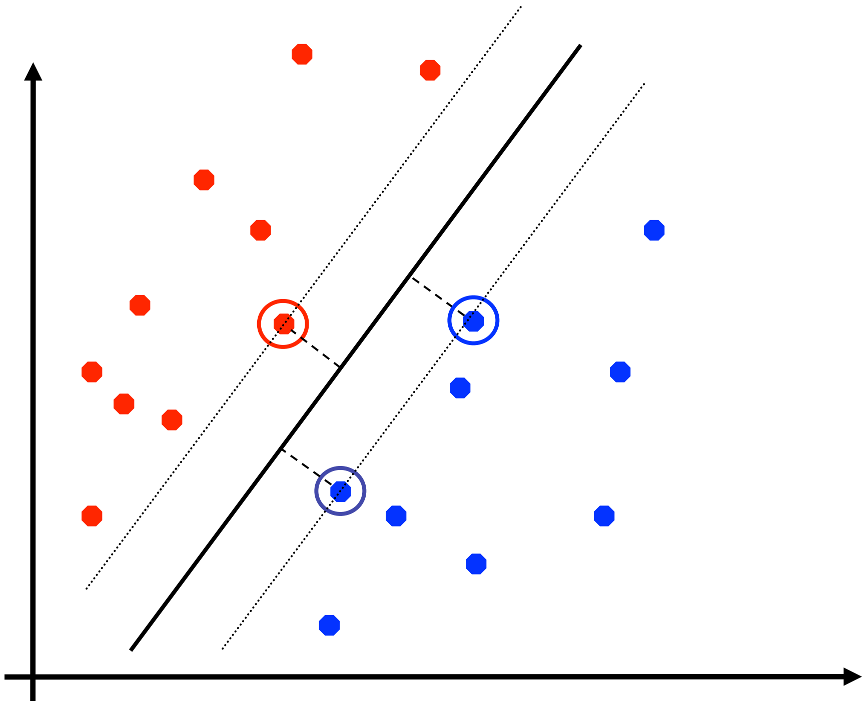
Linear Separators

- Which of these linear separators is optimal?



Support Vector Machines

- Maximizing the margin: good according to intuition, theory, practice
- Only support vectors matter; other training examples are ignorable
- Support vector machines (SVMs) find the separator with max margin
- Basically, SVMs are MIRA where you optimize over all examples at once



MIRA

$$\min_{w} \frac{1}{2}||w - w'||^2$$

$$w_{y^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

SVM

$$\min_{w} \frac{1}{2}||w||^2$$

$$\forall i, y \; w_{y^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

Classification: Comparison

- Naïve Bayes
  - Builds a model training data
  - Gives prediction probabilities
  - Strong assumptions about feature independence
  - One pass through data (counting)

- Perceptrons / MIRA:
  - Makes less assumptions about data
  - Mistake-driven learning
  - Multiple passes through data (prediction)
  - Often more accurate