1. (10 points) **State Space**

   Answer the following questions. Assume a minimal state representation.

   (a) (1 point) Consider a M by N grid of squares, where $M, N \geq 2$. Let there be a king on one of the squares. In terms of the variables introduced, what is the size of the world state space?

   (b) (1 point) From now on, each square can have a pawn on it. The king and a pawn can be on the same square. In terms of the variables introduced, what is the size of the world state space?

   (c) (1 point) From now on, the king cannot be on the same square as a pawn. In terms of the variables introduced, what is the size of the world state space?

   (d) From now on, the king can either move to any of its adjacent squares (including diagonal) on the grid or attack a pawn in one of those squares to reduce its health point by 1. A pawn will be removed from its square if it loses all its health points. For the following questions, your answer should be a number and not an expression.

      i. (1 point) What is the maximum number of actions available for the king at any state?

      ii. (1 point) What is the maximum number of actions available for the king at any state, if $M, N = 2$?

      iii. (1 point) What is the minimum number of actions available for the king at any state?

   (e) For this problem alone, the king wants to kill all pawns on the grid. Also assume that the king cannot move to any squares previously occupied by the pawns. In each of the following questions, consider the following instance of the problem where K represents the king and numbers represent the health points of the pawns at each square. A square with no numbers means there is no pawn present there:

K		3
	3	5

   In this instance we are considering a three by three board with three pawns, two of which start with health three and one of which starts with health five as well as one king.

i. (1 point) How many search states are there?

ii. (1 point) How many search states pass the goal test? That is, how many states are goal states?

(f) For this problem alone, the king wants to visit the maximum number of unique squares given the constraints of each sub-problem (he can revisit squares as he wants and move after visiting the maximum squares). In each of the following questions, consider the following instance of the problem where K represents the king and numbers represent the health points of the pawns at each square:

	1	
K	2	

i. (1 point) How many search states are there, given that the king cannot attack the pawns?

ii. (1 point) How many search states pass the goal test, given that the king cannot attack the pawns?

2. (5 points) **Uninformed Search**

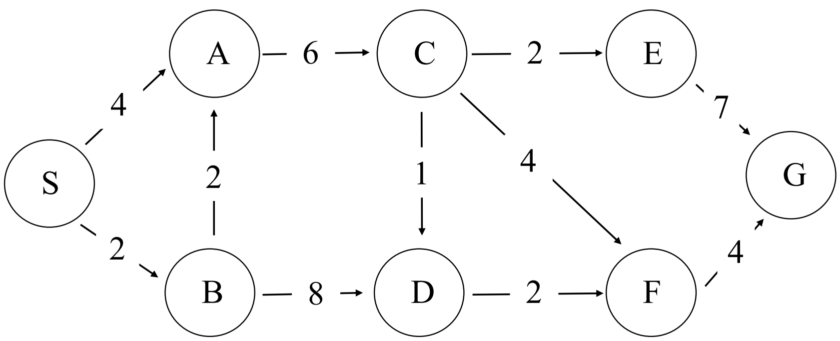
   (a) (3 points) Let's define the procedure of hill-climbing. You start at a random location on a hill, your goal is to get to the highest point on the hill. At each time step, you will take a step toward the location next to you that is higher than your current location.

   Is hill-climbing complete? Why? If not, is there any way to improve the performance in the discrete problem space?

   (b) (2 points) In what circumstances is Depth-first Search preferred over Breadth-first Search? Write down two circumstances.

3. (10 points) **Informed Search**

You are given a graph below, and two heuristics functions h_1 and h_2. The task requires to start from state S and arrive at state G. Break any ties alphabetically (e.g., if two nodes are enqueued at the same time or have the same priority, first deque the node that has the lowest value alphabetically).



State	h_1	h_2
S	12	15
A	11	12
B	11	13
C	6	7
D	4	6
E	5	5
F	3	4
G	0	0

Now answer the following questions:

(a) (1 point) What is the cost of the optimal path for uniform cost search from S to G?

(b) (2 points) If using A^* search, is h_1 admissible? If not, provide a state at which the heuristic is inadmissible. Is it consistent? If not, provide an arc at which the heuristic is inconsistent.

(c) (2 points) If using A^* search, is h_2 admissible? If not, provide a state at which the heuristic is inadmissible. Is it consistent? If not, provide an arc at which the heuristic is inconsistent.

(d) (3 points) List all the visited nodes in the order of visiting them during (1) uniform cost search, (2) greedy search using h_2, and (3) A^* search using h_2. List in the format of $S \to \cdots \to G$.

(e) (2 points) For any given graph, is the path returned by greedy search always more expensive than the path returned by A* search? If you answer yes, explain; If you answer no, provide a simple counterexample. (Assume the heuristic used for A* is consistent and admissible.)