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CSE 481C PSet 1

1) $x_{0_1} \ldots x_{0_n}$ = IDs of robot $0 \ldots n$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$ = global average, by defn of average

2) Let x_i, x_j any two current values $\quad \bar{x} = \frac{1}{n}\sum x_k \quad = \frac{1}{n}(x_i + x_j + x_0 + \ldots)$

Let x_i', x_j' vales after pairwise average $\quad \bar{x} = \frac{1}{n}\sum x_k' \quad ? = \frac{1}{n}(x_i' + x_j' + x_0 + \ldots)$

$$x_i' = \frac{x_i + x_j}{2} = x_j' = \frac{x_i + x_j}{2}$$

$$\boxed{x_i' + x_j' = \frac{x_i + x_j}{2} + \frac{x_i + x_j}{2} = x_i + x_j}$$ Sum is unchanged $\Rightarrow \bar{x}$ is unchanged

3) Average all nbrs simultaneously

x_1 — x_2 — x_3 $\qquad \Sigma = x_1 + x_2 + x_3$

$$x_1' = \frac{x_1 + x_2}{2} \qquad x_2' = \frac{x_1 + x_2 + x_3}{3} \qquad x_3' = \frac{x_2 + x_3}{2}$$

$$\Sigma = x_1' + x_2' + x_3' = \frac{x_1 + x_2}{2} + \frac{x_1 + x_2 + x_3}{3} + \frac{x_2 + x_3}{2} = \frac{3x_1 + 3x_2 + 2x_1 + 2x_2 + 2x_3 + 3x_2 + 3x_3}{6}$$

$$\Sigma' = \frac{5x_1 + 8x_2 + 5x_3}{6}$$

$\Sigma \neq \Sigma' \Rightarrow$ non-pairwise does not work

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4) Convergence $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2 = \frac{1}{n}\left[(x_i-\bar{x})^2 + (x_j-\bar{x})^2 + (x_0-\bar{x})^2 + \dots\right]$

$$\sigma^2 = \frac{1}{n}\left[x_i^2 - 2x_i\bar{x} + \bar{x}^2 + x_j^2 + 2x_j\bar{x} + \bar{x}^2\right] + \frac{1}{n}\left[\overset{c}{\cancel{\sim\sim}}\right]$$

$$\sigma^2 = \frac{1}{n}\left[x_i^2 - 2x_i\bar{x} + x_j^2 - 2x_j\bar{x} + 2\bar{x}^2\right] + c$$

$$x_i' = x_j' = \frac{x_i + x_j}{2}$$

$$\sigma^{2'} = \frac{1}{n}\left[\left(\frac{x_i+x_j}{2} - \bar{x}\right)^2 + \left(\frac{x_i+x_j}{2} - \bar{x}\right)^2\right] + \frac{1}{n}\left[\overset{c}{\cancel{\sim\sim}}\right]$$

$$\sigma^{2'} = \frac{1}{n}\left[2\left(\frac{x_i^2 + 2x_ix_j + x_j^2}{4} - \frac{2\bar{x}(x_i+x_j)}{2} + \bar{x}^2\right)\right] + c$$

$$\sigma^{2'} = \frac{1}{n}\left(\frac{x_i^2 + 2x_ix_j + x_j^2}{2} - 2\bar{x}x_i - 2\bar{x}x_j + 2\bar{x}^2\right) + c$$

Show that $\sigma^{2'} < \sigma^2$ if $x_i \neq x_j$

$$\frac{x_i^2 + 2x_ix_j + x_j^2}{2} \cancel{-2\bar{x}x_i} \cancel{-2\bar{x}x_j} \cancel{+2\bar{x}^2} < x_i^2 \cancel{-2x_i\bar{x}} + x_j^2 \cancel{-2x_j\bar{x}} \cancel{+2\bar{x}^2}$$

$$\frac{x_i^2 + 2x_ix_j + x_j^2}{2} < x_i^2 + x_j^2$$

Let $x_i = x_j + \Delta$ (Because $x_i \neq x_j$)

$$(x_j+\Delta)^2 + 2(x_j+\Delta)x_j + x_j^2 < 2(x_j+\Delta)^2 + 2x_j^2$$

$$\cancel{x_j^2} + \cancel{2x_j}\Delta + \Delta^2 + \cancel{2x_j^2} + \cancel{2x_j}\Delta + \cancel{x_j^2} < \cancel{2x_j^2} + \cancel{4x_j}\Delta + 2\Delta^2 + \cancel{2x_j^2}$$

$$\Delta^2 < 2\Delta^2$$

$\boxed{1 < 2}$ q.e.d.