Physical Modeling Synthesis of Sound

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One View of Sound

Sound is a waveform; we can record it, store it, and play it back accurately

PCM playback is all we need for interactions, movies, games, etc.

But, take one visual analogy:

"If I take lots of polaroid images, I can flip through them real fast and make any image sequence"

Interaction? We manipulate lots of PCM

Views of Sound

- Time Domain x(t)
  (from physics, and time's arrow)
- Frequency Domain X(f)
  (from math, and perception)
- Production what caused it
- Perception our "image" of it

Views of Sound

- The Time Domain
  is most closely related to

  Production
- The Frequency Domain
  is most closely related to

  Perception

Views of Sound: Time Domain

Sound is produced/modeled by physics, described by quantities of

- **Force** — **force = mass * acceleration**
- **Position** — ***x(t)*** — **actually *[x(t), y(t), z(t)]***
- **Velocity** — **Rate of change of position** — ***dx/dt***
- **Acceleration** — **Rate of change of velocity** — ***dv/dt***
  (2nd derivative of position) — d^2x/dt^2

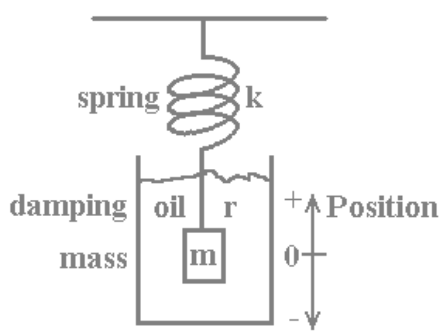
Examples: **Mass,Spring,Damper** **Wave Equation**

Mass/Spring/Damper

$F = ma = -ky - rv - mg$

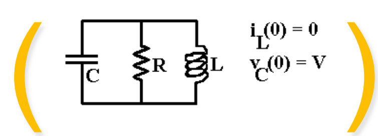
$ma = -ky - rv$

(if gravity negligible)



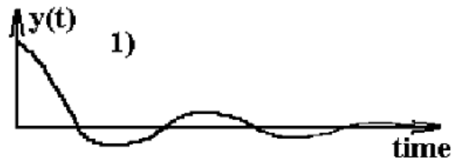
Solution:

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = 0$$

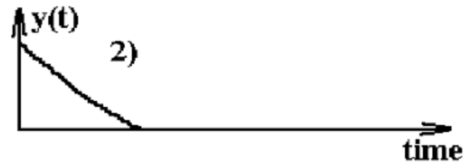


2nd Order Linear Diff Eq. Solution

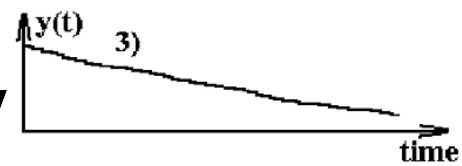
1) Underdamped:
$y(t) = Y_0 e^{-t/\tau} \cos(\omega t)$
exp. * oscillation



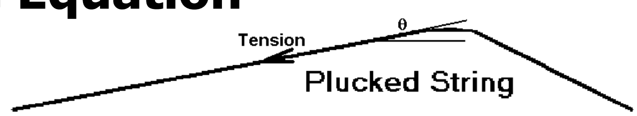
2) Critically damped:
fast exponential decay



3) Overdamped:
slow exponential decay



The Wave Equation



$df_y = (T \sin\theta)_{x+dx} - (T\sin\theta)_x$ **(for each dx of string)**

$f(x+dx) = f(x) + \delta f/\delta x \, dx + \ldots$ **(Taylor' s series in space)**

assume $\sin\theta = \theta$ (for small θ)

$F = ma = \rho \, dx \, d^2y/dt^2$ **(ρ = mass/length)**

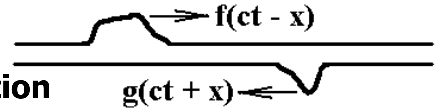
Solution:

The wave equation
($c^2 = T / \rho$)

$$\frac{d^2y}{dx^2} = \frac{1}{c^2}\frac{d^2y}{dt^2}$$

Traveling Wave String Solution

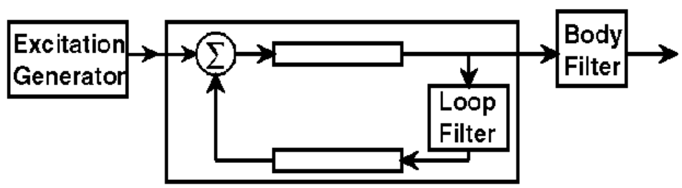
D' Alembert Solution of
2nd order wave equation
(left and right going waves)



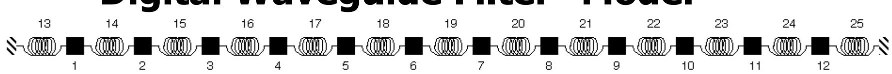
"Digital Waveguide Filter" Model (Smith)

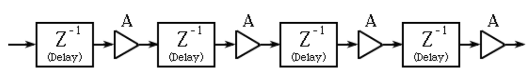
- **Bi-directional delay lines**
- **Filters for loss, radiation, other**

Simple Plucked String
Physical Model Block Diagram

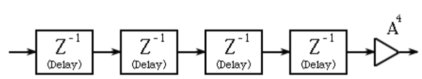


"Digital Waveguide Filter" Model

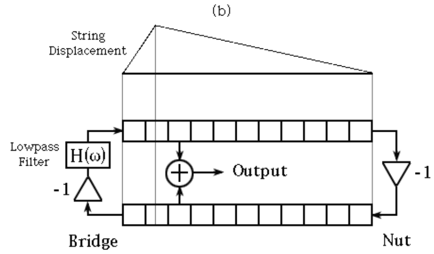




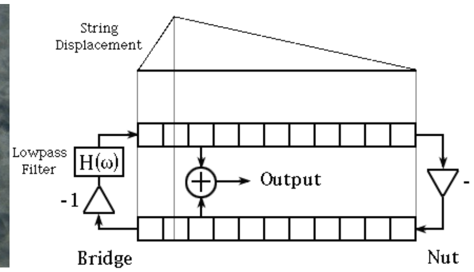
(a)

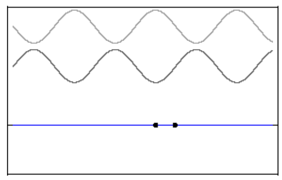


(b)



“Digital Waveguide Filter” Model





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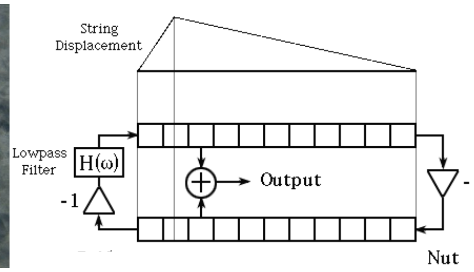
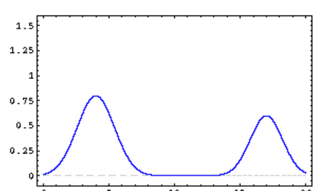
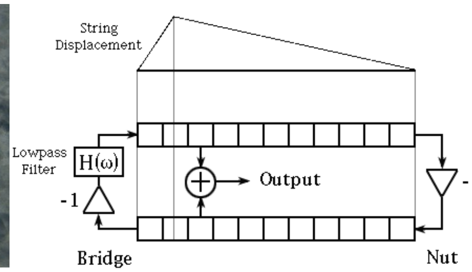
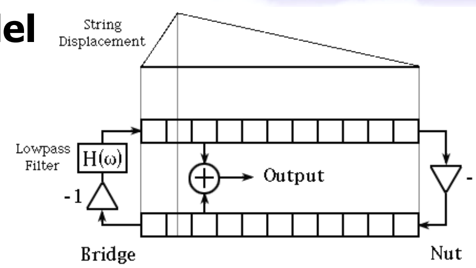




"Digital Waveguide Filter" Model



Karplus-Strong Model



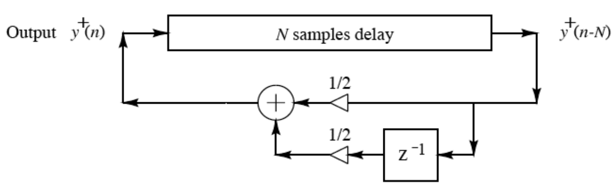
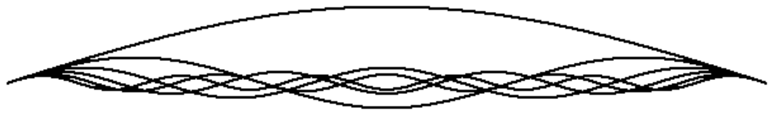


Figure 17. Rigidly terminated string with the simplest frequency-dependent loss filter. All N loss factors (possibly including losses due to yielding terminations) have been consolidated at a single point and replaced by a one-zero filter approximation.

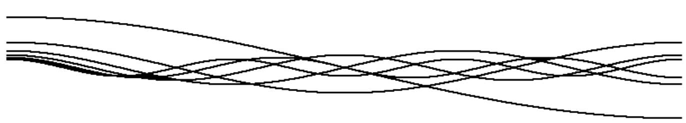
Modal String Solution



- **Superimposed <u>spatial</u> sine waves**
  (modes derive from spatial "boundary conditions")
- **Modes result in frequency "partials" (in time)**
- **Harmonic (*f, 2f, 3f*, etc.) relationship**
  (speed of sound *c* = constant)
- **Stiffness can cause minor stretching of harmonic frequencies (*c(f)*)**

Modal Solution for Bars

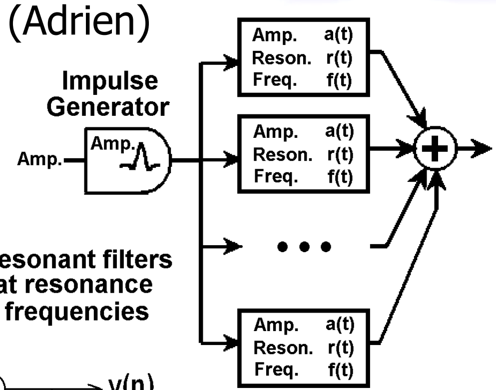
- **Bars are often free at one or both ends**

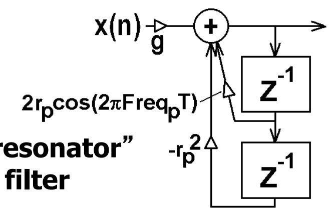


- **Spatial modal solution still holds**
- **Modes no longer harmonic. Stiffness of rigid bars "stretches" frequencies.**
- **Modes: *f, 2.765f, 5.404f, 8.933f,* etc.**

Modal Synthesis (Adrien)

- Impulse generator excites filters
- Filters shape spectrum, model eigenmodes
- Filter parameters can be time-varying



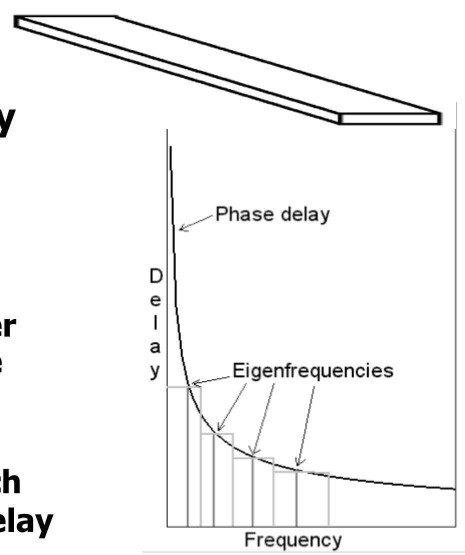


"2nd order resonator" digital filter

```
y[n]  = g*x[n];
y[n] += b1*y[n-1];
y[n] += b2*y[n-2];
n++;
```


Stiffness in Bars

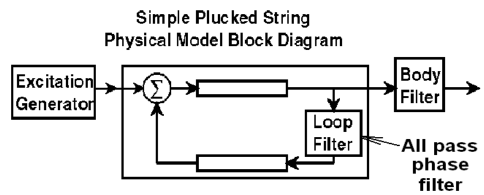
- Stiffness makes wave propagation frequency dependent (*c(f)*)
- Models:
  - Modal partials
  - Use all-pass phase filter to "stretch" waveguide harmonics
  - Merge waveguide with modal by modeling each mode with filter and delay



Stiffness

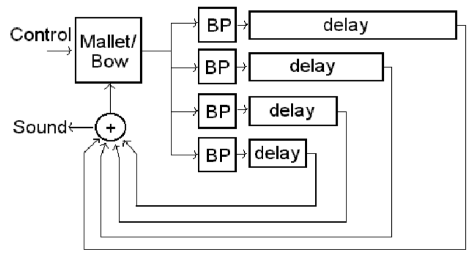
All-pass waveguide
(Smith & Jaffe)

- *Acoustics View: Frequency dependent propagation*
- *Filter View: Stretch comb filter harmonics*



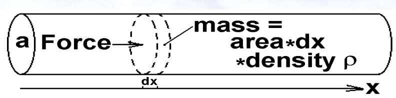
Banded waveguides (Essl)

- *Acoustics View: Wave train closures*
- *Filter View: Comb filters with one resonance each*



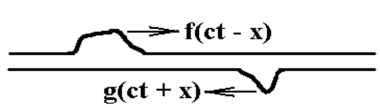
Or a purely modal model (lacks space and time)

Tubes

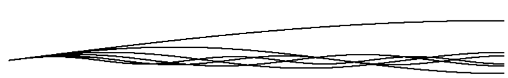


- **Open or closed at either end**
- **Wave equation solution same as strings**
- **Modes always harmonic because speed of sound is constant with frequency**
- **Solutions:**

Waveguide

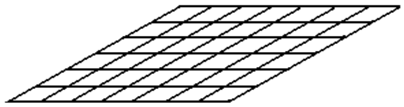


or Modal



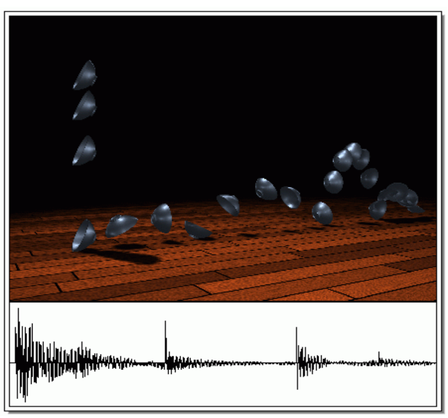
Open + Closed: odd 1/4 wavelengths

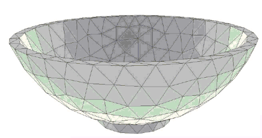
Two and Higher Dimensions

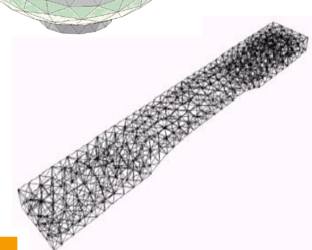


- 2 (N) Dimensional Waveguide Meshes
- or Finite Elements and Finite Differences
  - Discretize objects into cells (elements)
  - Express interactions between them
  - Express differential equation for system
  - Solve by discrete steps in space and time
- or Modal Solution

Finite Elements
(with O'Brien and Essl)

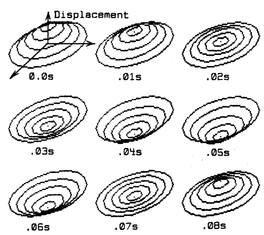


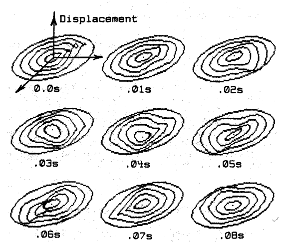


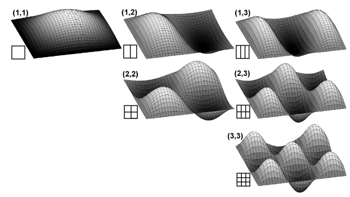


Hi-D Modal Solutions

Modes of Plates are inharmonic







Center strike **Edge strike**
round = Bessel function roots

Square Plate Modes
= sqrt(I) factors

Modes in higher dimensions are problematic
(impossible analytically except in very simple cases)

Where Are We So Far?

- **Physical descriptions (equations)**
- **Give rise to solutions:**
  1. **Traveling Waves**
  2. **Spatial/Frequency Modes**
- **We can solve the equations directly using**
  3. **Finite Elements/Meshes**
- **How to choose? Are there more?**

Waveguides

- **Strengths:**
  - Cheap in both computation and memory
  - Parametrically meaningful, extensible for more realism
- **Weaknesses:**
  - Little in the real world looks, behaves, or sounds exactly like a plucked string, flute, etc.
  - Each family needs a different model
  - No general blind signal model

Modal Modeling

- **Strengths:**
  - Generic, flexible, cheap if only a few modes
  - Great for modeling struck objects of metal, glass, wood
- **Weaknesses:**
  - No inherent spatial sampling
  - No (meaningful) phase delay
  - Hard to interact directly and continuously (rubbing, damping, etc).
  - No general blind signal model (closest)

Meshes, Finite Elements

- **Strengths**
  - (somewhat) arbitrary geometries
  - Fewer assumptions than parametric forms
  - Can strike, damp, rub, introduce non-linearities at arbitrary points
- **Weaknesses:**
  - Expensive
  - Don't know all the computational solutions
  - Sampling in space/time (high Q problems)
  - Dispersion is strange (diagonals vs. not)
  - No general blind signal model

Sound Views: Frequency Domain

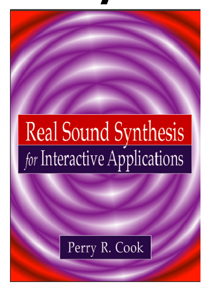
- Many physical systems have <u>modes</u> (damped oscillations)
- Wave equation (2nd order) or Bar equation (4th order) need 2 or 4 "boundary conditions" for solution
- Once boundary conditions are set solutions are sums of exponentially damped sinusoidal modes

References and Resources

Synthesis ToolKit in C++ (STK)

- **STK: a set of classes in C++ for rapid experimentation with sound synthesis. Available for free (source, multi-platform)**
  - **http://www.cs.princeton.edu/~prc**
  - **http://www-ccrma.stanford.edu/~gary**
  - **http://www-ccrma.stanford.edu/software/stk**
- **Based on “Unit Generators,” the classical computer music/sound building blocks:**
- **Oscillators, Filters, Delay Lines, etc.**
- **Build your own algorithms from these**

Book on interactive sound synthesis



Many examples and figures from these notes

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