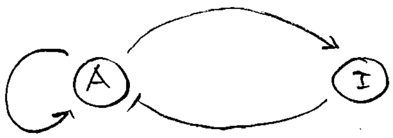
ACTIVATOR - INHIBITOR PATTERNS



A : ACTIVATOR (SHORT-RANGE)

I : INHIBITOR (LONG RANGE)

A , I ARE DIFFUSIBLE $D_I >> D_A$

$$\frac{\partial A}{\partial t} = F(A,I) + D_A \frac{\partial^2 A}{\partial x^2}$$

$$\frac{\partial I}{\partial t} = G(A,I) + D_I \frac{\partial^2 I}{\partial x^2}$$

EXAMPLES OF F, G :

(1) SCHNAKENBERG (NOT REALLY A-I)

$$F(A,I) = k_1 A^2 I - k_2 A + k_3$$

$$G(A,I) = - k_1 A^2 I + k_4$$

(2) GIERER MEINHARDT

$$F(A,I) = \frac{k_1 A^2}{I} - k_2 A + k_3$$

$$G(A,I) = k_4 A^2 - k_5 I$$

(3) DIAMBRA ET AL.

$$F(A,I) = \frac{\alpha_A (A/K_A)^n}{1 + (A/K_A)^n + (I/K_I)^m} - \gamma_A A$$

$$G(A,I) = \frac{\alpha_I (A/K_A)^n}{1 + (A/K_A)^n + (I/K_I)^m} - \gamma_I I$$

ASSUME THAT PROMOTERS ARE THE SAME $\alpha_I = \alpha_A$, $n = m$.

TURING PATTERN FORMATION RESULTS FROM DIFFUSION-DRIVEN INSTABILITY. STEADY STATE IS STABLE TO SPATIALLY HOMOGENEOUS PERTURBATIONS BUT MAY BE UNSTABLE TO INHOMOGENEOUS ONES.

$\Rightarrow$ THERE IS A STABLE STEADY STATE SUCH THAT

$$F(A_s, I_s) = 0$$

$$G(A_s, I_s) = 0$$

WHEN THE DIFFUSION TERM IS ADDED, A SMALL SPATIALLY VARYING PERTURBATION CAN SOMETIME RESULT IN STABLE PATTERNS IF CERTAIN CONDITIONS ON THE SYSTEMS PARAMETERS ARE MET. THE MOST IMPORTANT THOUGH NOT SUFFICIENT CONDITION IS THAT

$$\underline{D_A < D_I}$$

WHAT ARE TYPICAL DIFFUSION CONSTANTS FOR BIOMOLECULES?

GFP IN H_2O : $\sim 90 \mu m^2/s$

GFP IN MAMMALIAN CELL : $\sim 30 \mu m^2/s$

MORPHOGEN (BICOID) IN DROSOPHILA EMBRYO: $0.3 \mu m^2/s$

α-FACTOR IN H_2O(?) : $\sim 150 \mu m^2/s$

SMALL MOLECULE : $\sim 300 \mu m^2/s$

$\Rightarrow$ HARD TO GET DIFFERENT DIFFUSION COEFFICIENTS

<u>OTHER CONSTRAINTS IDENTIFIED BY DIAMBRA ET AL.</u>

- HIGH COOPERATIVITY / HILL COEFFICIENT $n_H, n_A > 2$
- MORE RAPID DEGRADATION OF ACTIVATOR $\gamma_A > \gamma_D$