



Physical Modeling Synthesis of Sound

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One View of Sound

Sound is a waveform,

it, store it,
back accurately

we can record
and play it

PCM playback is all we need for
interactions, movies, games, etc.

But, take one visual analogy:

*"If I take lots of polaroid images, I can flip through
them real fast and make any image sequence"*

Interaction? We manipulate lots of PCM

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Views of Sound

- **Time Domain** $x(t)$
 (from physics, and time's arrow)
- **Frequency Domain** $X(f)$
 (from math, and perception)
- **Production** **what caused it**
- **Perception** **our "image" of it**

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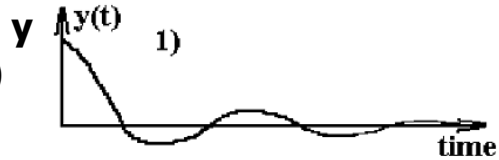
Views of Sound

- **The Time Domain**
 is most closely related to
 Production
- **The Frequency Domain**
 is most closely related to
 Perception

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2nd Order Linear Diff Eq. Solution

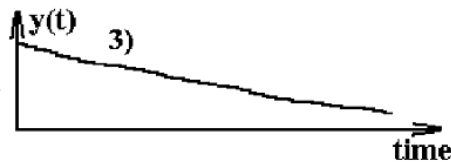
- 1) Underdamped:
 $y(t) = Y_0 e^{-t/\tau} \cos(\omega t)$
 exp. * oscillation



- 2) Critically damped:
 fast exponential decay

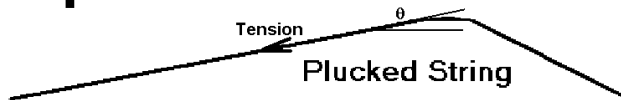


- 3) Overdamped:
 slow exponential decay



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The Wave Equation



$$df_y = (T \sin\theta)_{x+dx} - (T \sin\theta)_x \quad (\text{for each } dx \text{ of string})$$

$$f(x+dx) = f(x) + \delta f / \delta x \, dx + \dots \quad (\text{Taylor's series in space})$$

$$\text{assume } \sin \theta = \theta \quad (\text{for small } \theta)$$

$$F = ma = \rho \, dx \, d^2y/dt^2 \quad (\rho = \text{mass/length})$$

Solution:

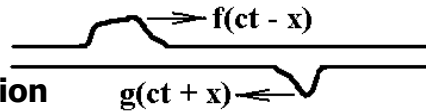
The wave equation
 $(c^2 = T / \rho)$

$$\frac{d^2 y}{dx^2} = \frac{1}{c^2} \frac{d^2 y}{dt^2}$$

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Traveling Wave String Solution

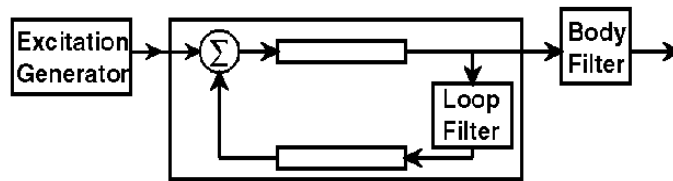
D'Alembert Solution of
2nd order wave equation
(left and right going waves)



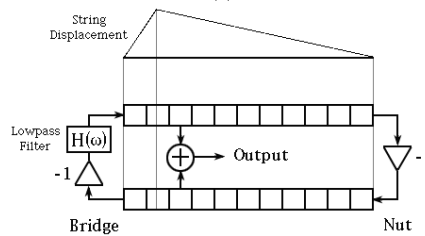
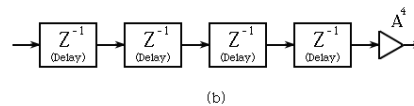
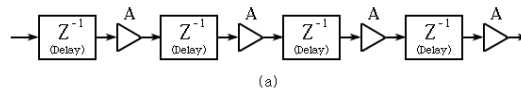
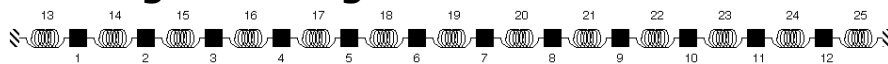
"Digital Waveguide Filter" Model (Smith)

- Bi-directional delay lines
- Filters for loss, radiation, other

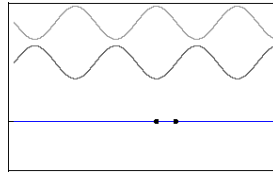
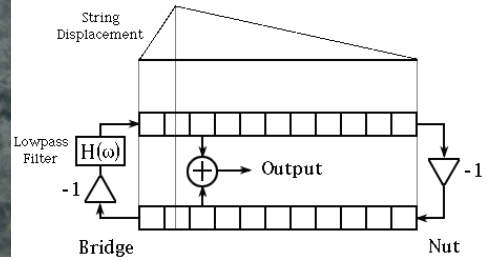
Simple Plucked String
Physical Model Block Diagram



"Digital Waveguide Filter" Model

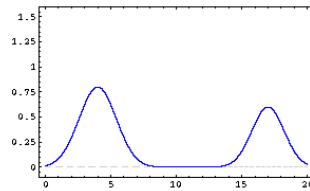
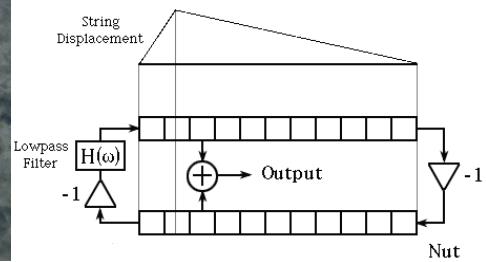


"Digital Waveguide Filter" Model



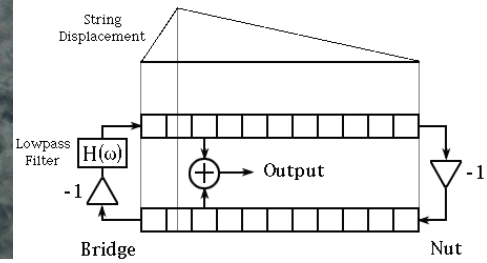
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"Digital Waveguide Filter" Model



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"Digital Waveguide Filter" Model



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Karplus-Strong Model

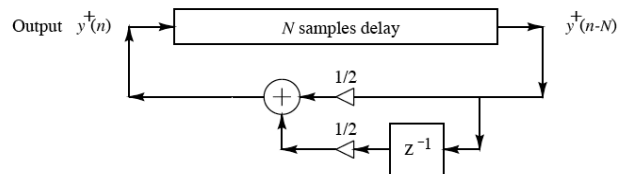
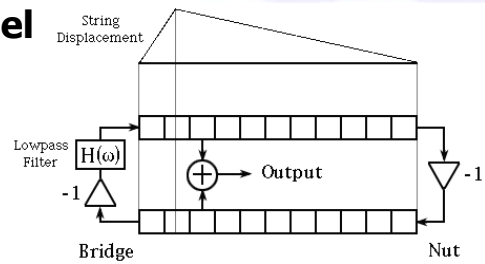


Figure 17. Rigidly terminated string with the simplest frequency-dependent loss filter. All N loss factors (possibly including losses due to yielding terminations) have been consolidated at a single point and replaced by a one-zero filter approximation.

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Modal String Solution

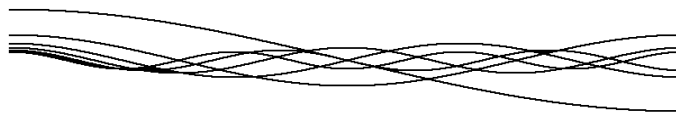


- **Superimposed spatial sine waves**
(modes derive from spatial "boundary conditions")
- **Modes result in frequency "partials" (in time)**
- **Harmonic (f , $2f$, $3f$, etc.) relationship**
(speed of sound $c = \text{constant}$)
- **Stiffness can cause minor stretching of harmonic frequencies ($c(f)$)**

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Modal Solution for Bars

- **Bars are often free at one or both ends**

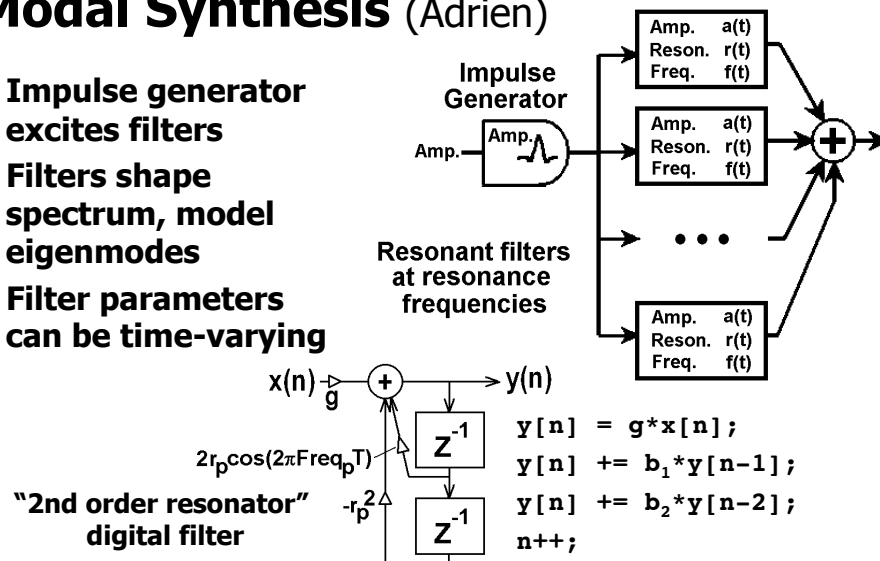


- **Spatial modal solution still holds**
- **Modes no longer harmonic. Stiffness of rigid bars "stretches" frequencies.**
- **Modes: f , $2.765f$, $5.404f$, $8.933f$, etc.**

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Modal Synthesis (Adrien)

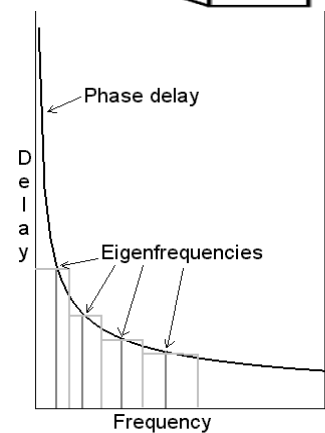
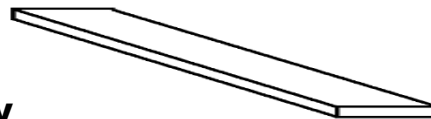
- Impulse generator excites filters
- Filters shape spectrum, model eigenmodes
- Filter parameters can be time-varying



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Stiffness in Bars

- Stiffness makes wave propagation frequency dependent ($c(f)$)
- Models:
 - Modal partials
 - Use all-pass phase filter to "stretch" waveguide harmonics
 - Merge waveguide with modal by modeling each mode with filter and delay

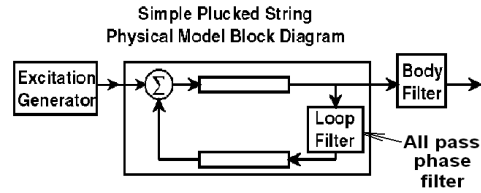


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Stiffness

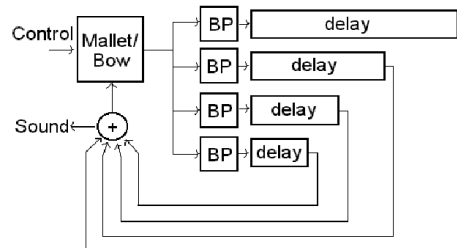
All-pass waveguide (Smith & Jaffe)

- *Acoustics View: Frequency dependent propagation*
- *Filter View: Stretch comb filter harmonics*



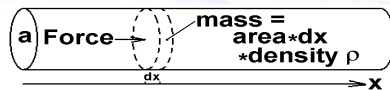
Banded waveguides (Essl)

- *Acoustics View: Wave train closures*
- *Filter View: Comb filters with one resonance each*

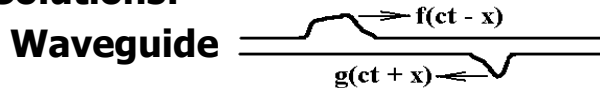


Or a purely modal model (lacks space and time)

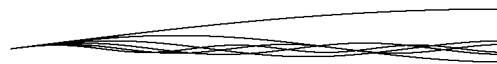
Tubes



- **Open or closed at either end**
- **Wave equation solution same as strings**
- **Modes always harmonic because speed of sound is constant with frequency**
- **Solutions:**

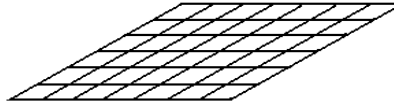


or Modal



Open + Closed: odd 1/4 wavelengths

Two and Higher Dimensions

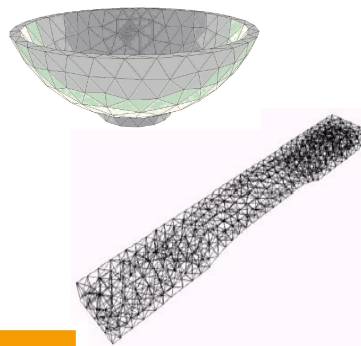
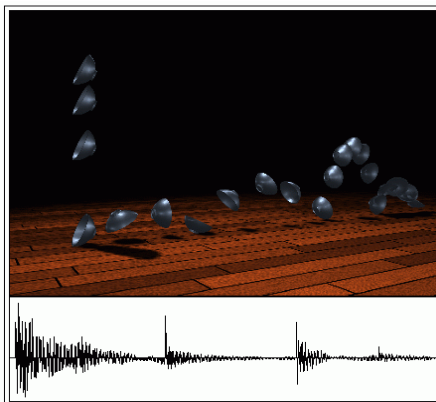


- **2 (N) Dimensional Waveguide Meshes**
- **or Finite Elements and Finite Differences**
 - Discretize objects into cells (elements)
 - Express interactions between them
 - Express differential equation for system
 - Solve by discrete steps in space and time
- **or Modal Solution**

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Finite Elements

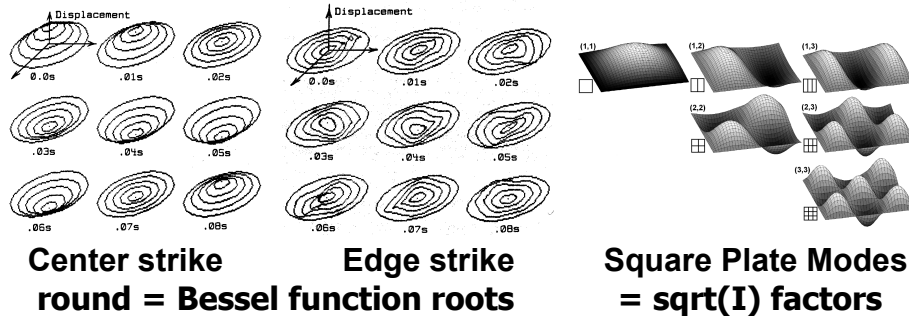
(with O'Brien and Essl)



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Hi-D Modal Solutions

Modes of Plates are inharmonic



**Modes in higher dimensions are problematic
(impossible analytically except in very simple cases)**

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Where Are We So Far?

- **Physical descriptions (equations)**
- **Give rise to solutions:**
 1. **Traveling Waves**
 2. **Spatial/Frequency Modes**
- **We can solve the equations directly using**
 3. **Finite Elements/Meshes**
- **How to choose? Are there more?**

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Waveguides

- **Strengths:**
 - Cheap in both computation and memory
 - Parametrically meaningful, extensible for more realism
- **Weaknesses:**
 - Little in the real world looks, behaves, or sounds exactly like a plucked string, flute, etc.
 - Each family needs a different model
 - No general blind signal model

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Modal Modeling

- **Strengths:**
 - Generic, flexible, cheap if only a few modes
 - Great for modeling struck objects of metal, glass, wood
- **Weaknesses:**
 - No inherent spatial sampling
 - No (meaningful) phase delay
 - Hard to interact directly and continuously (rubbing, damping, etc).
 - No general blind signal model (closest)

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Meshes, Finite Elements

- **Strengths**
 - (somewhat) arbitrary geometries
 - Less assumptions than parametric forms
 - Can strike, damp, rub, introduce non-linearities at arbitrary points
- **Weaknesses:**
 - Expensive
 - Don't know all the computational solutions
 - Sampling in space/time (high Q problems)
 - Dispersion is strange (diagonals vs. not)
 - No general blind signal model

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Sound Views: Frequency Domain

- Many physical systems have modes (damped oscillations)
- Wave equation (2nd order) or Bar equation (4th order) need 2 or 4 "boundary conditions" for solution
- Once boundary conditions are set solutions are sums of exponentially damped sinusoidal modes

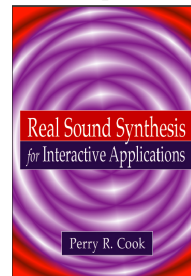
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References and Resources

Synthesis ToolKit in C++ (STK)

- STK: a set of classes in C++ for rapid experimentation with sound synthesis. Available for free (source, multi-platform)
 - <http://www.cs.princeton.edu/~prc>
 - <http://www-ccrma.stanford.edu/~gary>
 - <http://www-ccrma.stanford.edu/software/stk>
- Based on "Unit Generators," the classical computer music/sound building blocks:
- Oscillators, Filters, Delay Lines, etc.
- Build your own algorithms from these

Book on interactive sound synthesis



Many examples and figures from these notes

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