Representation of programs

Primary goals:

- analysis is easy & effective
  - just a few cases to handle
  - provide support for linking things of interest
- transformations are easy
- general, across input languages & target machines

Additional goals:

- compact in memory
- easy to translate to and from
- tracks info for source-level debugging, profiling, etc.
- extensible (new optimizations, targets, language features)
- displayable

Example IRs:

- C?
- Java bytecode?
- ...

High-level syntax-based representation

Represent source-level control structures & expressions directly

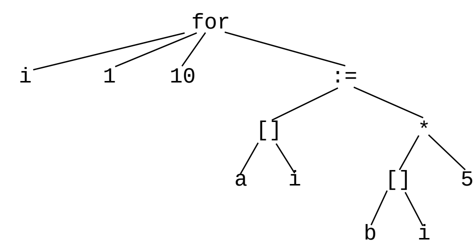
Examples

- (Attributed) AST
- Lisp S-expressions
- lambda calculus? Java bytecode?

Source:

```
for i := 1 to 10 do
  a[i] := b[i] * 5;
end
```

AST:



Low-level representation

Translate input programs into low-level primitive chunks, often close to the target machine

Examples

- assembly code, virtual machine code (e.g. stack machine)
- three address code, register transfer language (RTLs)
- lambda calculus? Java bytecode?

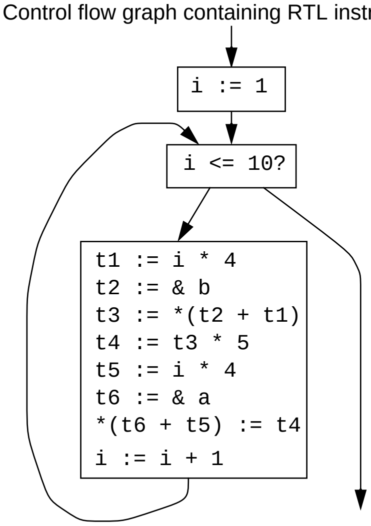
Standard RTL operators:

assignment	`x := y;`
unary op	`x := `*op*` y;`
binary op	`x := y `*op*` z;`
address-of	`p := &y;`
load	`x := *(p + o);`
store	`*(p + o) := x;`
call	`x := f(...);`
unary compare	*op* `x ?`
binary compare	`x `*op*` y ?`

Source:

```
for i := 1 to 10 do
  a[i] := b[i] * 5;
end
```

Control flow graph containing RTL instructions:



Comparison

Advantages of high-level rep:

- analysis can exploit high-level knowledge of constructs
  - probably faster to analyze
- supports semantics-based reasoning about correctness etc. of analysis
- easy to map to source code terms for debugging, profiling
- may be more compact

Advantages of low-level rep:

- can do low-level, machine-specific optimizations (if target-based representation)
  - high-level rep may not be able to express some transformations
- can have relatively few kinds of instructions to analyze
- can be language-independent

High-level rep suitable for a source-to-source or special-purpose optimizer, e.g. inliner, parallelizer

Can mix multiple representations in single compiler
Can sequence compilers using different reps

Components of representation

Operations

Dependences between operations

- **control** dependences: sequencing of operations
  - evaluation of then & else arms depends on result of test
  - side-effects of statements occur in right order
- **data** dependences: flow of values from **definitions** to **uses**
  - operands computed before operation
  - values read from variable before being overwritten

Ideal: represent just those dependences that matter

- dependences constrain transformations
- fewest dependences ⇒ most flexibility in implementation

Representing control dependences

Option 1: **high-level representation**

- control flow implicit in semantics of AST nodes

Option 2: **control flow graph**

- nodes are **basic blocks**
  - instructions in basic block sequence side-effects
- edges represent branches (control flow between basic blocks)

Some fancier options:

- **control dependence graph**, part of **program dependence graph** (PDG) [Ferrante *et al.* 87]
- convert into data dependences on a memory state, in **value dependence graph** (VDG) [Weise *et al. 94*]

Kinds of data dependences

read-after-write (RAW): **true/flow dependence**

- reflects real data flow, operands to operation

write-after-read (WAR): **anti-dependence**
write-after-write (WAW): **output dependence**

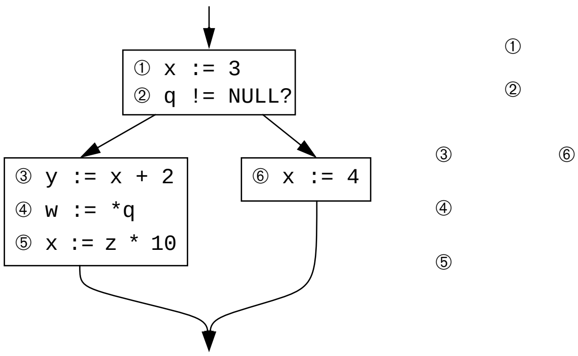
- reflects overwriting of memory, not real data flow
  ⇒ can sometimes be eliminated by optimization

read-after-read (RAR): no dependence

- can occur in any order

Example

```
x := 3
if q != NULL then
  y := x + 2
  w := *q
  x := z * 10
else
  x := 4
endif
```



Representing data dependences

Within basic block:

Option 1: sequence of instructions
(represent data flow as a kind of control dependence)

+ simple, source-like
+ fixed ordering supports easy analysis
– may overconstrain order of operations

Option 2: expression tree/DAG

+ natural, abstract
+ directly captures data dependences within basic block
+ DAG supports local CSE
+ can be compact
– conceptually harder to analyze, transform
– must linearize eventually

Example

Source:

```
x := (z/y) + (y*4)
y := x + (y*4)
z := z + (y*4)
```

Linear RTL:

```
t1 := z/y
t2 := y*4
x := t1 + t2

t3 := y*4
y := x + t3

t4 := y*4
z := z + t4
```


Representing data dependences, cont.

Across basic blocks:

Option 1: implicitly through variable defs/uses

+ simple
– analysis wants important things explicit ⇒
  analysis can be slow

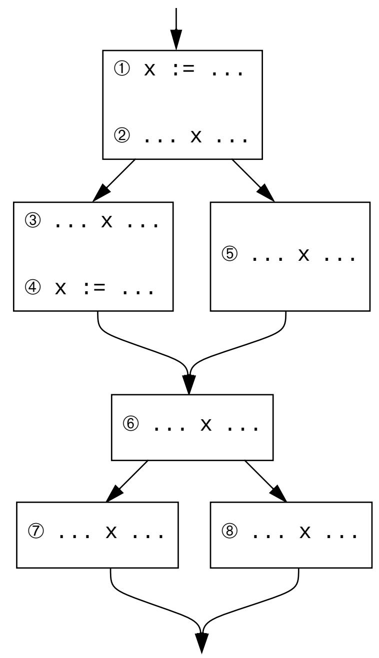
Option 2: def/use chains, linking each def with each use

+ explicit ⇒ analysis can be fast
– must be computed, maintained after transformations
– may be space-consuming

Fancier options:

- **static single assignment** (SSA) form [Alpern *et al.* 88]
- value dependence graphs (VDGs)
- **dependence flow graphs** (DFGs)
- ...

Example



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④

⑥

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Data flow analysis

Want to compute some info about program

- at **program points**
- to identify opportunities for improving transformations

Can model data flow analysis as solving a system of constraints

- each node in CFG imposes a constraint relating info at predecessor and successor points
- solution to constraints is result of analysis

Solution must be **safe**/**sound**

Solution can be **conservative**

Key issues:

- how to know if constraint system defines the analysis correctly?
- how to represent info efficiently?
- how to represent & solve constraints efficiently?
  - how long does constraint solving take? finite time?
- what if multiple solutions are possible?
- how to synchronize transformations with analysis?

Example: reaching definitions

For each program point,
want to compute set of definitions (statements) that *may reach* that point

- reach: are the last definition of some variable

Info ≡ set of *var*→*rtl* bindings

E.g.:
$\{x \rightarrow s_1, y \rightarrow s_5, y \rightarrow s_8\}$

Can use reaching definition info to:

- build def-use chains
- do constant & copy propagation
- ...

Safety rule (for these intended uses of this info):
can have more bindings than the “true” answer,
but can’t miss any

Constraints for reaching definitions

Main constraints:

A simple assignment removes any old reaching defs for the lhs and replaces them with this stmt:

- **strong update**

s: `x := ...`:

$$info_{succ} = info_{pred} - \{x \rightarrow s' \mid \forall s'\} \cup \{x \rightarrow s\}$$

A pointer assignment may modify anything, but doesn’t definitely replace anything

- **weak update**

s: `*p := ...`:

$$info_{succ} = info_{pred} \cup \{x \rightarrow s \mid \forall x \in \text{may-point-to}(p)\}$$

Other statements: do nothing

$$info_{succ} = info_{pred}$$

Constraints for reaching definitions, continued

Branches pass through reaching defs to both successors

$\mathrm{info}_{succ[i]} = \mathrm{info}_{pred}$

Merges take the union of all incoming reaching defs

- we don't know which path is being taken at run-time
  $\Rightarrow$ be conservative

$\mathrm{info}_{succ} = \bigcup_i \mathrm{info}_{pred[i]}$

Conditions at entry to CFG: definitions of formals

$\mathrm{info}_{entry} = \{x \rightarrow \mathit{entry} \mid \forall x \in \text{formals}\}$

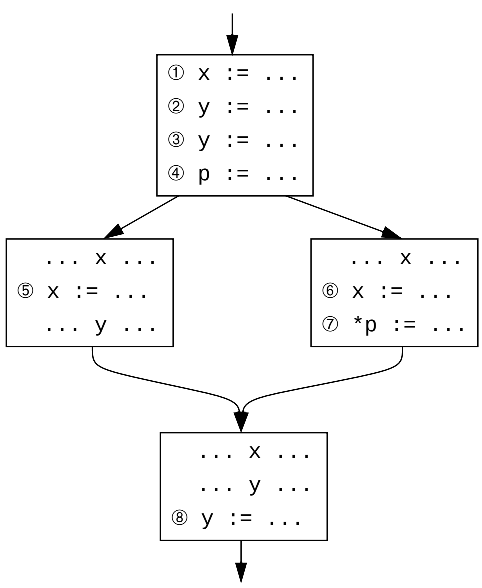
Solving constraints

A given program yields a system of constraints

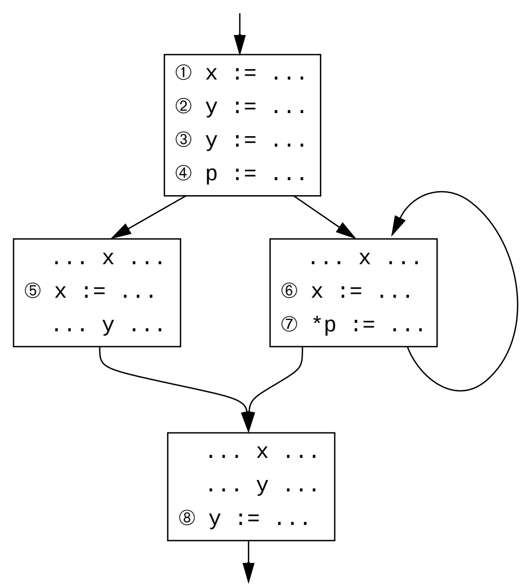
Need to solve constraints

For reaching definitions,
can traverse instructions in forward topological order,
computing successor info from predecessor info

Example



Another example



Topological order not defined!

Loop terminology

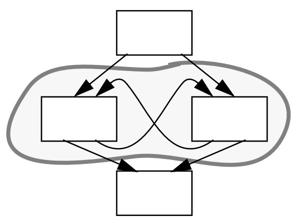
loop: strongly-connected component in CFG with single entry

loop entry edge: source not in loop, target in loop
loop exit edge: the reverse
back edge: target is loop head node

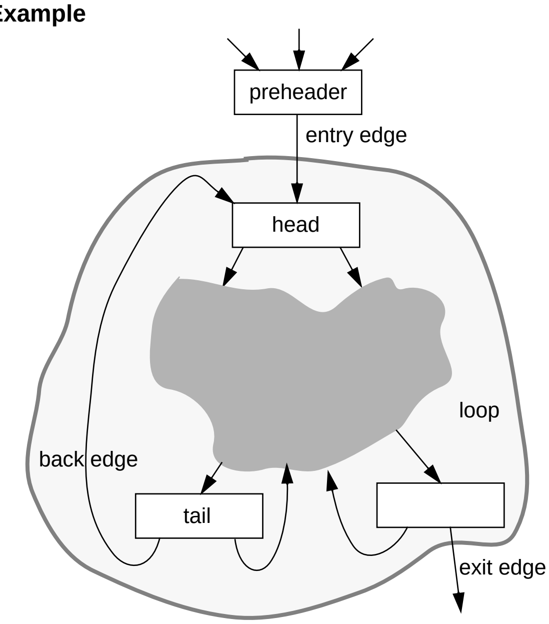
loop head node: target of loop entry edge
loop tail node: source of back edge
loop preheader node:
single node that's source of loop entry edge

nested loop: loop whose head is inside another loop

reducible flow graph: all SCC's have single entry



Example



Analysis of loops

If CFG has a loop, data flow constraints are recursively defined:
$info_{loop\text{-}head} = info_{loop\text{-}entry} \cup info_{back\text{-}edge}$
$info_{back\text{-}edge} = \ldots\ info_{loop\text{-}head} \cdots$

Substituting definition of $info_{back\text{-}edge}$:
$info_{loop\text{-}head} = info_{loop\text{-}entry} \cup (\ldots\ info_{loop\text{-}head} \ldots)$

Summarizing r.h.s. as F:
$info_{loop\text{-}head} = F(info_{loop\text{-}head})$

Legal solution to constraints is a **fixed-point** of F

Recursive constraints can have many solutions
- want **least** or **greatest** fixed-point,
  whichever corresponds to the most precise answer

How to find least/greatest fixed-point of F?
- for restricted CFGs can use specialized methods
  - e.g. **interval analysis** for **reducible** CFGs
- for arbitrary CFGs, can use **iterative** approximation

Iterative data flow analysis

1. Start with initial guess of info at loop head:
   $info_{loop\text{-}head} = \mathit{guess}$

2. Solve equations for loop body:
   $info_{back\text{-}edge} = F_{body}(info_{loop\text{-}head})$
   $info_{loop\text{-}head}' = info_{loop\text{-}entry} \cup info_{back\text{-}edge}$

3. Test if found fixed-point:
   $info_{loop\text{-}head}' = info_{loop\text{-}head}$?

   A. if same, then done

   B. if not, then adopt result as (better) guess and repeat:

   $info_{back\text{-}edge}' = F_{body}(info_{loop\text{-}head}')$
   $info_{loop\text{-}head}'' = info_{loop\text{-}entry} \cup info_{back\text{-}edge}'$
   $info_{loop\text{-}head}'' = info_{loop\text{-}head}'$?
   ...

When does iterating work?

1. need to be able to make an initial guess
2. info^{n+1} must be closer to the fixed-point than info^{n} (constraints must be **monotonic**)
3. must eventually reach the fixed-point in a finite number of iterations (info must be drawn from a **finite-height domain**)

To reach best fixed-point, initial guess for loop head should be **optimistic**

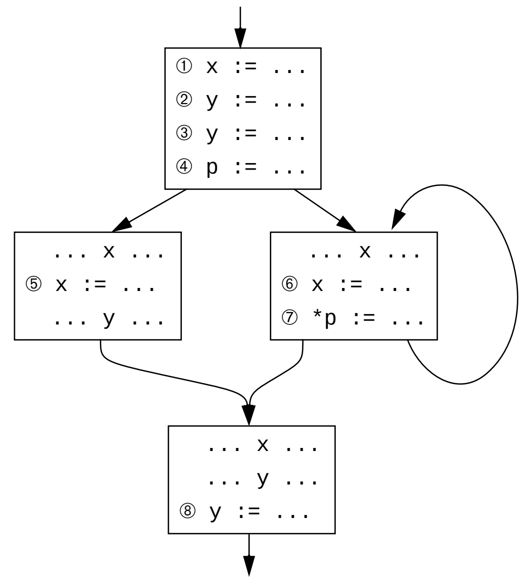
- easy choice: $\text{info}_{\text{loop-head}} = \text{info}_{\text{loop-entry}}$

(Even if guess is overly optimistic, iteration will ensure we won't stop analysis until the answer is safe.)

To speed iterative analysis, want to test guess ASAP

- ideal: solve constraints along shortest path from loop head to loop tail
- practical: avoid solving constraints outside of loop until fixed-point is reached within loop

Another example, again



Direction of dataflow analysis

In what order are constraints solved, in general?

Constraints are declarative, not directional/procedural, so may require mixing forward & backward solving, or other more global solution methods

But often constraints can be solved by (directional) propagation & iteration

- may be **forward** or **backward** propagation of info
- topological traversals of acyclic subgraphs minimize analysis time

Directional constraints often called **flow functions**

- often written as functions on input info to compute output

$RD_{s:\ \texttt{x := ...}}(in) = in - \{x \rightarrow s' \mid \forall s'\} \cup \{x \rightarrow s\}$

$RD_{s:\ \texttt{*p := ...}}(in) = in \cup \{x \rightarrow s \mid \forall x \in \text{may-point-to}(p)\}$

GEN and KILL sets

For even more structure, can often think of flow functions in terms of each's GEN set and KILL set

- GEN = new information added
- KILL = old information removed

Then

$F_{instr}(in) = in - KILL_{instr} \cup GEN_{instr}$

E.g., for reaching defs:

$RD_{s:\ \texttt{x := ...}}(in) = in - \{x \rightarrow s' \mid \forall s'\} \cup \{x \rightarrow s\}$

$RD_{s:\ \texttt{*p := ...}}(in) = in \cup \{x \rightarrow s \mid \forall x \in \text{mpt}(p)\}$

Bit vectors

Can sometimes represent info/KILL/GEN sets as **bit vectors**

- if can express abstractly as set of things
  (e.g. statements, vars),
  drawn from a statically known set of things,
  each thing getting a statically determined bit position
- bitvector encodes **characteristic function** of set

E.g., for reaching defs:

info = bitvector over statements,
each stmt getting a distinct bit position

- statement implies which variable is defined

Bit vectors compactly represent sets

Bit-vector operations efficiently perform set difference & union

Flow function may be able to be represented simply by a pair of bit vectors, if they don't depend on input bit vector

- can merge the KILL and GEN bit vectors of a whole block of instructions into a single overall KILL and GEN set, for faster iterating

Another example: constant propagation

Goal: data flow analysis that implements constant propagation

What info computed for each program point?

I is a conservative approximation to true info I_{true} iff:

$CP_{\mathtt{x\ :=\ }N}$:

$CP_{\mathtt{x\ :=\ y\ +\ z}}$:

$CP_{\mathtt{*p\ :=\ *q\ +\ *r}}$:

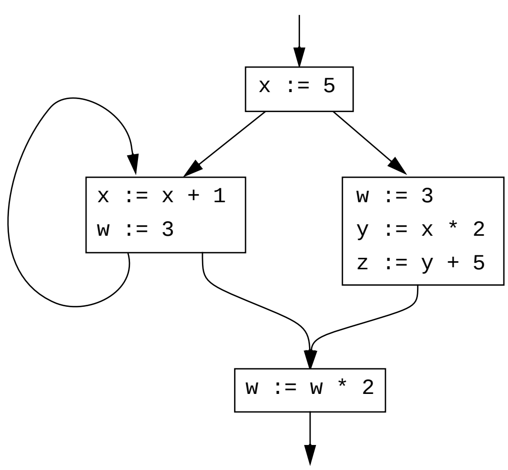
Merge function?

Initial info at program points?

Direction of analysis?

Can use bit vectors?

Example



May vs. must info

Some kinds of info imply guarantees: **must** info

Some kinds of info imply possibilities: **may** info

- the complement of **may** info is **must not** info

	May	**Must**
desired info	small set	big set
safe	overly big set	overly small set
GEN	add everything that might be true	add only if guaranteed true
KILL	remove only if guaranteed wrong	remove everything possibly wrong
MERGE	$\cup$	$\cap$

Another example: live variables

Want the set of variables that are **live** at each pt. in program

- live: might be used later in the program

Supports dead assignment elimination, register allocation

What info computed for each program point?

May or must info?

I is a conservative approximation to true info I_{true} iff:

LV`x := y + z`:

LV`*p := *q + *r`:

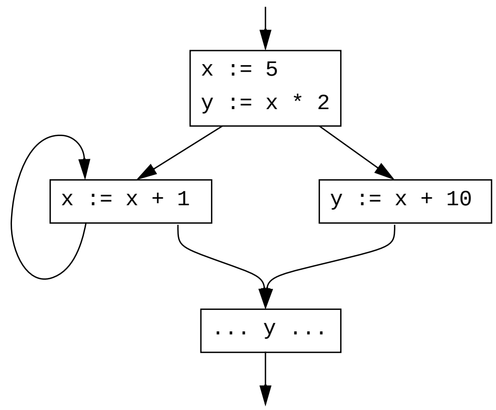
Merge function?

Initial info at program points?

Direction of analysis?

Can use bit vectors?

Example



Lattice-Theoretic Data Flow Analysis Framework

Goals:

- provide a single, formal model that describes all DFAs
- formalize notions of “safe”, “conservative”, “optimistic”
- place precise bounds on time complexity of DF analysis
- enable connecting analysis to underlying semantics for correctness proofs

Plan:

- define **domain** of program properties computed by DFA
  - domain has a set of elements
  - each element represents one possible value of the property
  - order elements to reflect their relative precision
  - domain = set of elements + order over elements = **lattice**
- define flow functions & merge function over this domain, using standard lattice operators
- benefit from lattice theory in attacking above issues

History: Kildall [POPL 73], Kam & Ullman [JACM 76]

Lattices

Define lattice $D = (S, \leq)$:

- S is a set of elements of the lattice
- $\leq$ is a binary relation over elements of S

Required properties of $\leq$:

- $\leq$ induces a **partial order** over S
  - reflexive, transitive, & anti-symmetric
- every pair of elements of S has a unique **greatest lower bound** (a.k.a. meet) and a unique **least upper bound** (a.k.a. join)

Height of D = longest path through partial order from greatest to least

- infinite lattice can have finite height (but infinite width)

Top (T) = unique element of S that’s greatest, if exists

Bottom ($\perp$) = unique element of S that’s least, if exists

Lattice models in data flow analysis

Model data flow information by elements of a lattice domain

- top = best case info
- bottom = worst case info
- if $a \leq b$, then a is a conservative approximation to b
- merge function = g.l.b. (meet) on lattice elements
  (the most precise element that's a conservative approximation to both input elements)
- initial info for optimistic analysis (at least back edges): top

(Opposite up/down conventions used in PL semantics!)

Examples

Reaching definitions:

- elements:
- ≤:
- top:
- bottom:
- meet:

Reaching constants:

- elements:
- ≤:
- top:
- bottom:
- meet:

Tuples of lattices

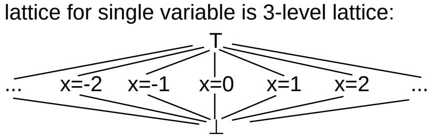
Often helpful to break down a complex lattice into a tuple of lattices, one per variable being analyzed

Formally: $D_T = \langle S_T, \leq_T \rangle \ = \ (D = \langle S, \leq \rangle)^N$

- $S_T = S_1 \times S_2 \times \ldots \times S_N$
  - element of tuple domain is a tuple of elements from each variable's domain
  - i^{th} component of tuple is info about i^{th} variable
- $\langle \ldots, d_{1i}, \ldots \rangle \ \leq_T \ \langle \ldots, d_{2i}, \ldots \rangle \ \equiv \ d_{1i} \leq d_{2i}, \forall i$
  - i.e. pointwise ordering
- meet: pointwise meet
- top: tuple of tops
- bottom: tuple of bottoms
- height(D_T) = N * height(D)

E.g. reaching constants

- lattice for single variable is 3-level lattice:



- whole problem is tuple of individual lattices

Some typical lattice domains

Single-point lattice: just bottom

- trivial do-nothing analysis

Two-point lattice: top and bottom

- computes a boolean property
- a tuple of two-point lattices ⇔ a bit-vector

A lifted set: a set of incomparable values, plus top & bottom

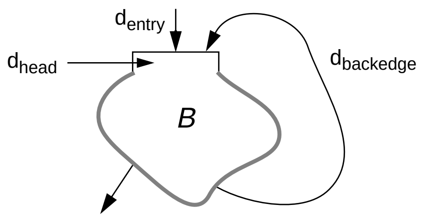
- e.g. reaching constants domain

Powerset lattice: set of all subsets of a set S, ordered somehow

- top & bottom = ∅ & S, or vice versa
- "a collecting analysis"
- isomorphic to tuple of booleans
  indicating membership in subset of elements of S

Analysis of loops in lattice model

Consider:



Assume B(d) computes info at back-edge given d, info at head

Want solution to constraints:

$d_{head} = d_{entry} \cap d_{backedge}$
$d_{backedge} = B(d_{head})$

Let $F(d) = d_{entry} \cap B(d)$

Then want fixed-point of F:

$d_{head} = F(d_{head})$

Iterative analysis in lattice model

Iterative analysis computes fixed-point
by iterative approximation:

$F^0 = d_{entry} \cap T$

$F^1 = d_{entry} \cap B(F^0) = F(F^0) = F(d_{entry})$

$F^2 = d_{entry} \cap B(F^1) = F(F^1) = F(F(F^0)) = F(F(d_{entry}))$

. . .

$F^k = d_{entry} \cap B(F^{k\text{-}1}) = F(F^{k\text{-}1}) = F(F(...(F(d_{entry}))...))$

until

$F^{k+1} = d_{entry} \cap B(F^k) = F(F^k) \;\; = \;\; F^k$

Is k finite?
If so, how big can it be?

Termination of iterative analysis

In general, k need not be finite

Sufficient conditions for finiteness:

- flow functions (e.g. F) are **monotonic**
- lattice is of finite height

A function F is monotonic iff:

$d_1 \le d_2 \;\Rightarrow\; F(d_1) \le F(d_2)$

- for application of DFA, this means that giving a flow function at least as conservative inputs ($d_1 \le d_2$) leads to at least as conservative outputs ($F(d_1) \le F(d_2)$)

For monotonic F over domain D, the maximum number of times that F can be applied to itself, starting w/ any element of D, w/o reaching fixed-point, is height(D)-1

- start at top of D
- go down one level in lattice each application of F
- eventually must hit fixed-point or bottom (which is guaranteed to be a fixed-point), if D of finite height

Complexity of iterative analysis

How long does iterative analysis take?

`l`: depth of loop nesting
`n`: # of stmts in loop
`t`: time to execute one flow function
`k`: height of lattice

Another example: integer range analysis

For each program point,
for each integer-typed variable,
calculate (an approximation to) the set of integer values
that can be taken on by the variable

- use info for constant folding comparisons,
  for eliminating array bounds checks,
  for (in)dependence testing of array accesses,
  for eliminating overflow checks

What domain to use?

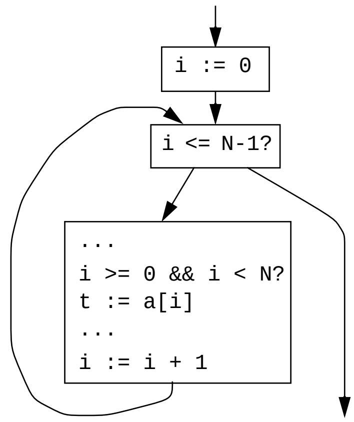
- what is its height?

What flow functions to use?

- are they monotonic?

Example

```
for i := 0 to N-1
  ... a[i] ...
end
```



Widening operators

If domain is tall, then can introduce artificial generalizations
(called **widenings**) when merging at loop heads

- ensure that only a finite number of widenings are possible

Sharlit

A data flow analyzer generator [Tjiang & Hennessy 92]

- analogous to YACC

User writes basic primitives:

- control flow graph representation
  - nodes are instructions, not basic blocks
- domain (“flow value”) representation and key operations
  - `init`
  - `copy`
  - `is_equal`
  - `meet`
- flow functions for each kind of instruction
- action routines to optimize after analysis

Sharlit constructs iterative dataflow analyzer from these pieces

+ easy to build, extend

– not highly efficient, in this first mode of use

Path compression

Can improve analysis efficiency by
summarizing effect of sequences of nodes

User can define path compression operations to collapse nodes together

- linear joining of sequential nodes
  $\Rightarrow$ summarizes effect of whole BB
  - presumes a fixed GEN/KILL bit-vector structure to be effective
- merge trees into extended BB's
- merge merges, loops as in interval analysis
  - simplifies reducible parts, applies iteration to nonreducible parts

+ gets efficiency, preserves modularity & generality
– doesn't support data-dependent flow functions, cannot simulate optimizations during analysis

Performance results for code quality of generated optimizer, but not for compilation speed of optimizer

Vortex IDFA framework

Like Sharlit,
except a compiler library rather than a compiler-compiler

User defines a subclass of `AnalysisInfo` to represent elements of domain

- `copy`
- `merge` (lattice g.l.b. operator)
- `generalizing_merge` (g.l.b. with optional widening)
- `as_general_as` (lattice ≤ operator)

User invokes `traverse` to perform analysis:

```
cfg.traverse(direction, is_iterative?,
  initial_analysis_info,
  λ(rtl, info){ rtl.flow_fn(info) })
```

Flow function returns an `AnalysisResult`: one of

- keep instruction and continue analysis w/ updated info(s)
- delete instruction/constant-fold branch
- replace instruction with instruction or subgraph

`ComposedAnalysis` supports running multiple analyses interleaved at each instruction

Features of Vortex IDFA

Big idea: separate analyses and transformations, make framework compose them appropriately

- don't have to simulate the effect of transformations during analysis
- can run analyses in parallel if each provides opportunities for the other
  - sometimes can achieve strictly better results this way than if run separately in a loop
- more general transformations supported (e.g. inlining) than Sharlit

Exploit inheritance & closures

Analysis speed is not stressed

- no path compression
- no "compilation" of analysis with framework

[Vortex's interprocedural analysis support discussed later]