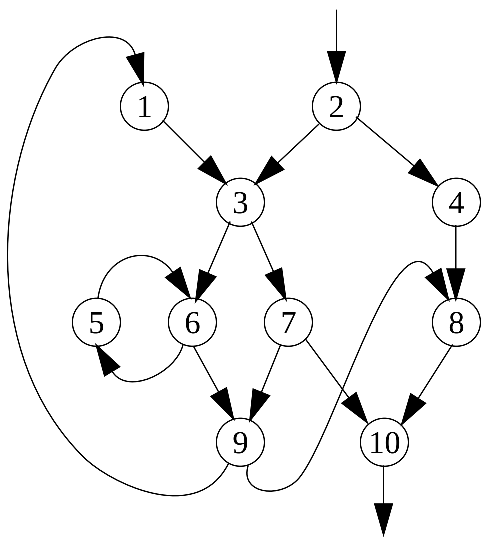
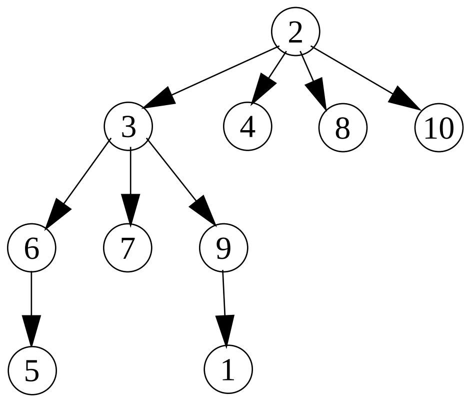
Homework Assignment #1 Sample Solutions

Due Monday 1/22, at the start of lecture

1. For the following control flow graph, for the purposes of a forward data flow analysis (such as available expressions), show the dominator tree, identify any loops, and for each loop, identify the loop head node and the loop entry, exit, and back edges. Is this graph reducible?



Dominator tree:



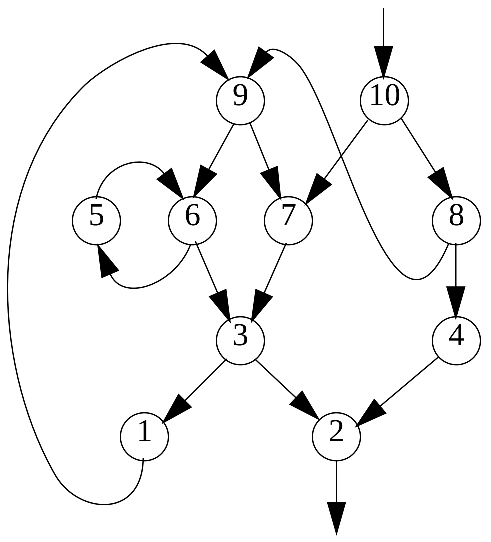
Loop 1: nodes: 1, 3, 5, 6, 7, 9; head node: 3; back edge: 1→3; loop entry edge: 2→3; loop exit edges: 7→10, 9→8

Loop 2: nodes: 5, 6; head node: 6, back edge: 5→6; loop entry edge: 3→6; loop exit edge: 6→9

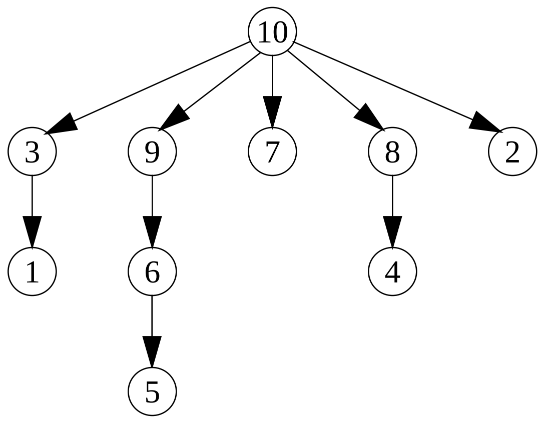
The graph is reducible.

2. Repeat problem 1 for purposes of a backwards data flow analysis (such as live variables).

Reversed control flow graph:



Dominator tree:



Loop: nodes: 5, 6; head node: 6, back edge: 5→6; loop entry edge: 9→6; loop exit edge: 6→3

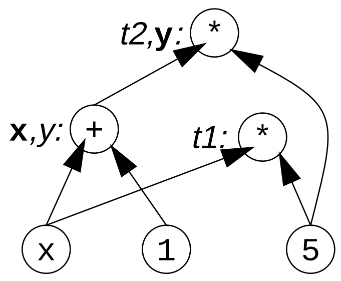
(There's also a larger strongly connected component, but it has two loop head nodes, so it's not a true natural loop: nodes: 1, 3, 5, 6, 7, 9; head nodes: 7, 10; back edges (i.e., in-loop edges to a head): 1→9, 9→7; loop entry edges: 10→7, 8→9; loop exit edge: 3→2)

The graph is not reducible.

3. Build the DAG representation of the following basic block (where variables whose names start with `t` are assumed not to be referenced outside of the basic block). You should perform common subexpression elimination as part of this process, exploiting commutativity of operators. You should mark which nodes in the DAG are stored in which user variables at the end of the block.

```
t1 := x * 5
x  := x + 1
y  := x
```

```
t2 := x * 5
y  := 5 * y
```



(Final assignments to source variables in bold.)

4. Why can we not simply apply this same DAG-construction algorithm over the whole procedure body, across basic blocks?

   Because we'd lose track of the control dependence conditions under which different computations are executed. E.g. operations may appear redundant (i.e., common subexpressions), but be executed in different control conditions (e.g. inside of different conditionals), and so we can't just combine them into a single operation without some sort of extra work. Also, the BB algorithm didn't address constructing cyclic graphs, as would be needed to represent loops, although it could be extended to handle loops without too much trouble. Stay tuned for the VDG representation, which can be thought of as one way to extend the DAG model to the whole procedure.

5. Consider the case where a forward analysis's flow functions for each kind of statement are represented in terms of constant KILL and GEN bit-vectors. Thus, each flow function has the form *out* = *in* - KILL + GEN, with KILL and GEN being constants independent of *in* or *out*. The combined effect of a whole basic block of instructions can be calculated by applying each instruction's flow function in turn, starting with the in bit-vector at the start of the basic block, and computing the out vector at the end of the basic block. I.e., for instruction *i* of the basic block, $out_i = in_i$ - KILL_i + GEN_i, and for all instructions other than the first, $in_i = out_{i-1}$. However, in this special case where KILL and GEN do not depend on *in*, it is possible to compute a single summary flow function with constant KILL and GEN bit-vectors, of the form $out_n = in_1$ - SUMMARY-KILL + SUMMARY-GEN (where in_1 and out_n are the points before and after the basic block, respectively).

   Give a pair of simple inductive equations for computing the SUMMARY-KILL_i and SUMMARY-GEN_i bit-vectors, representing the summary KILL and GEN bit-vectors for the first *i* instructions of a basic block, in terms of the $\text{SUMMARY-KILL}_{i-1}$ and SUMMARY-GEN_{i-1} bit-vectors (if *i*>1) and the KILL_i and GEN_i bit-vectors for the flow function of the *i*th instruction, $1 \le i \le n$. SUMMARY-KILL and SUMMARY-GEN for the whole basic block are then SUMMARY-KILL_n and SUMMARY-GEN_n, respectively.

   $SUMMARY\text{-}KILL_i = SUMMARY\text{-}KILL_{i-1} \cup KILL_i$

   $SUMMARY\text{-}GEN_i = SUMMARY\text{-}GEN_{i-1} - KILL_i \cup GEN_i$

where $SUMMARY\text{-}KILL_0$ and $SUMMARY\text{-}GEN_0$ are $\emptyset$

(Note that $SUMMARY\text{-}KILL_i = SUMMARY\text{-}KILL_{i-1} - GEN_{i-1} \cup KILL_i$ looks more correct than the above equation, but they have the same end effect.)

What is the chief advantage of this SUMMARY flow function over the original flow functions?

It allows the effect of a whole basic block to be computed with a single flow function (that runs a lot faster than the sequence of per-instruction flow functions). This is useful during iterative analysis where we can recompute the effect of a basic block several times.

6. Define a data flow analysis that can be used to compute two sets of variables for each procedure: those which are definitely not defined before use, and those which may not be defined before use; these sets can be used to provide extra error checking of programs. Strive for the highest quality information while maintaining safety. Use a lattice-theoretic formalism to define your analysis. Argue that your analysis terminates. Explain how to use the information computed by your analysis to construct the two sets specified above for output to the user.

It is possible to construct either a forward or a reverse dataflow problem to answer this question. We sketch out both possibilities; either one is a sufficient answer. We'll assume that any formal parameters are initialized via explicit assignments in the flow graph.

The first approach is to perform a forward dataflow analysis computing for each program point a set of definitely defined variables and a set of possibly defined variables. After these sets are computed, we can then examine all of the uses in the program and report any errors. A variable is definitely not defined before it is used when the variable does not appear in the possibly defined variable set associated with the program point immediately preceding a statement in which the variable is used. A variable may not be defined before it is used if at the program point proceeding a statement in which it is used it is not an element of the definitely defined variable set. Note that a read through a pointer must be considered to be a use of every variable, due to the lack of more detailed pointer analysis.

To simplify the formal definition we'll define two lattices, and then use the 2-Tuple constructor to compose them into a single lattice.

Definitely Defined Variables:

Elements:	*Pow(Variables), where Variables is the set of all variables in the CFG*
$\leq$:	$\subseteq$
meet:	$\cap$
top:	*Variables*
bottom:	$\emptyset$

Flow Functions:

$F_{x := \dots}$ $\quad Info_{succ} = Info_{pred} \cup \{x\}$

$F_{*x := \ldots}$ $Info_{succ} = Info_{pred}$

Possibly Defined Variables:

Elements:	*Pow(Variables)*
$\leq$:	$\supseteq$
meet:	$\cup$
top:	$\emptyset$
bottom:	*Variables*

Flow Functions:

$F_{x := \ldots}$	$Info_{succ} = Info_{pred} \cup \{x\}$
$F_{*x := \ldots}$	$Info_{succ} = Info_{pred} \cup Variables = Variables$

The flow functions are monotonic and the lattices are finite height, so the analysis will terminate. (Each individual lattice has height O(N); when we compose them into a 2-Tuple we still have a O(N) height lattice. Each flow function can run in O(N) time (or less). So, we get an O(N^3) analysis to compute the sets. The post-pass to examine the sets and produce warnings can run in O(N^2) time (O(N) statements, O(N) work for each (O(N) for reads through a pointer).)

Alternatively, one could define a reverse dataflow analysis that computes the set of possibly live and definitely live variables at each program point. All variables that are definitely live at the flow graph entry point are definitely used before they are defined; all variables that are possibly live at the flow graph entry point may be used before they are defined.

Just as before, we'll define two lattices and then combine them using the 2-Tuple operator:

Definitely Live Variables:

Elements:	*Pow(Variables)*
$\leq$:	$\subseteq$
meet:	$\cap$
top:	*Variables*
bottom:	$\emptyset$

Example Flow Functions:

$F_{x := y + z}$	$Info_{succ} = Info_{pred} - \{x\} \cup \{y,z\}$
$F_{*x := y + z}$	$Info_{succ} = Info_{pred} - Variables \cup \{y,z\} = \{y,z\}$
$F_{x := *y + *z}$	$Info_{succ} = Info_{pred} - \{x\} \cup \emptyset = Info_{pred} - \{x\}$
$F_{*x := *y + *z}$	$Info_{succ} = Info_{pred} - Variables \cup \emptyset = \emptyset$

Possibly Live Variables:

Elements:	*Pow(Variables)*
≤:	$\supseteq$
meet:	$\cup$
top:	$\emptyset$
bottom:	*Variables*

Example Flow Functions:

$F_{x := y + z}$	$Info_{succ} = Info_{pred} - \{x\} \cup \{y,z\}$
$F_{*x := y + z}$	$Info_{succ} = Info_{pred} - \emptyset \cup \{y,z\} = Info_{pred} \cup \{y,z\}$
$F_{x := *y + *z}$	$Info_{succ} = Info_{pred} - \{x\} \cup Variables = Variables$
$F_{*x := *y + *z}$	$Info_{succ} = Info_{pred} - \emptyset \cup Variables = Variables$

Termination and complexity are identical to the forward analysis.